# Hydrodynamics in a Degenerate Strongly-Interacting Fermi Gas

Duke Physics
Atom Cooling and Trapping

John E. Thomas

Department of Physics, Duke University

**Students: Joseph Kinast, Staci Hemmer,**

**Post Doc: Andrey Turlapov**

**Ken O'Hara*, Mike Gehm*, Stephen Granade***

**Sponsored by: DOE, NSF, ARO, NASA**

# Outline

Duke Physics
Atom Cooling and Trapping

- Strongly-Interacting Fermi Gases
- All-Optical Cooling Method
- Hydrodynamic Expansion
- Trapped Atom Hydrodynamics – Fermi superfluid?
- Conclusions

# Strongly-Interacting Fermi Gases Mimic Exotic Systems in Nature



- High-Temperature Superconductors
- Neutron Stars
- Strongly-Interacting Matter
- Quark-Gluon Plasma – Elliptic Flow

## A BUNCH OF DEGENERATES

A degenerate gas of fermions occurs in diverse situations, as described below:

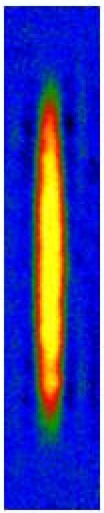
- **Superconductors:** The electrons are degenerate and form loosely correlated Cooper pairs, which produce the superconductivity. Something similar must happen in high-temperature superconductors, but that process remains a mystery.
- **Neutron stars:** The refusal of neutrons (which are fermions) to occupy identical quantum states generates a repulsion that prevents the star from collapsing under its own immense weight. A similar repulsion stabilizes the laboratory-made degenerate fermi gases against collapse.
- **Quark-gluon plasma:** As created at the Relativistic Heavy Ion Collider at Brookhaven National Laboratory, the exploding cloud of free quarks (which are fermions) and gluons has properties similar to a gas of fermionic atoms released from the confines of a trap.

26 SCIENTIFIC AMERICAN

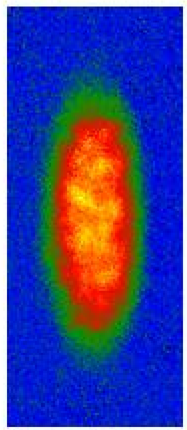
# “Shape-shifting” Fermi Gas

Duke Physics
Atom Cooling and Trapping

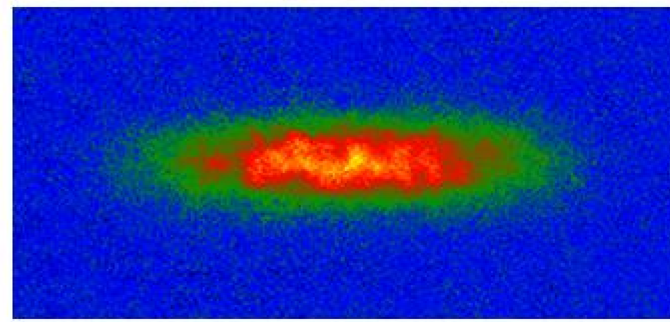
Trapped Gas:
0 ms



Released Gas:
after 2.1 ms



Released Gas: 12 ms

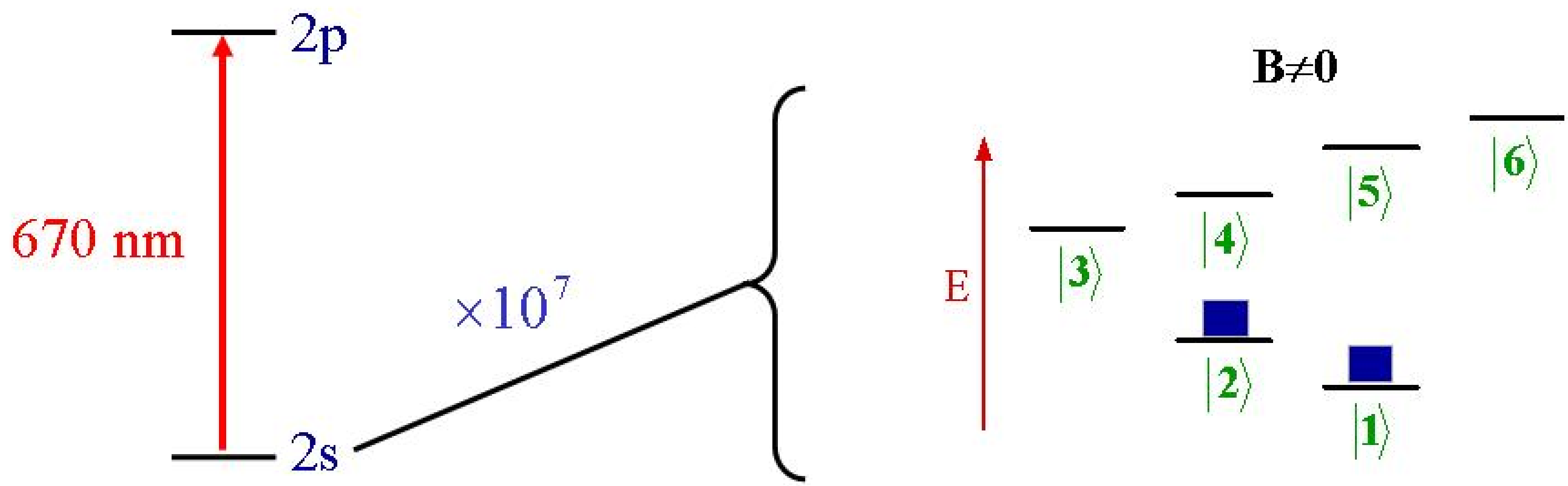


Fermi energy 700 nK
T < 35 nK

# Mixture of Spin Up/Down $^6$Li Atoms

Duke Physics
Atom Cooling and Trapping

## Ground State Hyperfine Structure in a Magnetic Field B




Ground State: $1s^2 2s^1$

Nuclear Spin: $I_N=1$

Level 1 Spin ½ Up $\quad |1\rangle = |\uparrow\rangle = \left|-\frac{1}{2}, 1\right\rangle$

Level 2 Spin ½ Down $\quad |2\rangle = |\downarrow\rangle = \left|-\frac{1}{2}, 0\right\rangle$

