BEC
BOSE EINSTEIN CONDENSATION

Inauguration Meeting &
Celebration of Lev Pitaevskii's 70^{th} Birthday

Bogoliubov excitations

with and without an optical lattice

Chiara Menotti

OUTLINE OF THE TALK

- Bogoliubov theory:
  - ▷ uniform system
  - ▷ harmonic trap
  - ▷ 1D optical lattice
- How to reveal Bogoliubov excitations experimentally:
  - ▷ collective oscillations
  - ▷ sound propagation
  - ▷ structure factor and Bragg spectroscopy

BOGOLIUBOV THEORY

$$\hat{H} - \mu\hat{N} = \int d\mathbf{r}\ \hat{\Psi}^\dagger \left[-\frac{\hbar^2\nabla^2}{2m} + V \right] \hat{\Psi} + \frac{g}{2} \int d\mathbf{r}\ \hat{\Psi}^\dagger\hat{\Psi}^\dagger\hat{\Psi}\hat{\Psi} - \mu \int d\mathbf{r}\ \hat{\Psi}^\dagger\hat{\Psi}$$

$$\hat{\Psi} = \sqrt{N}\varphi + \widehat{\delta\Psi}$$

Diagonalization of $\hat{H} - \mu\hat{N}$:

1^{st} order: $$\left[-\frac{\hbar^2\nabla^2}{2m} + V(\mathbf{r}) + gN|\varphi(\mathbf{r})|^2 - \mu \right] \varphi(\mathbf{r}) = 0 \longrightarrow$$ GPE

2^{nd} order: $$\widehat{\delta\Psi} = \sum_j \hat{b}_j = \sum_j u_j(\mathbf{r})\hat{a}_j + v_j^*(\mathbf{r})\hat{a}_j^\dagger \rightsquigarrow$$ Bogoliubov eqs.

$$E = E_0 + \sum_j \hbar\omega_j \hat{b}_j^\dagger \hat{b}_j$$

BEC BOSE EINSTEIN CONDENSATION

LINEARIZED GROSS-PITAEVSKII EQUATION

$$\varphi(\mathbf{r},t) = e^{-i\mu t/\hbar}\left[\varphi(\mathbf{r}) + u(\mathbf{r})e^{-i\omega t} + v^*(\mathbf{r})e^{i\omega t}\right]$$

$$i\hbar\frac{\partial\,\varphi(\mathbf{r},t)}{\partial\,t} = \mu\varphi(\mathbf{r},t)$$

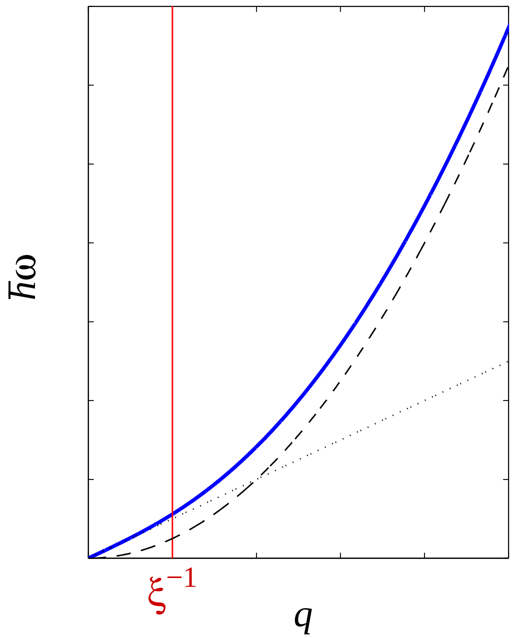
$$\left[-\frac{\hbar^2\nabla^2}{2m} + V(\mathbf{r}) - \mu + 2gN|\varphi(\mathbf{r})|^2\right] u(\mathbf{r}) + gN\varphi^2(\mathbf{r})\,v = \hbar\omega\; u(\mathbf{r})$$

$$\left[-\frac{\hbar^2\nabla^2}{2m} + V(\mathbf{r}) - \mu + 2gN|\varphi(\mathbf{r})|^2\right] v(\mathbf{r}) + gN\varphi^{*2}(\mathbf{r})\,u(\mathbf{r}) = -\hbar\omega\; v(\mathbf{r})$$

$$\int d\mathbf{r}\,\left[u_i^*(\mathbf{r})u_j(\mathbf{r}) - v_i^*(\mathbf{r})v_j(\mathbf{r})\right] = \delta_{ij}$$

BOGOLIUBOV SPECTRUM IN THE UNIFORM SYSTEM

$$\hbar\omega(q) = \sqrt{\frac{q^2}{2m}\left[\frac{q^2}{2m} + 2gn\right]} \longrightarrow \sqrt{(cq)^2 + \left(\frac{q^2}{2m}\right)^2}$$



sound velocity $c = \sqrt{\frac{gn}{m}}$

healing length $\xi = \frac{\hbar}{\sqrt{4mgn}}$

for $q \longrightarrow 0$: phononic regime

for $q \longrightarrow \infty$: free − particle regime

HARMONIC TRAP

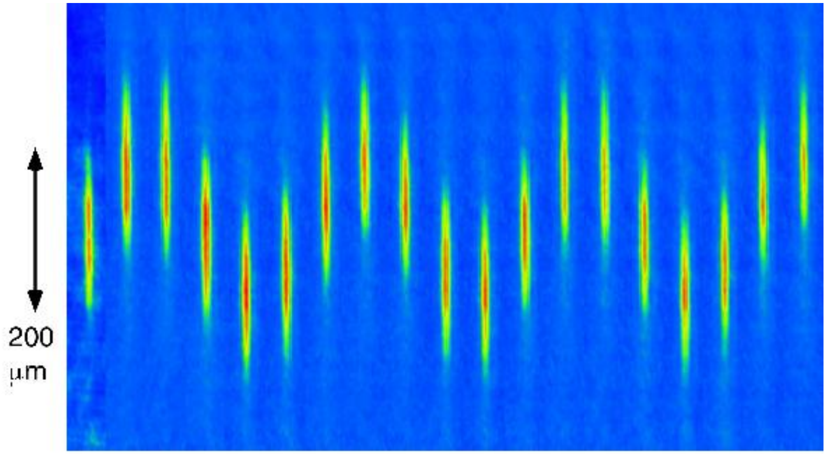
- ▷ Classification with the harmonic oscillator quantum numbers
- ▷ $Na/a_{ho} > 1$: collisionless hydrodynamics

collective oscillations	sound waves	single particle
$\lambda \approx R$	$\lambda \ll R$	(i) $\lambda < \xi$ (ii) $\lambda < d$
$\omega \approx \omega_{ho}$	$\omega_{ho} < \omega < \mu$	(i) $\omega > \mu$ (ii) $\omega > \mu(a_{ho}/R)^{4/3}$

