

# BCS-BEC CROSSOVER:

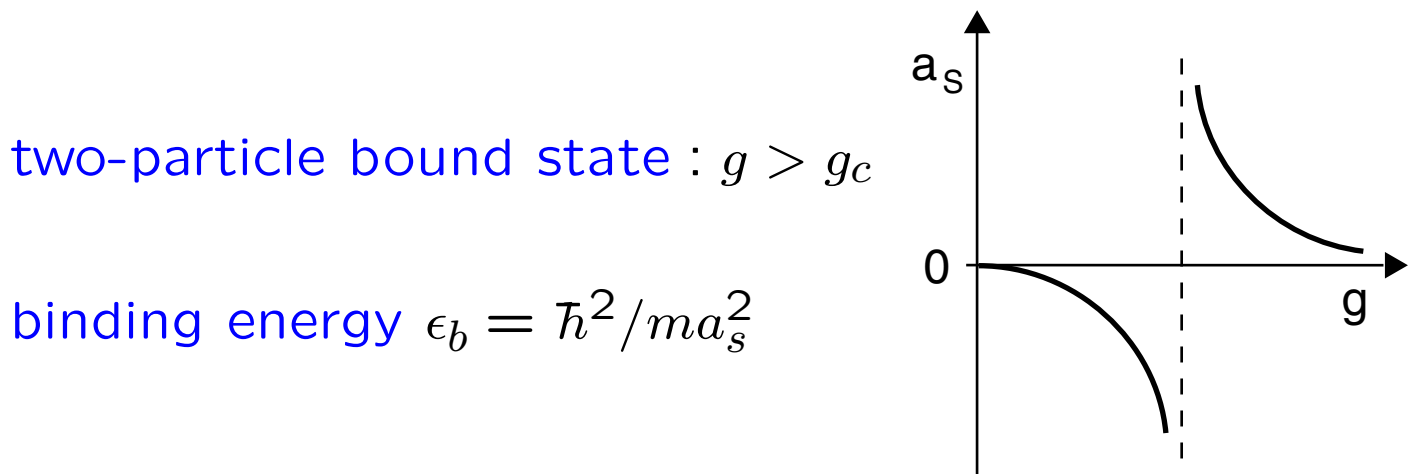
## BASICS AND AN EXACT SOLUTION IN 1D

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attractive fermions ( ${}^6\text{Li}$ ,  ${}^{40}\text{K}$ ) scattering length  $a_s$

BCS-BEC crossover via Feshbach resonance



BCS groundstate (variational Ansatz for arbitrary  $g$ )

$$\psi_{\text{BCS},N} = \hat{A} \{ \underbrace{\phi(1\,2) \phi(3\,4) \cdots \phi(N-1\,N)} \}$$

ideal Bose gas ground state

pair-wavefunction  $\phi(1, 2) = \psi(\vec{x}) \cdot \chi(\vec{r}) \cdot \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

rel. coordinate  $\vec{r} \rightarrow$  pair radius  $\xi_b$  + internal structure

COM-coord.  $\vec{x} \rightarrow$  inhomogeneous pair-amplitude  $\psi(\vec{x})$

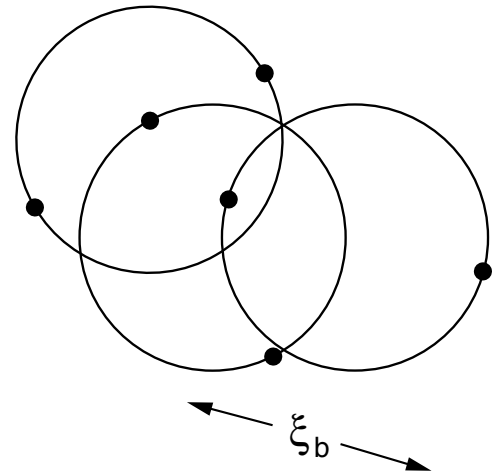
BCS-LIMIT: weak coupling if  $k_F |a_s| \ll 1$

pairs form and condense at

$$T_c \approx 0.3 T_F \exp -\frac{\pi}{2k_F |a_s|} \ll T_F \quad 2\Delta_0 = 3.52 k_B T_c$$

pair size  $\xi_b = \xi_0 = \frac{\hbar v_F}{\pi \Delta_0}$  is large  $k_F \xi_b \gg 1$

$$k_F \xi_0 \approx \begin{cases} 10^3 & \text{supercond.} \\ 10^1 & \text{high-}T_c \end{cases}$$



are Cooper pairs Bosons ?

antisymmetrization  $\hat{A}$  reduces condensate fraction

$N_0/N \approx \Delta_0/\epsilon_F \ll 1$  in weak coupling, nevertheless

BCS-superfluidity is BEC of pairs !