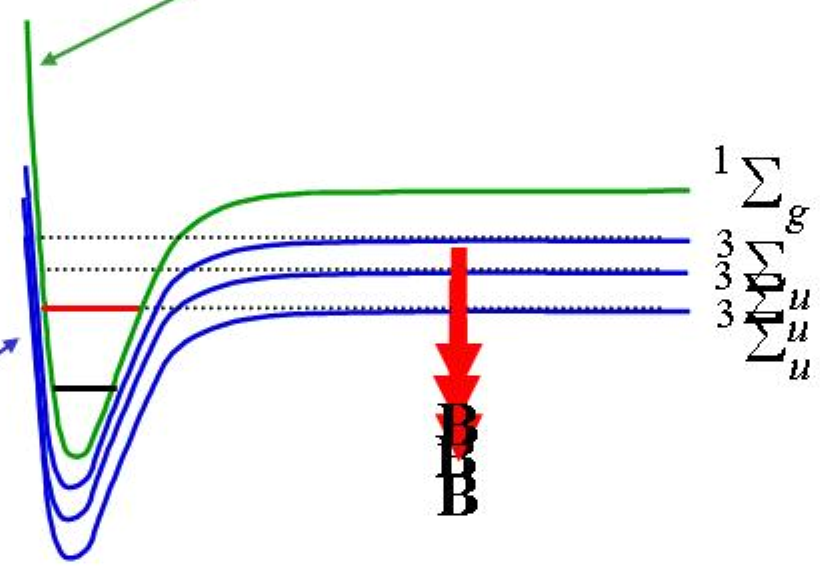
1,2 States High B-Field Seeking-Requires Optical Trap