MEASUREMENT OF THE COLLECTIVE OSCILLATIONS

Dipole oscillation:

$$\omega_D = \omega_z$$

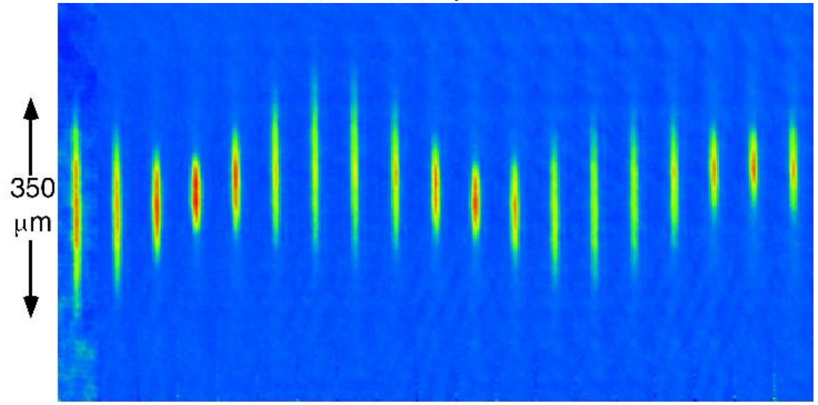


10 milliseconds per frame

Quadrupole oscillation:

$$(\omega_z \ll \omega_\perp)$$

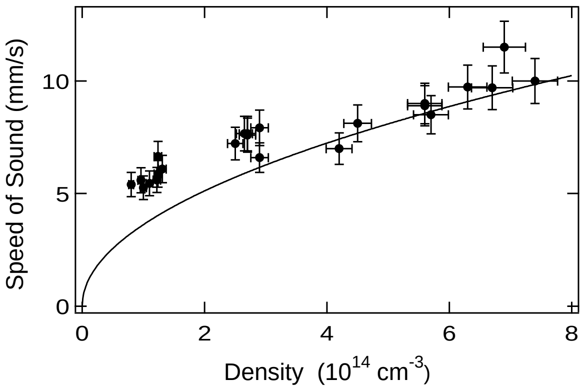
$$\omega_Q = \sqrt{\frac{5}{2}}\omega_z$$