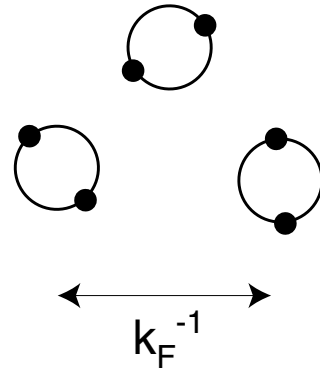
BEC limit: strong coupling  $|\epsilon_b| \gg \epsilon_F \rightarrow k_F a_s \ll 1$

pair size  $\xi_b \approx a_s \rightarrow 0$  point bosons

gap parameter  $\Delta_0 \rightarrow \sqrt{2|\epsilon_b|\epsilon_F} \gg \epsilon_F$  is irrelevant

Bose-limit  $k_F \xi_b \ll 1$

$\xi_b \neq$  coherence length  $\xi_0$



critical temperature  $T_c = 0.218 T_F$

ideal Bose gas with  $n_B = n/2$  and  $m_B = 2m$

crossover is smooth (Leggett, Nozieres/Schmitt-Rink)

BCS-transition in a harmonic trap

local density approx.  $\xi_b \ll R_{TF}$  ( $\Delta_0 \gg \hbar\omega$ ) valid

if  $N \gg N^* \approx \exp 3\pi/2 k_F |a_s| = 10^5$  at  $k_F |a_s| = 0.4$

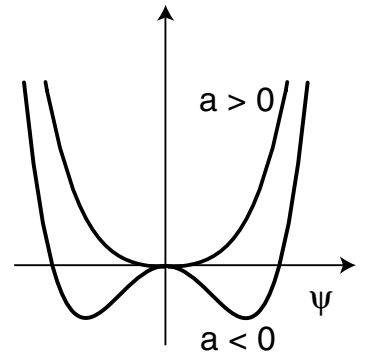
$N \ll N^*$ : pairs formed within a single shell  $\Delta_0 \ll \hbar\omega$

as in atomic nuclei (Bruun, Heiselberg, Mottelson 02)

# Statistical Ginzburg–Landau theory (Drechsler/Zw.92)

idea: condensation is associated with broken  
gauge symmetry  $\rightarrow$  complex order parameter  $\psi$

$$F[\psi] = \int d^d x \left[ a |\psi|^2 + \frac{b}{2} |\psi|^4 + c |\vec{\nabla} \psi|^2 \right]$$



Thermodynamics  $Z = \text{Tr} \exp -\beta(\hat{H} - \mu\hat{N})$

Gorkov Hamiltonian:  $(b_q^\dagger = \sum_k c_{k+q\downarrow}^\dagger c_{-k\uparrow})$

$$\hat{H} - \mu\hat{N} = \sum_{k\sigma} (\epsilon_k - \mu) c_{k\sigma}^\dagger c_{k\sigma} - \frac{g}{V} \sum_q b_q^\dagger b_q$$

dilute system with interaction range  $r_0 \ll k_F^{-1}$

linearize  $\sum_q b_q^\dagger b_q$  via Hubbard-Stratonovich

order parameter  $\psi(\vec{x}, \tau) \rightarrow$  quantum GL-functional  $F[\psi] =$

$$\frac{1}{\hbar} \int_0^{\beta \hbar} d\tau \int d^d x \left[ -\mu^* |\psi|^2 + \frac{\hbar^2}{2m^*} |\vec{\nabla} \psi|^2 + \frac{g^*}{2} |\psi|^4 + \hbar \psi^* \partial_\tau \psi \right]$$