# Producing Strong Interactions with a Feshbach Resonance

Duke Physics
Atom Cooling and Trapping

Resonant Coupling between Colliding Atom Pair – Bound Molecular State

Singlet Diatomic Potential: Electron Spins Not Parallel

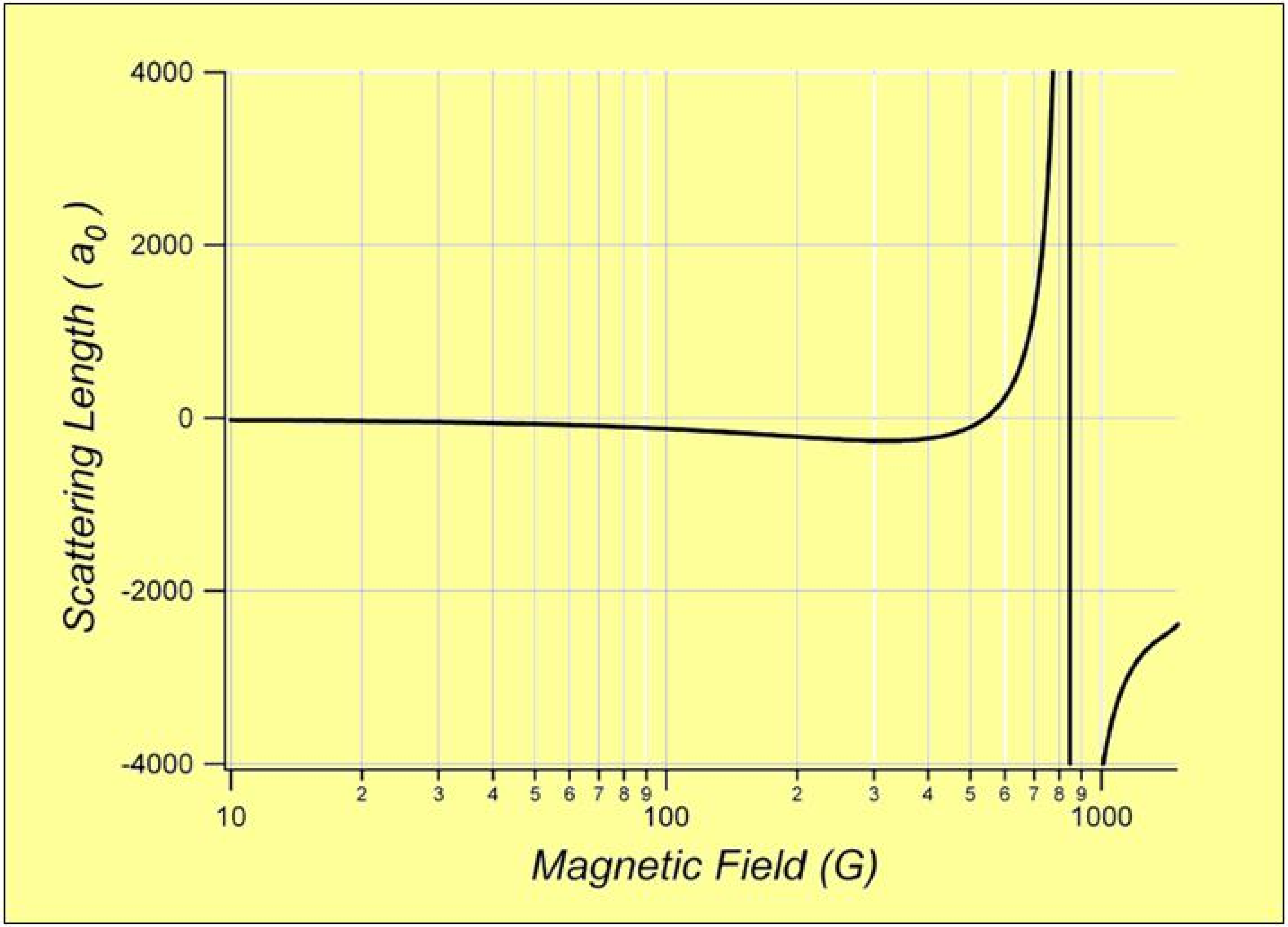



Triplet Diatomic Potential: Electron Spins Parallel

Zero Energy Scattering Length $a_S \rightarrow \pm\infty$

# *Feshbach Resonance for 1-2 Mixture

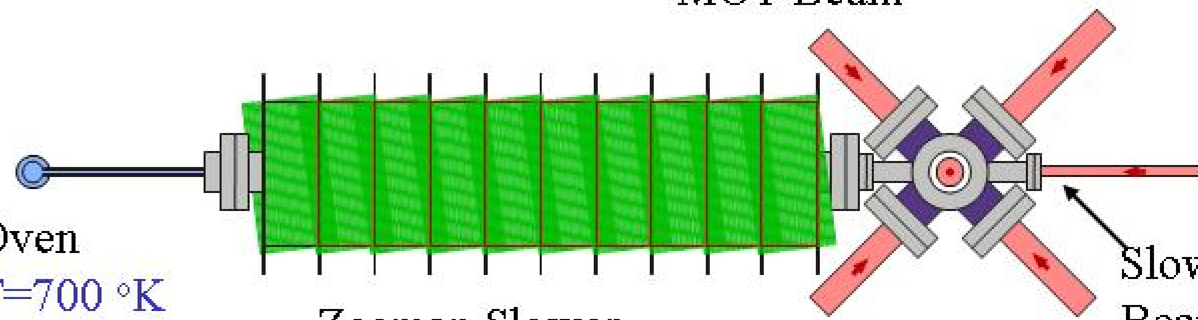
Duke Physics
Atom Cooling and Trapping




*M. Houbiers, *et al.*, Phys. Rev. A **57**, 1497 (1998)

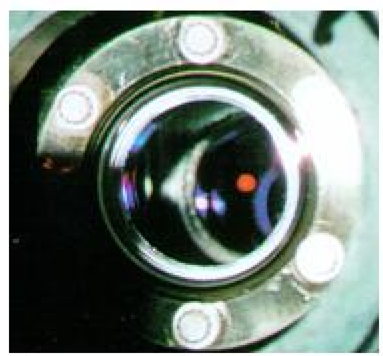
# Cooling Fermi Gases in Optical Traps

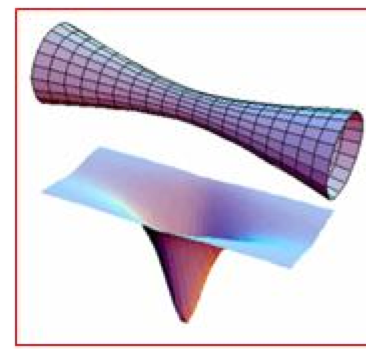
Duke Physics
Atom Cooling and Trapping




T < 100 nK
$\rho = 1$

P = $10^{-11}$ Torr
T = 150 $\mu$K
$\rho = 10^{-5}$





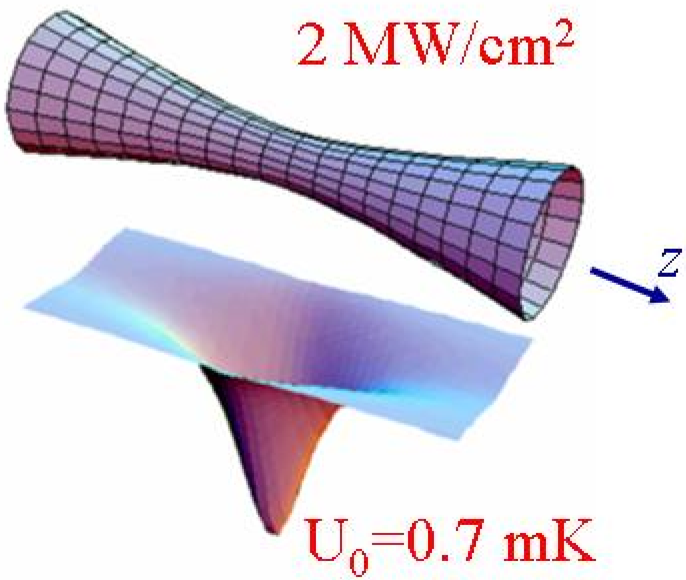
Optical Trap

# Optical Trap

Duke Physics
Atom Cooling and Trapping

Focused Gaussian Laser Beam

$$U = -\frac{1}{2}\alpha \overline{E_0^2}\,\frac{1}{1+(z/z_0)^2}\,e^{-2r^2/w_0^2}$$



# Ultrastable $CO_2$ Laser Trap



- Stable Commercial Laser


- Typical Trap Parameters

$P = 65\ \mathrm{W}$ $\qquad \omega_0 = 47\ \mu\mathrm{m}$, $z_0 = 0.7\ \mathrm{mm}$

$I_0 = 2.0\ \mathrm{MW/cm^2}$ $\qquad U_0 = 0.7\ \mathrm{mK}$

$\nu_r \cong 6.6\ \mathrm{kHz}$ $\qquad \nu_z \cong 340\ \mathrm{Hz}$

- Negligible Optical Heating
  - Scattering Time $\cong$ 1/2 hour
  - Optical Heating $\cong$ 18 pK/s