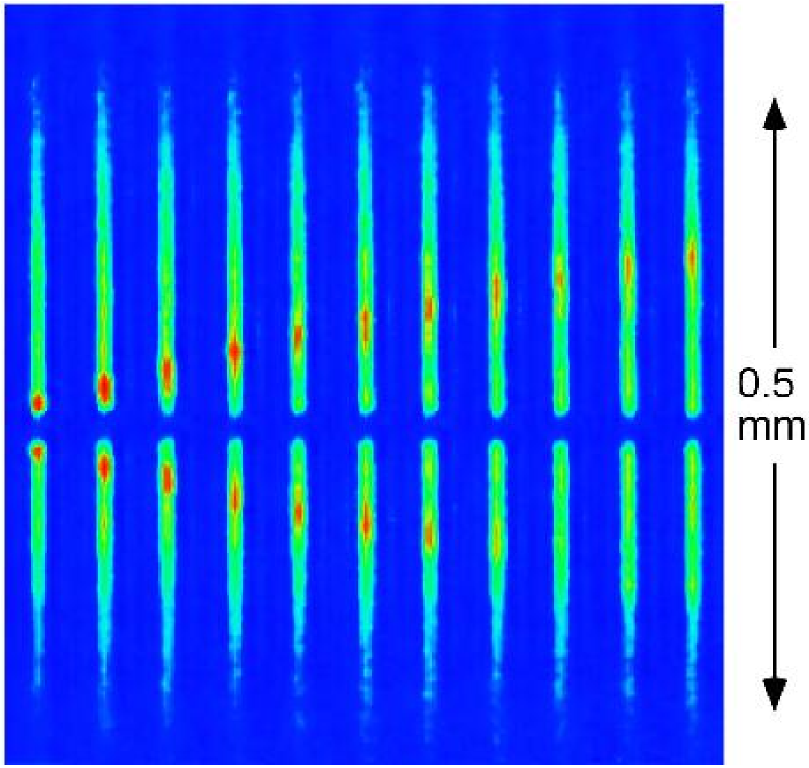


5 milliseconds per frame

MIT Group

OBSERVATION OF SOUND PROPAGATION



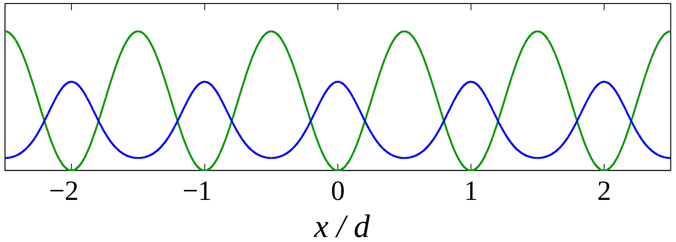


M. R. Andrews, D. M. Kurn, H.-J. Miesner, D. S. Durfee, C. G. Townsend, S. Inouye, and W. Ketterle (1997)

1D OPTICAL LATTICE

Geometry and parameters

$$V(z) = s\, E_R \sin^2\left(\frac{\pi z}{d}\right)$$



d : lattice spacing

$q_B = \hbar\pi/d$: Bragg momentum

$E_R = q_B^2/2m$: recoil energy

Parameters of the problem: s : optical lattice depth

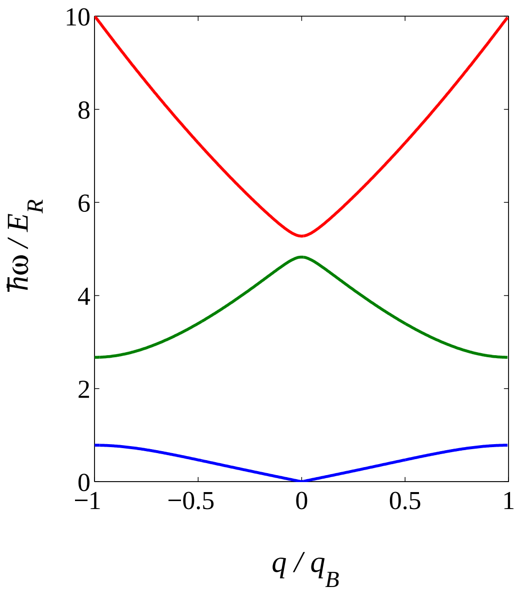
gn : interaction parameter

where $g = 4\pi\hbar^2 a/m$ and n is the 3D average density

BOGOLIUBOV SPECTRUM IN PRESENCE OF THE LATTICE

Bloch ansatz for the Bogoliubov amplitudes:

$$\left.\begin{aligned} u_{jq}(z) &= e^{iqz/\hbar}\tilde{u}_{jq}(z) \qquad (q = \text{quasi} - \text{momentum}) \\ v_{jq}(z) &= e^{iqz/\hbar}\tilde{v}_{jq}(z) \qquad (j = \text{band index}) \end{aligned}\right\} \Longrightarrow \quad \omega_j(q)$$



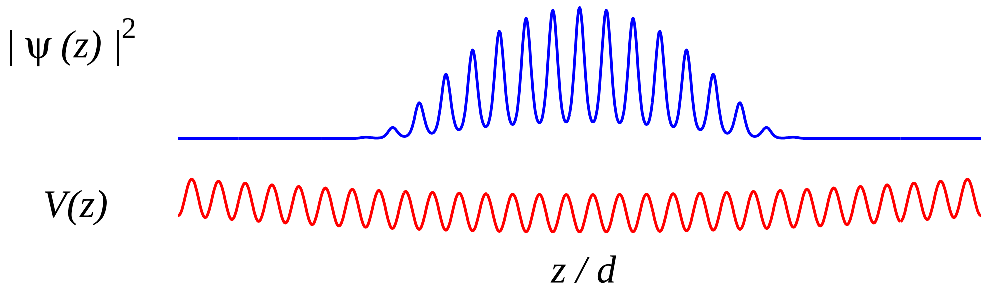
IMPORTANT!!!

The Bogoliubov spectrum is different from the Bloch energy bands, defined as the energy per particle of a condensate moving as a whole with quasi-momentum q

COLLECTIVE OSCILLATIONS IN A 1D OPTICAL LATTICE

F. Cataliotti, S. Burger, C. Fort, P. Maddaloni, F. Minardi, A. Trombettoni, A. Smerzi, M. Inguscio, Science (2001)

M. Krämer, L. Pitaevskii, and S. Stringari, PRL (2002)



Dipole oscillation : $\omega_D = \sqrt{\frac{m}{m^*}}\,\omega_z$

Quadrupole oscillation : $\omega_Q = \sqrt{\frac{m}{m^*}}\,\sqrt{\frac{5}{2}}\,\omega_z$

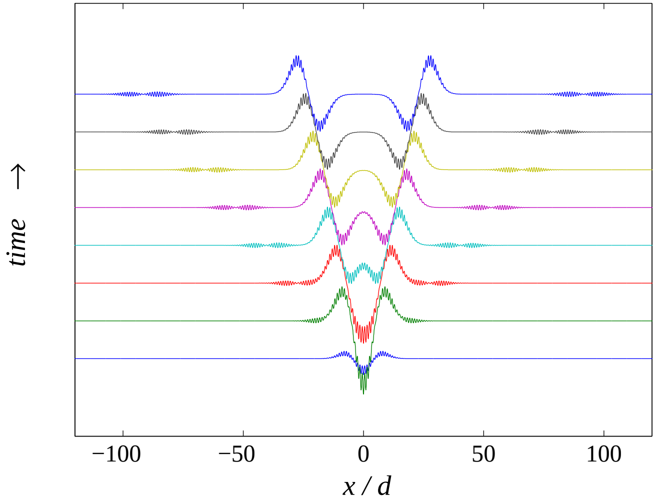
!! optical lattice + interactions might give rise to dynamical instabilities !!

A. Smerzi, A. Trombettoni, A.R. Bishop and P. Kevrekidis, PRL (2002)

SOUND PROPAGATION IN A 1D OPTICAL LATTICE

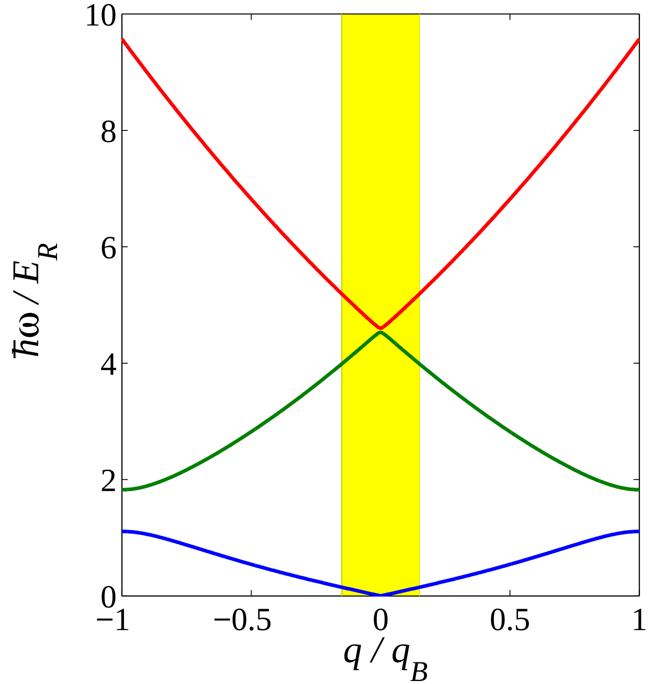
F. Dalfovo, M. Krämer, C. Menotti, L. Pitaevskii, A. Smerzi and S. Stringari