GL-theory is coherent state representation of

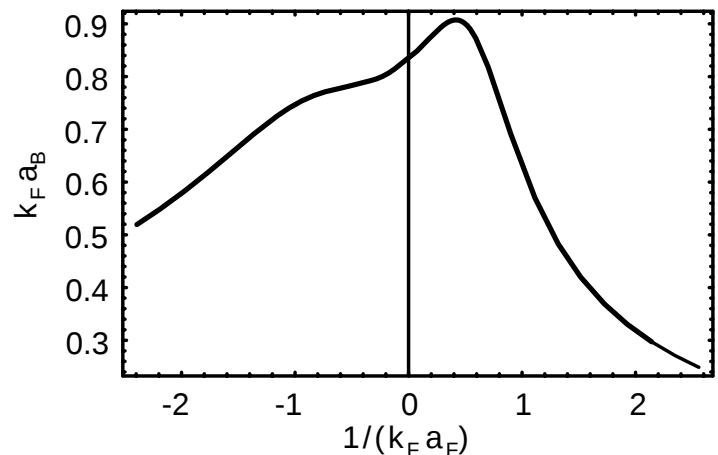
interacting Bosons, smooth crossover from Gorkov's

BCS GL-functional to Gross-Pitaevski for BEC's

$g^* = 4\pi \hbar^2 a_B / m^*$  defines dimer scatt. length  $a_B$

Born approximation

$$a_B = \begin{cases} 6|a_F| & \text{BCS} \\ 2a_F & \text{Bose} \end{cases}$$

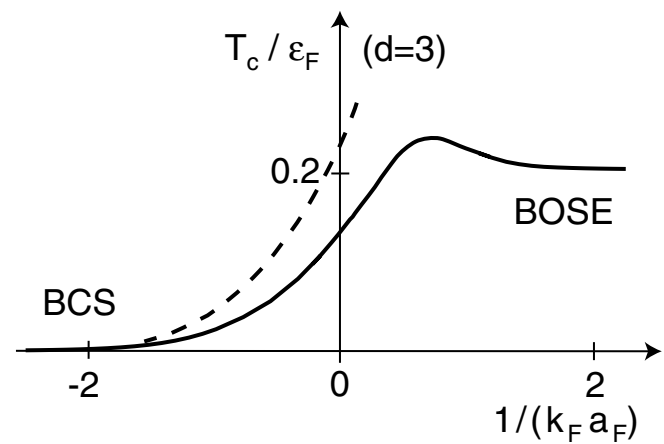


Pauli principle gives statistical, repulsive interaction between composite Bosons

Thouless criterion

$\mu^* = 0$  gives  $T_c$

in Gaussian approx.



universal numbers at resonance:  $(k_F a)^{-1} = 0$

$$E_0/N = \frac{3}{5}\epsilon_F(1 + \beta), \quad T_c/\epsilon_F, \quad v_s/v_F \dots = ?$$

pair formation is just a matter of shifting

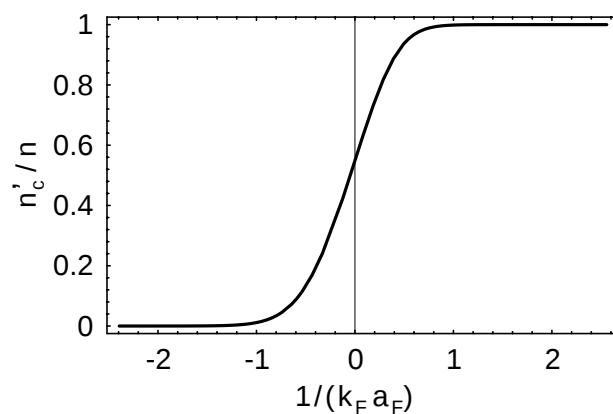
the chemical equilibrium from  $\bullet + \bullet$  to  $\bullet\bullet$

it coincides with the transition only in

weak coupling

preformed pairs

at condensation



nature of the transition changes in a narrow

range of coupling

## BCS–BEC crossover in 1D

arbitr. small attraction gives two-particle binding

Gaudin/Yang 67: attractive fermions in 1D

$$H = -\frac{\hbar^2}{2m} \sum_{i=1}^N \frac{\partial^2}{\partial x_i^2} + g_1 \sum_{i < j} \delta(x_i - x_j)$$

dim.less interaction constant  $\gamma = mg_1/\hbar^2 n < 0$

$H$  describes a Luther-Emery liquid : extended

pairs if  $|\gamma| \ll 1$  transform to local pairs if  $|\gamma| \gg 1$

BCS–BEC crossover trivial in 1D ? (no FB–reson.)