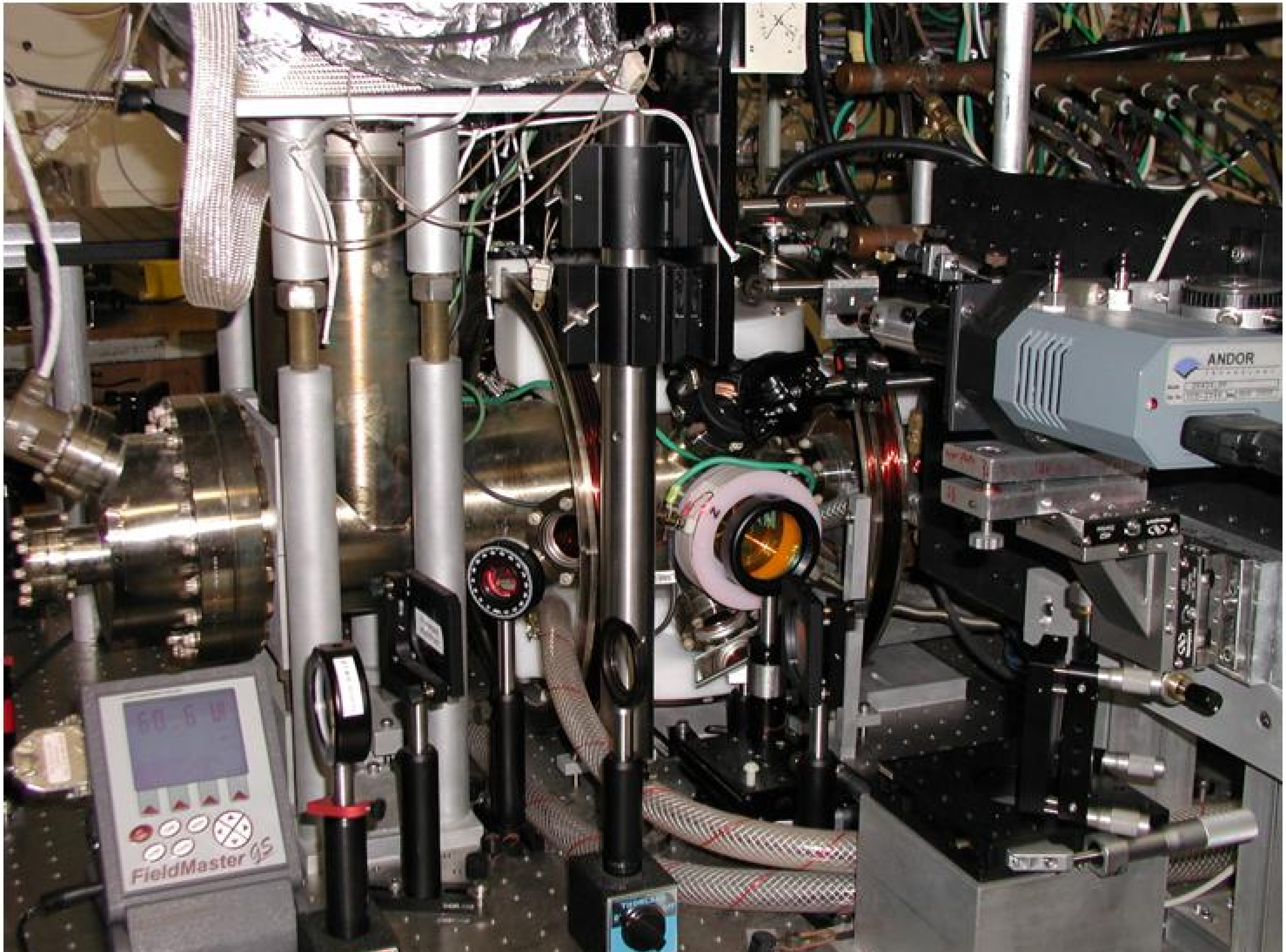
- Extremely Low Noise
  - Intensity Noise Heating

$$\boxed{\Gamma^{-1} \geq 2.3 \times 10^4\ \mathrm{sec}}$$

- Ultra-High Vacuum
  - Pressure: $< 10^{-11}$ Torr
  - Heating: $< 5$ nK/sec
  - Lifetime: 400 sec