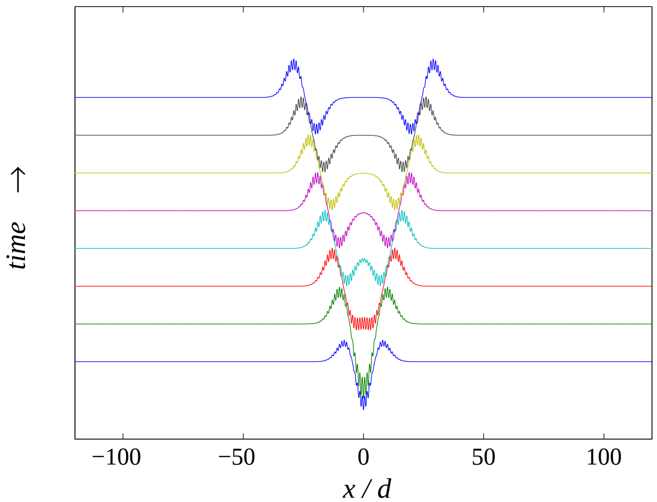
work in progress



← time-dependent GPE in the linear regime

$s = 2,\ \tau = 1 \ \Rightarrow$ higher bands excitations:





$s = 2,\ \tau = 10$

OPEN QUESTIONS

- on-set of the non-linear regime
- behaviour in the non-linear regime
- dynamical instabilities in large amplitude sound waves?

DIRECT MEASUREMENT OF BOGOLIUBOV EXCITATIONS

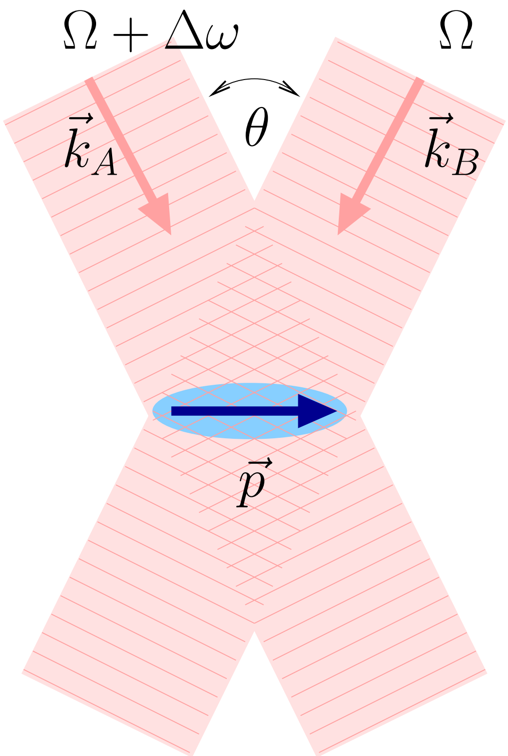
▷ collective oscillations

▷ sound propagation

▷ structure factor and Bragg spectroscopy:

- N-photon Bragg scattering
- $S(p, \omega)$ in the uniform system and in a 1D optical lattice
- measurement of $S(p, \omega)$ of a BEC trapped in a harmonic trap
- under which conditions one measures $S(p, \omega)$?

N-PHOTON STIMULATED SCATTERING

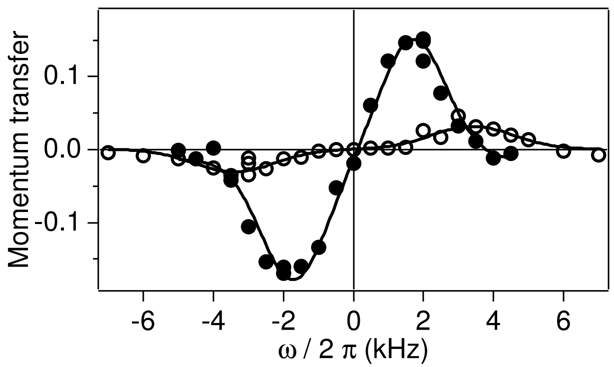


$$|\vec{p}| = N\,\hbar|\vec{k}_A - \vec{k}_B|$$

$$= N\,2\hbar k \sin(\theta/2)$$

$$\omega = N\Delta\omega$$

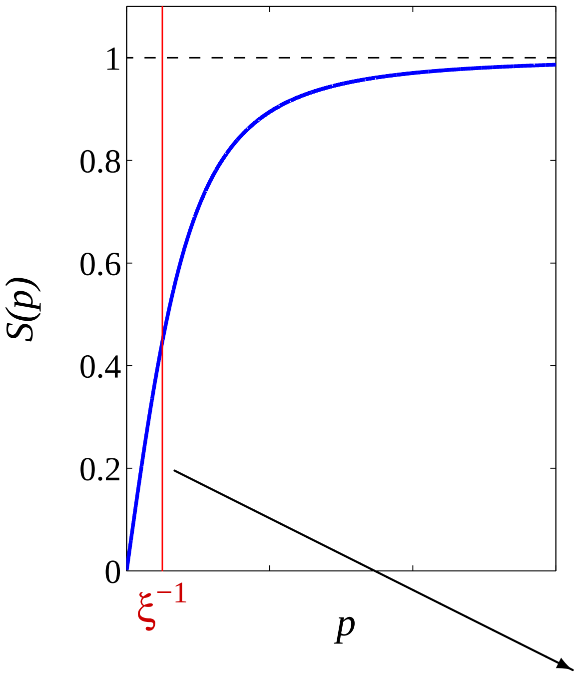
one measures



D.M. Stamper-Kurn, A.P. Chikkatur, A. Görlitz, S. Inouye, S. Gupta, D.E. Pritchard and W. Ketterle, PRL (1999)