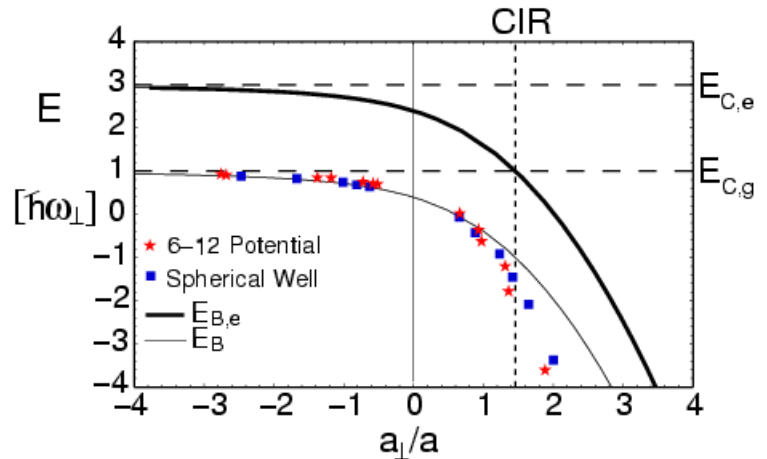
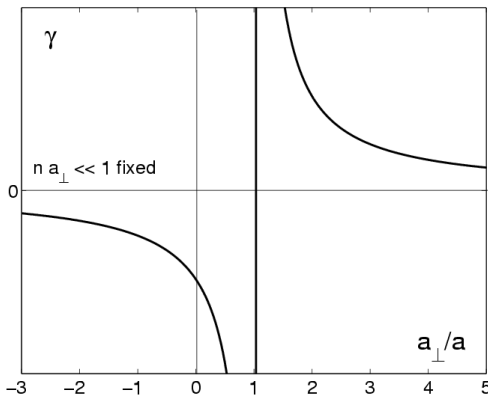
Olshanii 98: scattering theory in a single channel wire

$$\hbar\omega_{\perp} \gg \epsilon_F \gg k_B T, \hbar\omega_{\parallel}$$

eff. pot. between fermions with opposite spin  $g_1\delta(x)$

$$g_1(a) = 2\hbar\omega_{\perp} \cdot \frac{a}{1 - Aa/a_{\perp}}$$

confinement induced resonance (CIR) at  $a = a_{\perp}/A \approx a_{\perp}$   
 plus a bound state for arbitrary sign of  $a$



binding energy at CIR:  $|\epsilon_b| = 2\hbar\omega_{\perp} \gg \epsilon_F \rightarrow$

BEC-side  $g_1 > 0$  has unbreakable bosons

extension of Gaudin/Yang to Lieb/Liniger ( $g_B = g_1$ )

with dim.less coupling  $\gamma = mg_B/\hbar^2 n > 0$

repulsion between dimers in a tight waveguide

$$g_B(a < a_{\perp}) = \sqrt{2}\hbar\omega_{\perp} \cdot \frac{a}{1 - A a/a_{\perp}}$$

fixed by coincidence of resonance for fermions and dimers

gives  $a_B \rightarrow a/\sqrt{2}$  for  $a \ll a_{\perp}$  (Petrov :  $a_B = 0.6 a$ )