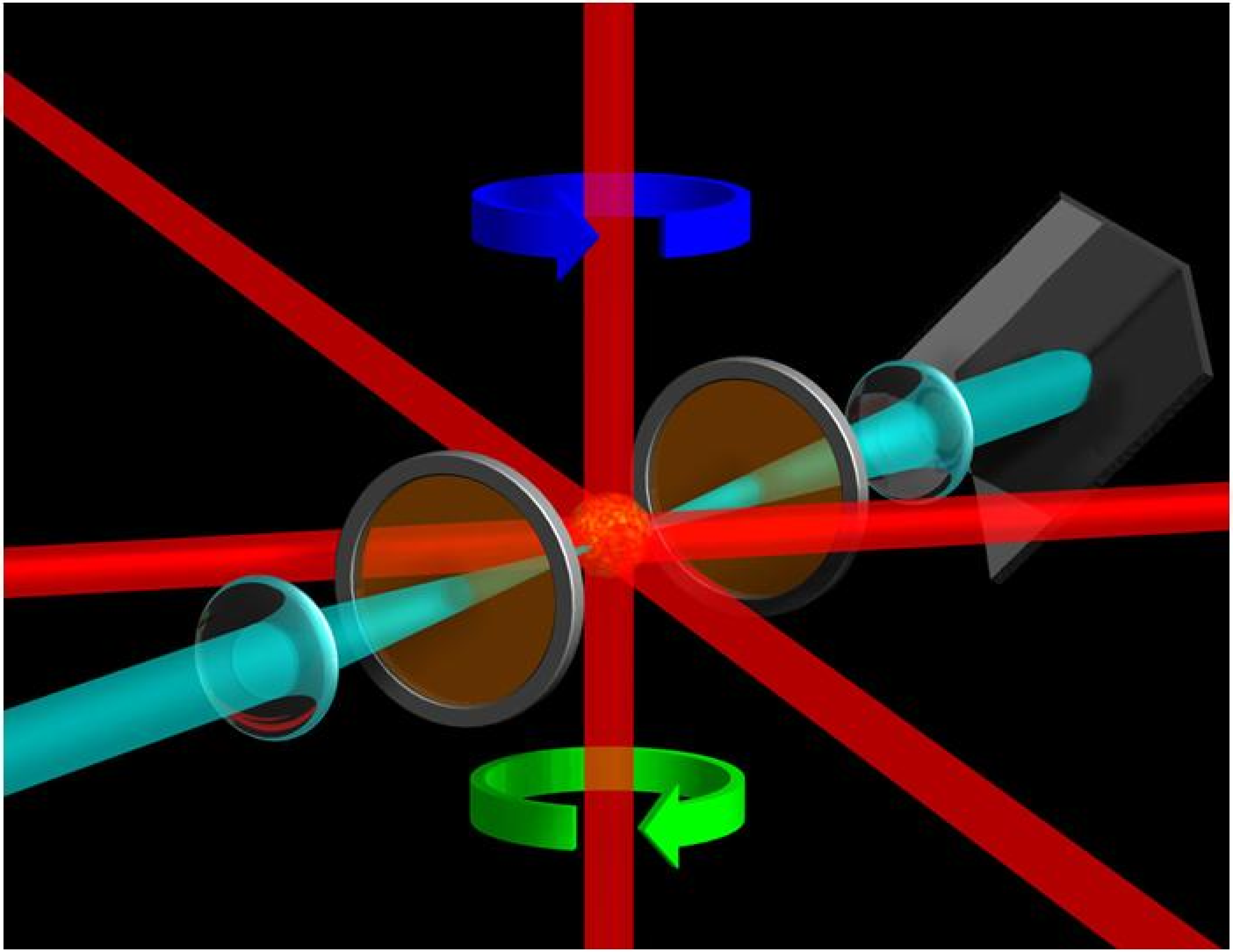
# Experimental Apparatus





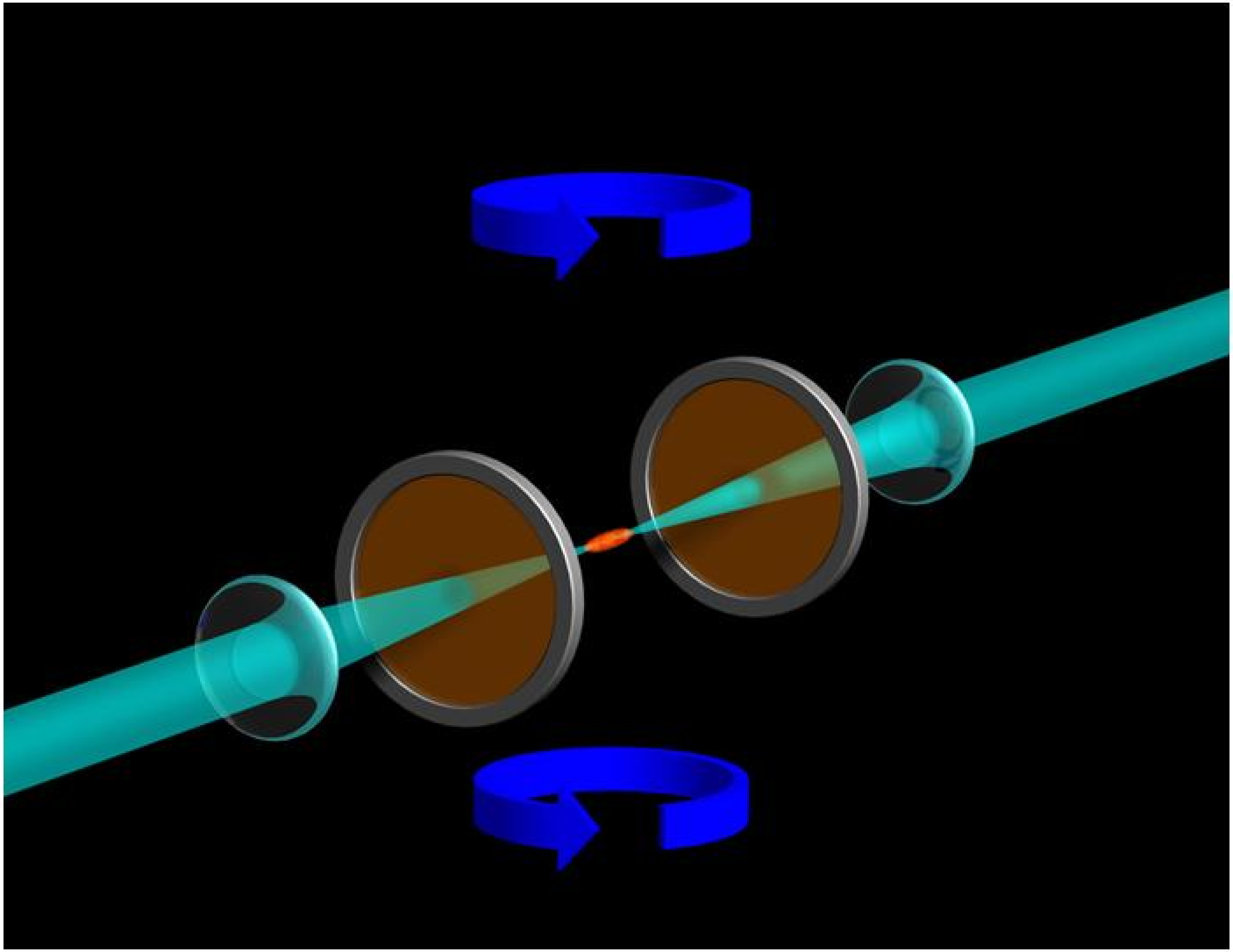
# Optical Trap Loading





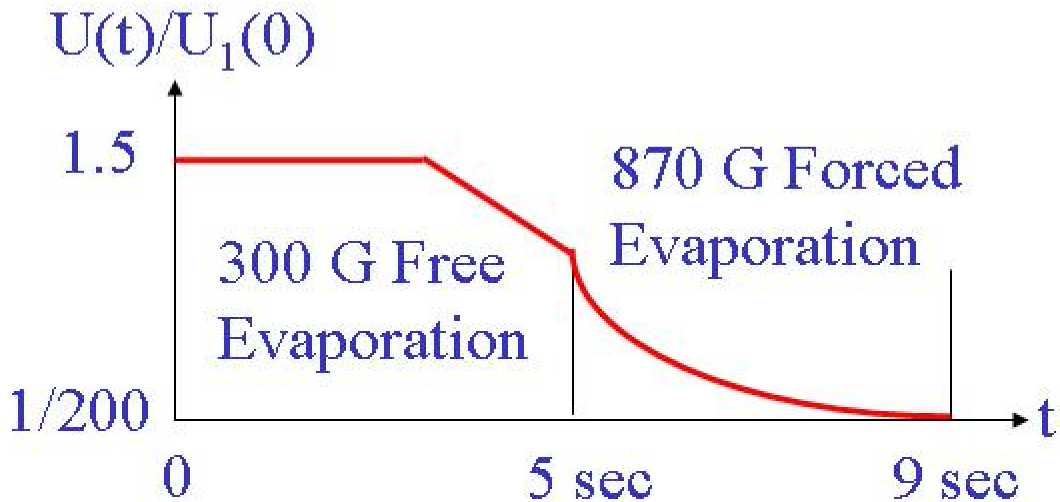
# Forced Evaporation

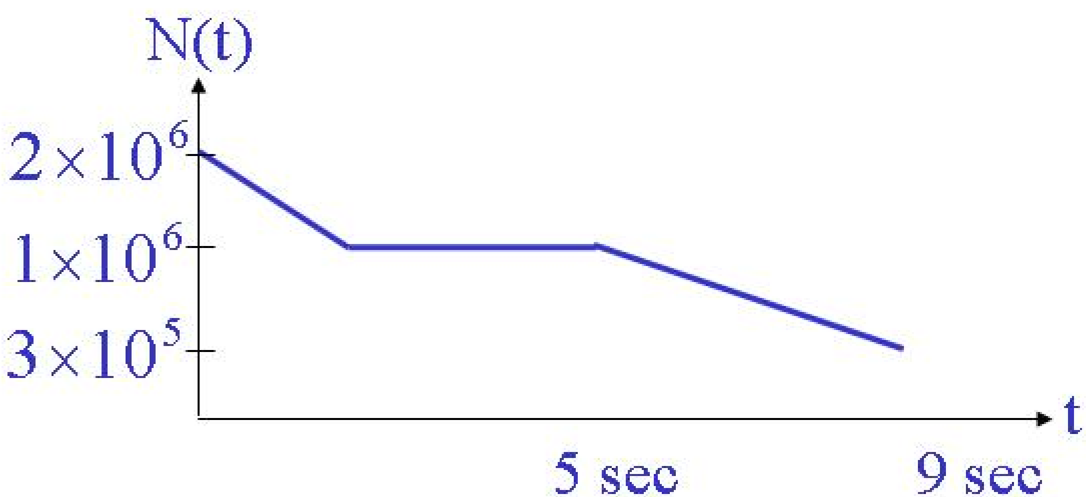




# Timing Sequence for Evaporation






<u>Scaling Laws:</u>

$$\frac{\rho_f}{\rho_i} = \left(\frac{U_i}{U_f}\right)^{1.3} = 10^3$$

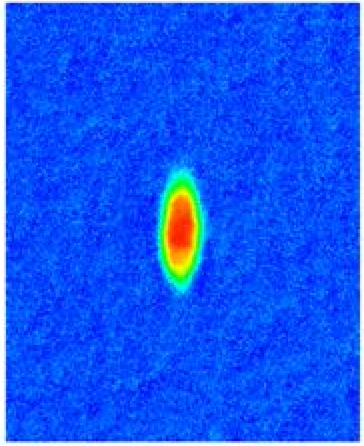
$$\frac{N_f}{N_i} = \left(\frac{U_f}{U_i}\right)^{\frac{3}{16}} = \frac{1}{2.7}$$

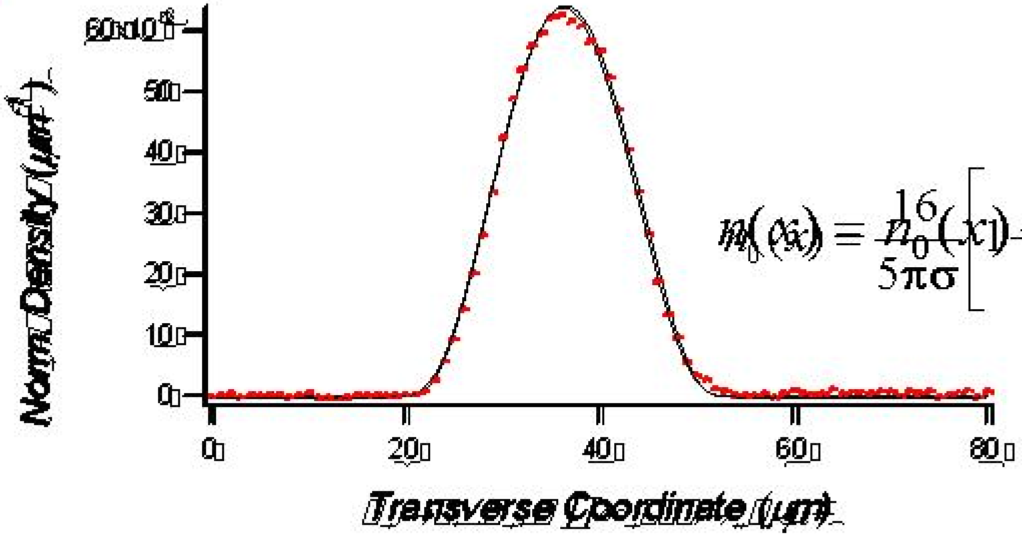
# Cooling a Strongly-Interacting Fermi Gas

Duke Physics