$S(p,\omega)$ IN THE UNIFORM SYSTEM

$$S(p,\omega) = \sum_n \left|\langle n|\hat{\rho}_p^\dagger|g\rangle\right|^2 \delta(\omega - \omega_{ng}) =$$
$$= \left|U_p + V_p\right|^2 \delta(\omega - \omega(p))$$



$$S(p) = \frac{p^2/2m}{\hbar\omega(p)} \quad =$$

$$= \begin{cases} |p|/2mc\,; & |p| \longrightarrow 0 \\ 1\,; & |p| \longrightarrow \infty \end{cases}$$

suppression at small p $\equiv$ indication of the phononic regime

BEC BOSE EINSTEIN CONDENSATION

PHONONIC CORRELATIONS AND INTERFERENCE

$$S(p) = \frac{1}{N}\langle g|\hat{\rho}(p)\hat{\rho}^\dagger(p)|g\rangle$$

$\hat{\rho}^\dagger(p) = \sum_k \hat{a}^\dagger_{k+p}\hat{a}_p$ $\quad\mathcal{F}$ – transform of the atomic density operator

in Bogoliubov theory:
$$\begin{cases} \hat{a}^\dagger_p = u_p\hat{b}^\dagger_p - v_{-p}\hat{b}_{-p} \\ \hat{a}_p = u_p\hat{b}_p - v_{-p}\hat{b}^\dagger_{-p} \end{cases}$$

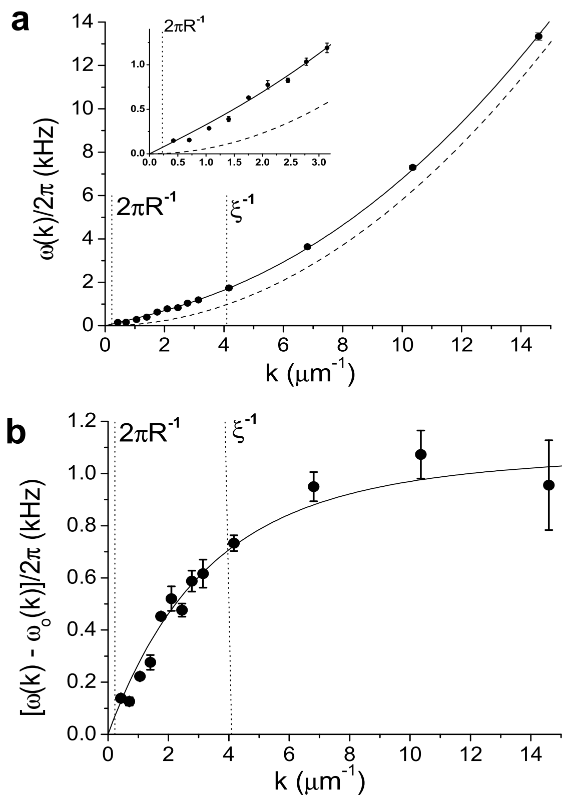
$$\hat{\rho}^\dagger(p)|g\rangle \approx (\hat{a}^\dagger_p\hat{a}_0 + \hat{a}^\dagger_0\hat{a}_{-p})|g\rangle = \sqrt{N}(\hat{a}^\dagger_p + \hat{a}_{-p})|g\rangle$$

$$S(p) = \langle g|(\hat{a}^\dagger_p\hat{a}_p + \hat{a}^\dagger_{-p}\hat{a}_{-p} + \hat{a}^\dagger_p\hat{a}^\dagger_p + \hat{a}_p\hat{a}_{-p})|g\rangle = (u_p + v_p)^2$$

when $p \to 0$, $v_p \to -u_p \implies S(p) \to 0 \quad \rightsquigarrow$ due to phonons