qualitative physics of the crossover in 1D

$\gamma < 0$ : increasing  $|\gamma|$  transforms Cooper-pairs

to tightly bound bosons; size at CIR is  $\approx a_{\perp} \rightarrow$

hard core bosons form Tonks gas

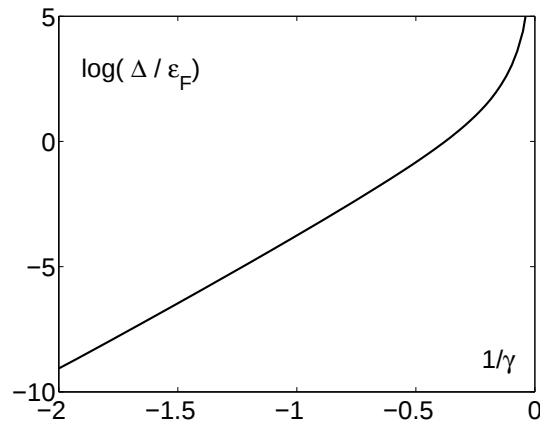
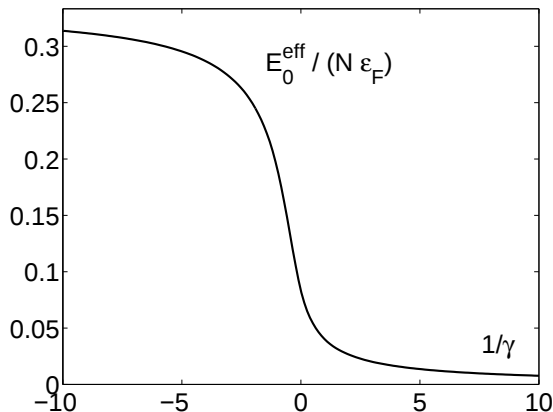
$\gamma > 0$ : boson size shrinks  $\rightarrow$  weakly interacting BEC

Bethe-Ansatz determines exact ground state

$$\pi\rho(k) = 1 + \int_{-K}^K \frac{dq}{n} \frac{\gamma\rho(k)}{\gamma^2 + [(k-q)/n]^2}$$

for both  $\gamma < 0$  and  $\gamma > 0$

ground state energy  $E_0^{eff}/N = 2 \int \frac{dq}{n} \rho(q) \epsilon(q)$



excitations: breaking of pairs for  $\gamma < 0$

triplet excit. (2 spinons):  $\omega^s(q) = \sqrt{(\Delta/2\hbar)^2 + (v_s q)^2}$

energy gap in weak coupling

$$\Delta \rightarrow 2\Delta_{BCS} = \epsilon_F 16\pi^{-3/2} |\gamma|^{1/2} \exp -\pi^2/2 |\gamma|$$

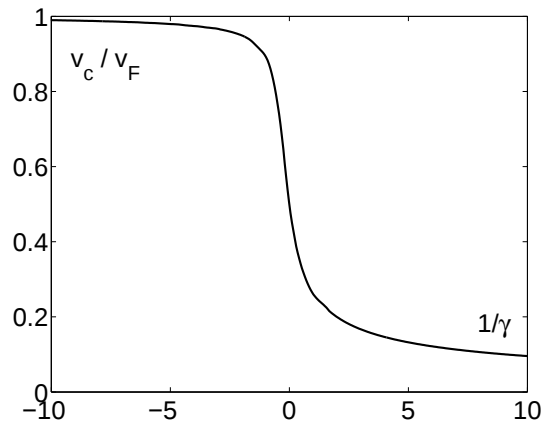
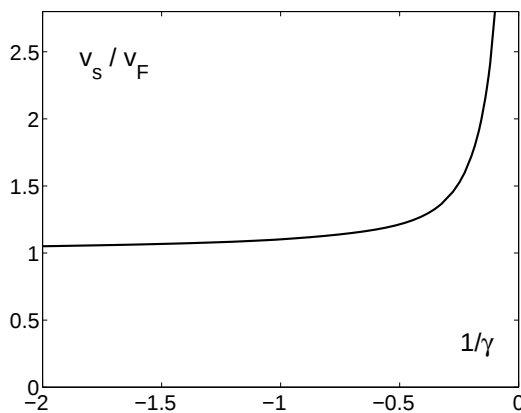
pair size reaches  $n^{-1}$  at  $\gamma \approx -2$

spin gap at strong coupling  $\Delta \rightarrow |\epsilon_b|$  diverges

collective excitations of a neutral superfluid

Bogoliubov-Anderson sound mode:  $\omega^c(q) = v_c \cdot q$

Luttinger parameter  $K = v_F/(2)v_c > 1$  implies sf prop.



exp. realization:  ${}^6\text{Li}$ ,  ${}^{40}\text{K}$  in atomic quantum wires

H. Moritz et.al. ETHZ

## CONCLUSION AND OPEN QUESTIONS

1) The BCS–BEC crossover is smooth both in the ground state and at finite  $T$  in **any** dimension

2) An exact solution is possible in **one** dimension and appears realizable experimentally.

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3) proper detection of a BCS correlations ?

4) quantitative theory in the crossover region including the dynamics of pair formation ?