Atom Cooling and Trapping

## Evaporate for 3.5 s at 910 G:

t = 0.2 ms after release






$$n(x) = n_0(x) + \left(\frac{T}{T_F}\right)^2 n_2(x)$$

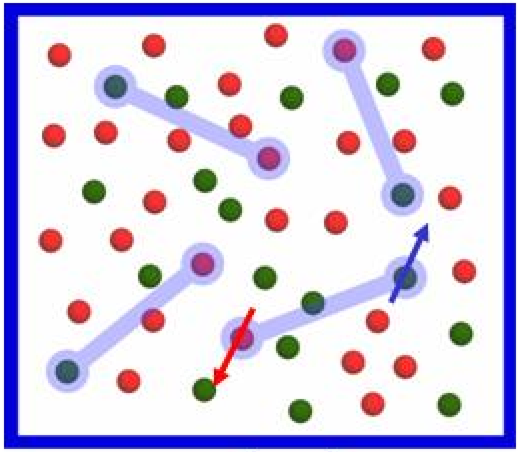
$$n_0(x) = \frac{16}{5\pi\sigma}\left[1-\left(\frac{x}{\sigma}\right)^2\right]^{\frac{5}{2}}$$

$T/T_F = 0.09$
$T = 0.7\ \mu K$ at full trap depth $U_0$
$T = 50\ nK$ at $U_0/200$

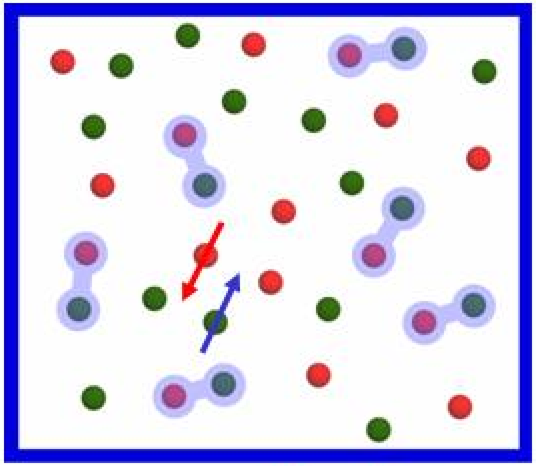
# Superfluidity in Atomic Fermi Gases

Duke Physics
Atom Cooling and Trapping

- Magnetically tunable interactions via Feshbach Resonance
- Theory BCS Pairing $^6$Li: Houbiers, et al., PRA **56**, 4864 (1997).