when $p \to \infty$, $v_p \to 0 \implies S(p) \to 1$

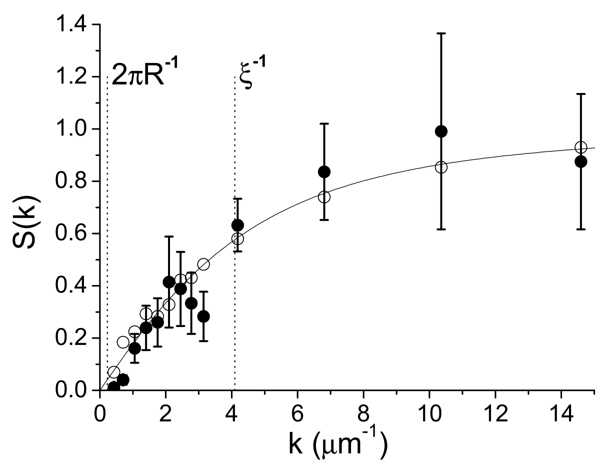
FIRST EXPERIMENTAL RESULTS



← Bogoliubov spectrum

Static structure factor

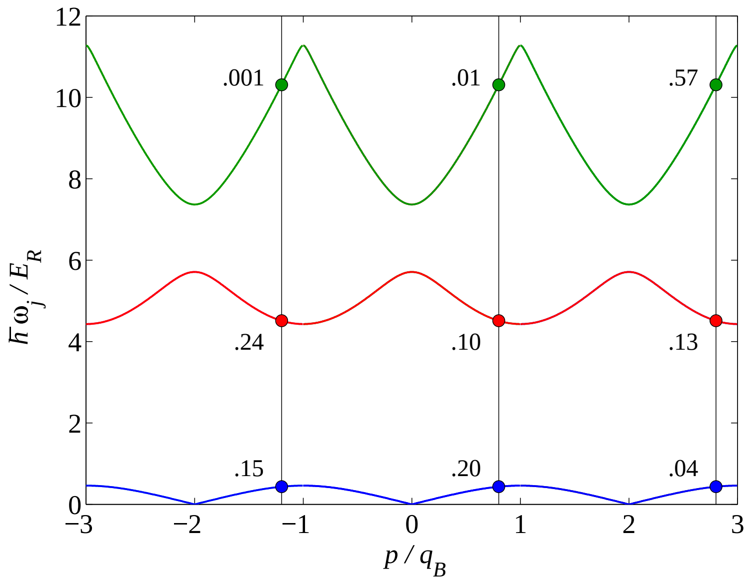
↓



J. Steinhauer, R. Ozeri, N. Katz, and N. Davidson, PRL (2002)

$S(p,\omega)$ OF A BEC IN A 1D OPTICAL LATTICE

C. Menotti, M. Krämer, L. Pitaevskii and S. Stringari, cond-mat/0212299



$$S(p,\omega) = \sum_j Z_j(p)\delta(\omega - \omega_j(p))$$

$$Z_j(p) = \left| \int_{-d/2}^{d/2} \left[u^*_{jq}(z) + v^*_{jq}(z)\right] e^{ipz/\hbar}\varphi(z)dz \right|^2$$

where $q = p + 2\ell q_B$ with ℓ integer

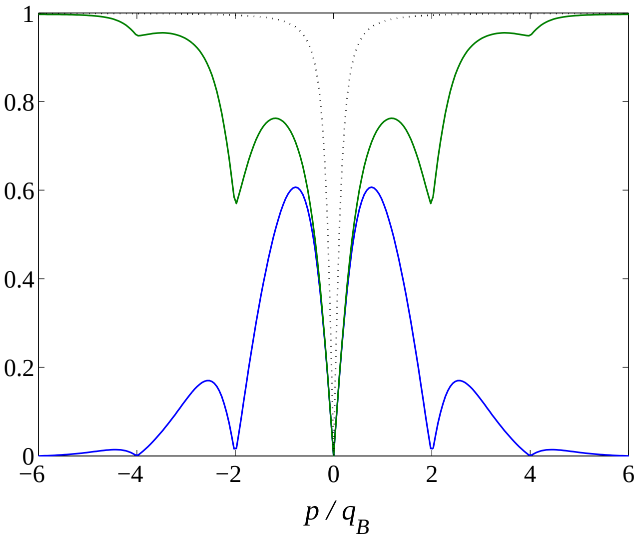
and $q \in 1^{st} BZ$

(i) possibility of exciting low bands with high p

(ii) possibility of exciting high excited states with low p

(iii) phononic regime at every p/q_B even

NUMERICAL RESULTS



$\cdots\ S_0$

$-\ S$

$-\ Z_1$

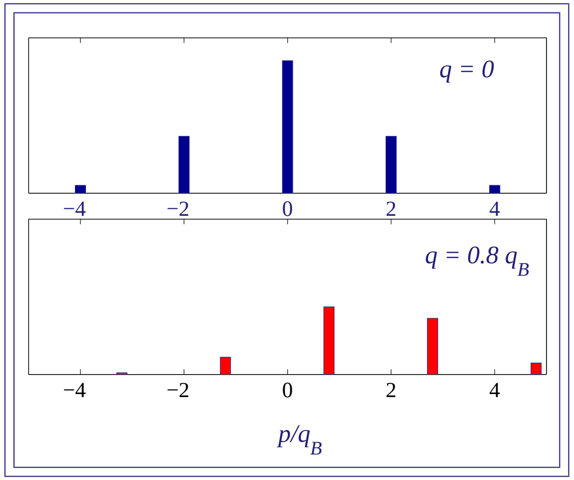
main features:

$\triangleright$ $Z_1 \neq 0$ for p out of 1^{st} BZ

$\triangleright$ suppression of Z_1 at large p

$\triangleright\triangleright$ $Z_1 = 0$ for $p = 2\ell q_B$

$\triangleright$ effect of the lattice $\longleftrightarrow$

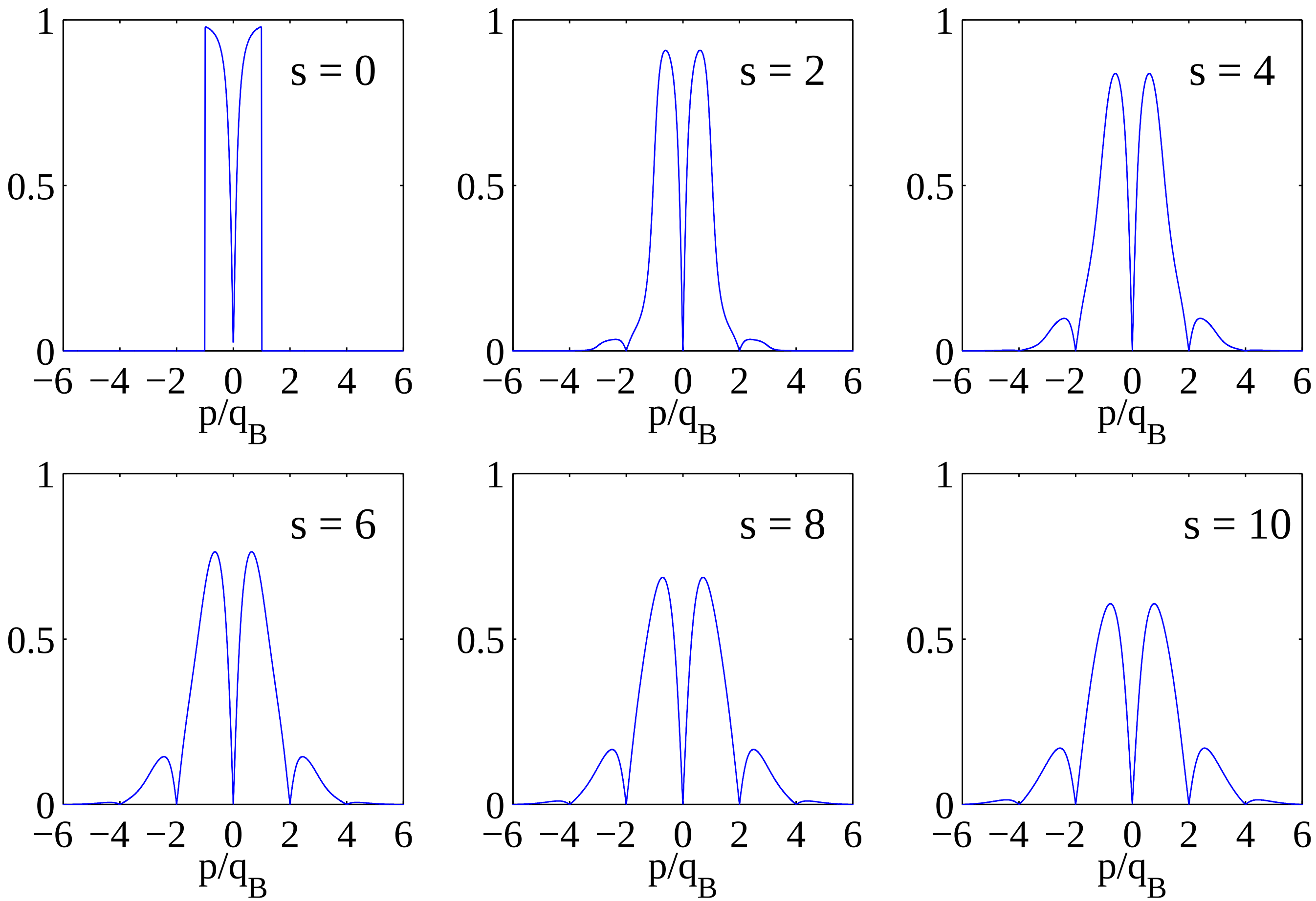


$\triangleright\triangleright$ effect of interactions:

introduce phononic correlations

($\Rightarrow Z_1 \to 0$ for s large)

DEPENDENCE OF $Z_1(p)$ on s



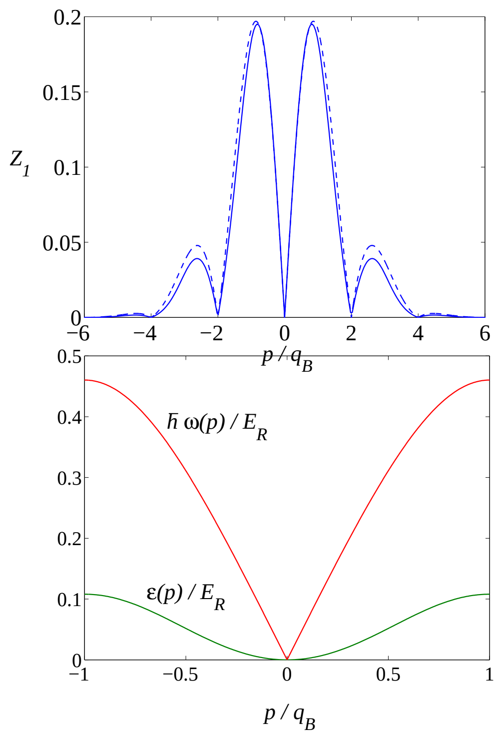
DEPENDENCE ON INTERACTIONS AND s

Tight binding analytic solution:

$$Z_1(p) = \frac{\varepsilon(p)}{\hbar\omega(p)} e^{-\pi^2\sigma^2 p^2/2d^2 q_B^2}$$

$$\varepsilon(p) = 2\,\delta\,\sin^2\left(\frac{pd}{2\hbar}\right)$$

$$\hbar\omega(p) = \sqrt{\varepsilon(p)\left[\varepsilon(p) + 2\kappa^{-1}\right]}$$



$$Z_1(q_B) \approx \sqrt{\kappa\delta/(\kappa\delta + 1)}$$

- increasing s : $\delta \rightarrow 0 \Rightarrow Z_1(q_B) \rightarrow 0$
- increasing gn : $\kappa \rightarrow 0 \Rightarrow Z_1(q_B) \rightarrow 0$
- non interacting ($gn = 0$): $\kappa^{-1} = 0 \Rightarrow Z_1(q_B) \rightarrow 1$

CONNECTION WITH EXPERIMENTS

C. Tozzo and F. Dalfovo

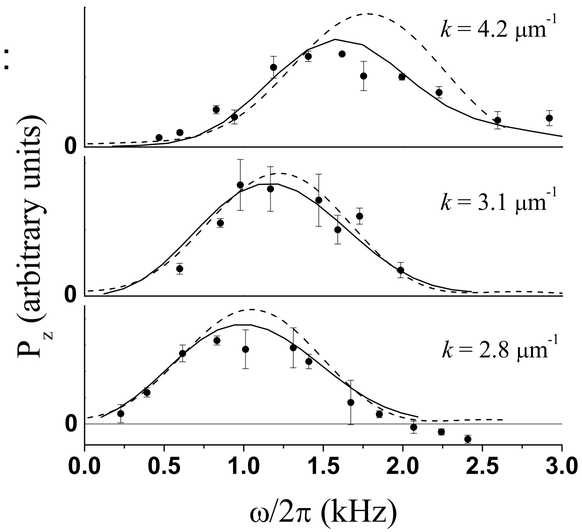
- ▷ Local Density Approximation and beyond
- ▷ long time measurements and effect of radial excitations
- ▷ what does one really measure?