$$T_C = \eta_C T_F$$



$$\eta_C \approx \exp\left(\frac{-L}{|a_S|}\right) << 1$$

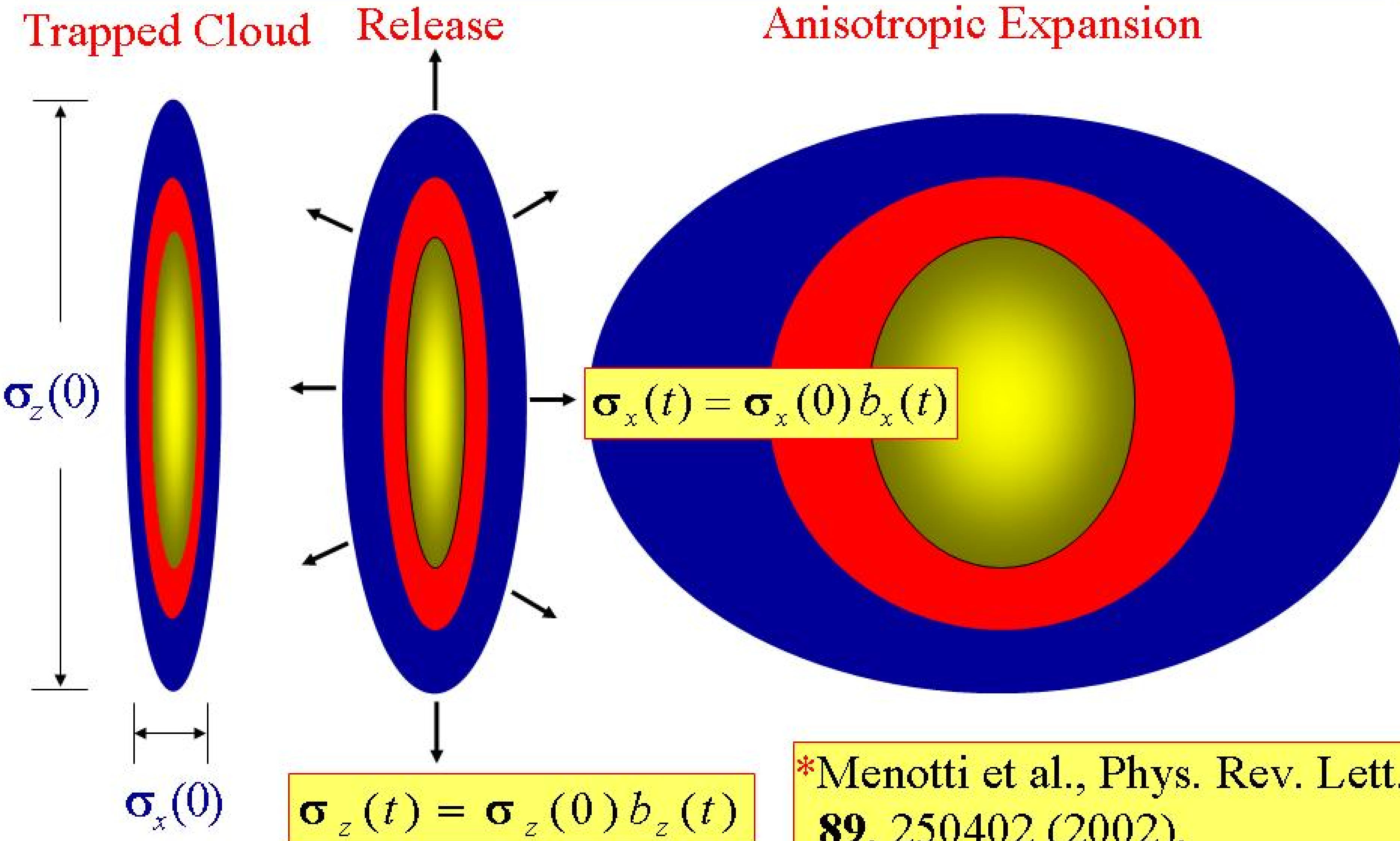
$$\eta_C \approx 0.2\text{–}0.5$$

- On Resonance: Super-High $T_C$ Superconductivity

M. Holland, *et al.* Phys. Rev. Lett. **87**, 120406 (2001)
E. Timmermans, *et al.* Phys. Lett. A **285**, 228 (2001)
Y. Ohashi *et al.* cond-mat/0201262 (2002)