LOCAL DENSITY APPROXIMATION AND BEYOND

J. Steinhauer, N. Katz, R. Ozeri, N. Davidson, C. Tozzo and F. Dalfovo, PRL (2003)

real systems are inhomogeneous and finite:

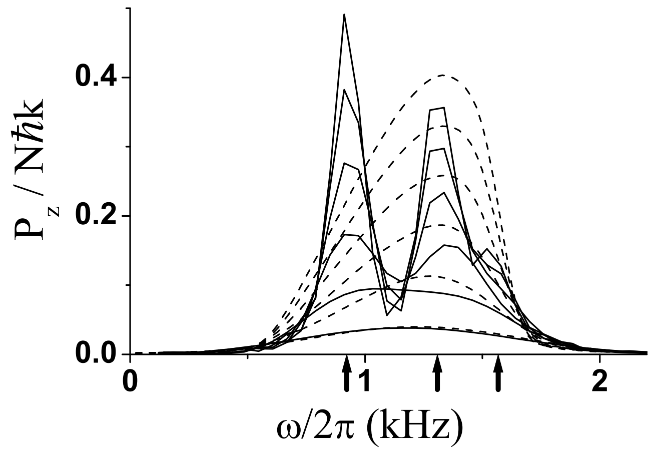
▷ Local Density Approximation



$$\tau < \frac{2\pi}{\omega_\perp}$$

▷ time-dependent GPE with

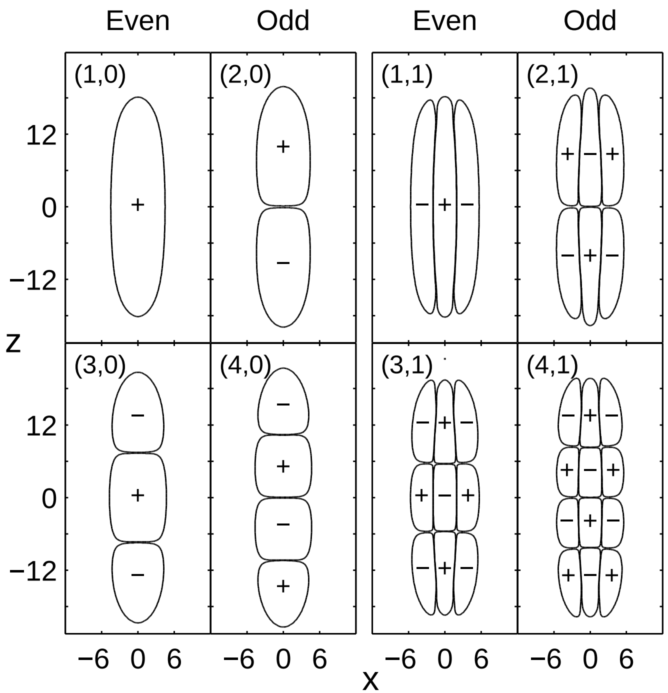
$$V(\mathbf{r}, t) = V_{ho} + \theta(t) V_B \cos(kz - \omega t)$$

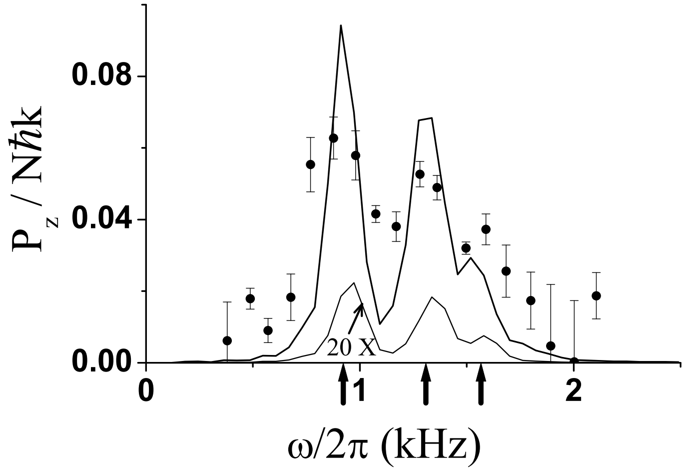


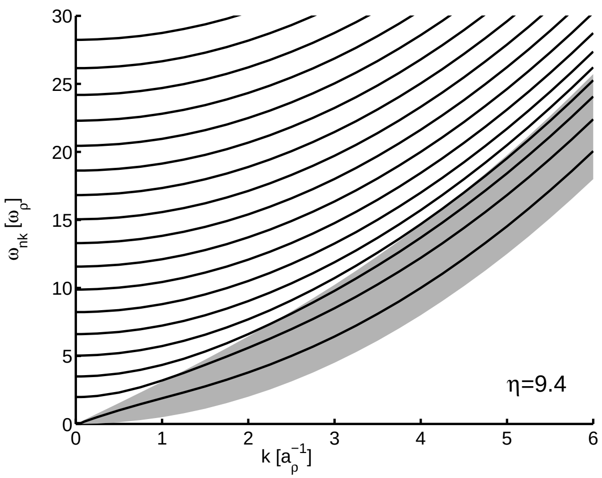
$$\tau > \frac{2\pi}{\omega_\perp}$$

EFFECT OF RADIAL EXCITATIONS

C.Tozzo and F. Dalfovo, cond-mat/0303206







↪ A.A. Penckwitt and R.J. Ballagh (2001)

WHAT DOES ONE REALLY MEASURE IN BRAGG SCATTERING EXPERIMENTS?

in a cylindrical condensate ($\omega_z = 0$):

$$S(p,\omega) - S(-p,-\omega) = \frac{2\,\hbar}{\pi\,p\,{V_B}^2\,\tau} \lim_{\tau\to\infty} P_z(p,\omega,\tau)$$

otherwise

$$S(p,\omega) - S(-p,-\omega) = \frac{2\,\hbar}{\pi\,p\,{V_B}^2\,\tau}\,\omega_z^2 \int_0^\infty P_z(p,\omega,\tau)\,\tau\,d\tau$$

P.B. Blakie, R.J. Ballagh and C.W. Gardiner, PRA (2002)

if $\omega_z << \omega_\perp$:

- the cylindrical approximation is good for intermediate τ_B
- the Bogoliubov branches are observable as distinguishable peaks in $P_z(t)$

SUMMARY AND OUTLOOK

- Bogoliubov excitations in various geometries:

— theory and experiment in the harmonic trap and 1D optical lattices

▷▷ sound propagation in a 1D optical lattice

- addressing the different excitations through Bragg-scattering exps.

— calculation of $S(p, \omega)$ for a BEC in a 1D optical lattice

— deep understanding of Bragg experiments in elongated harmonic traps

▷▷ low-q phonon spectroscopy by revealing density fluctuations

after expansion