# Prediction of Anisotropic Expansion*

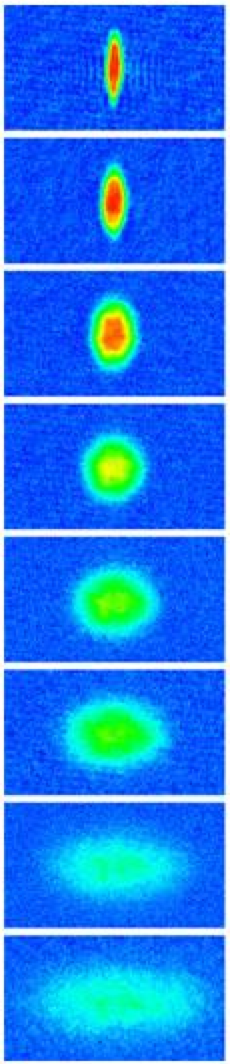




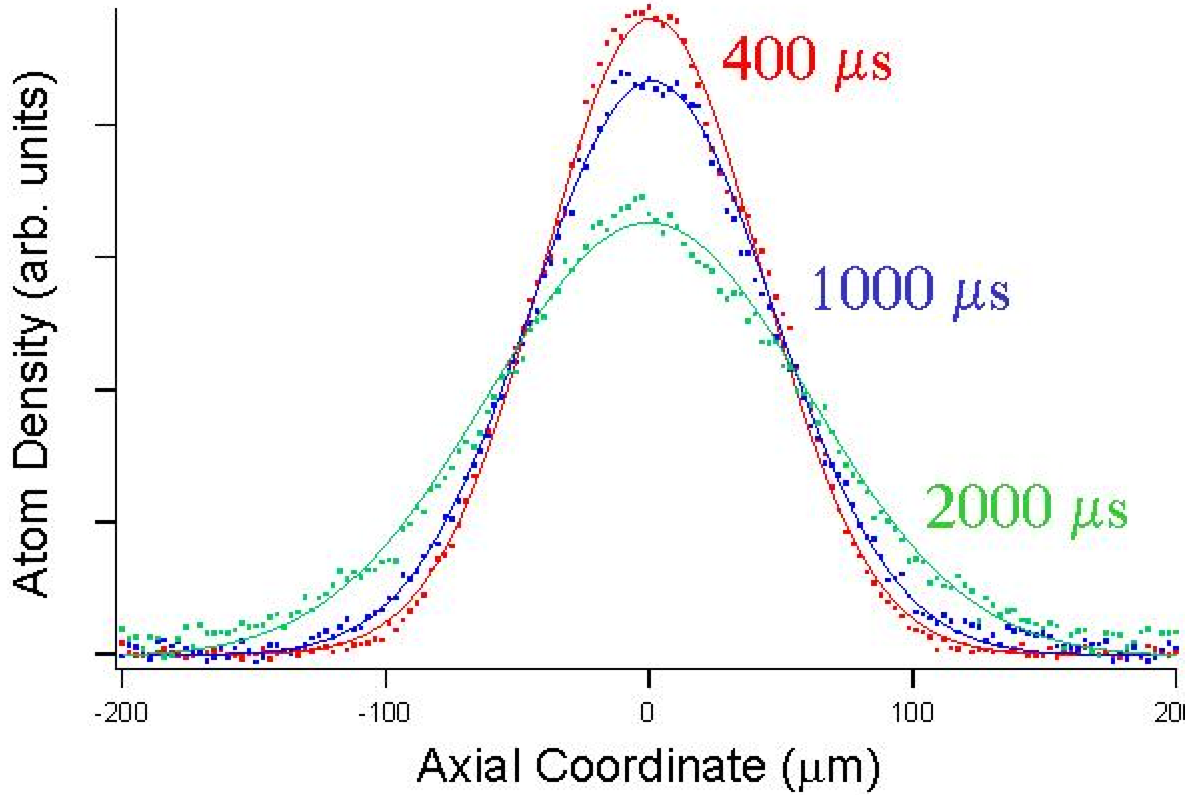

*Menotti et al., Phys. Rev. Lett. **89**, 250402 (2002).

# Axial Expansion at 910 G

Duke Physics
Atom Cooling and Trapping

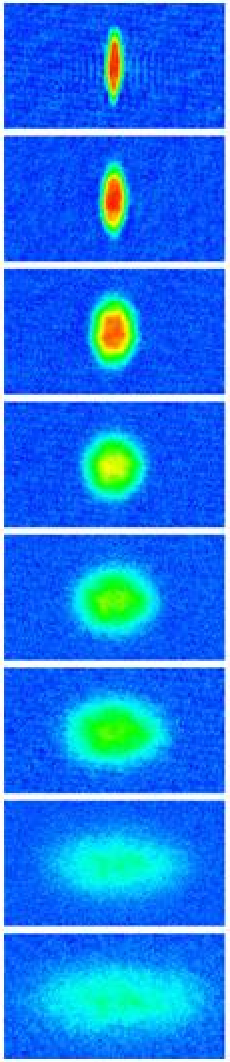


100 μs
200 μs
400 μs
600 μs
800 μs
1000 μs
1500 μs
2000 μs

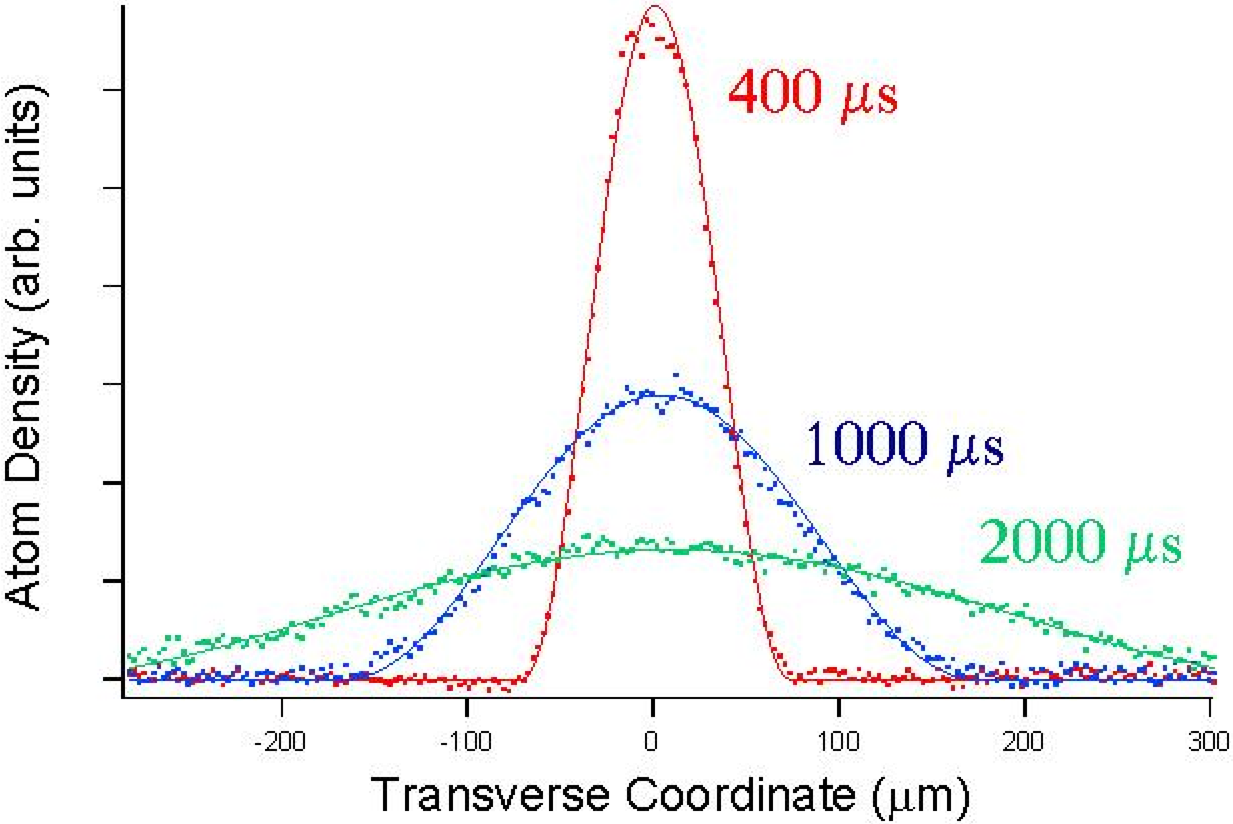


# Transverse Expansion at 910 G



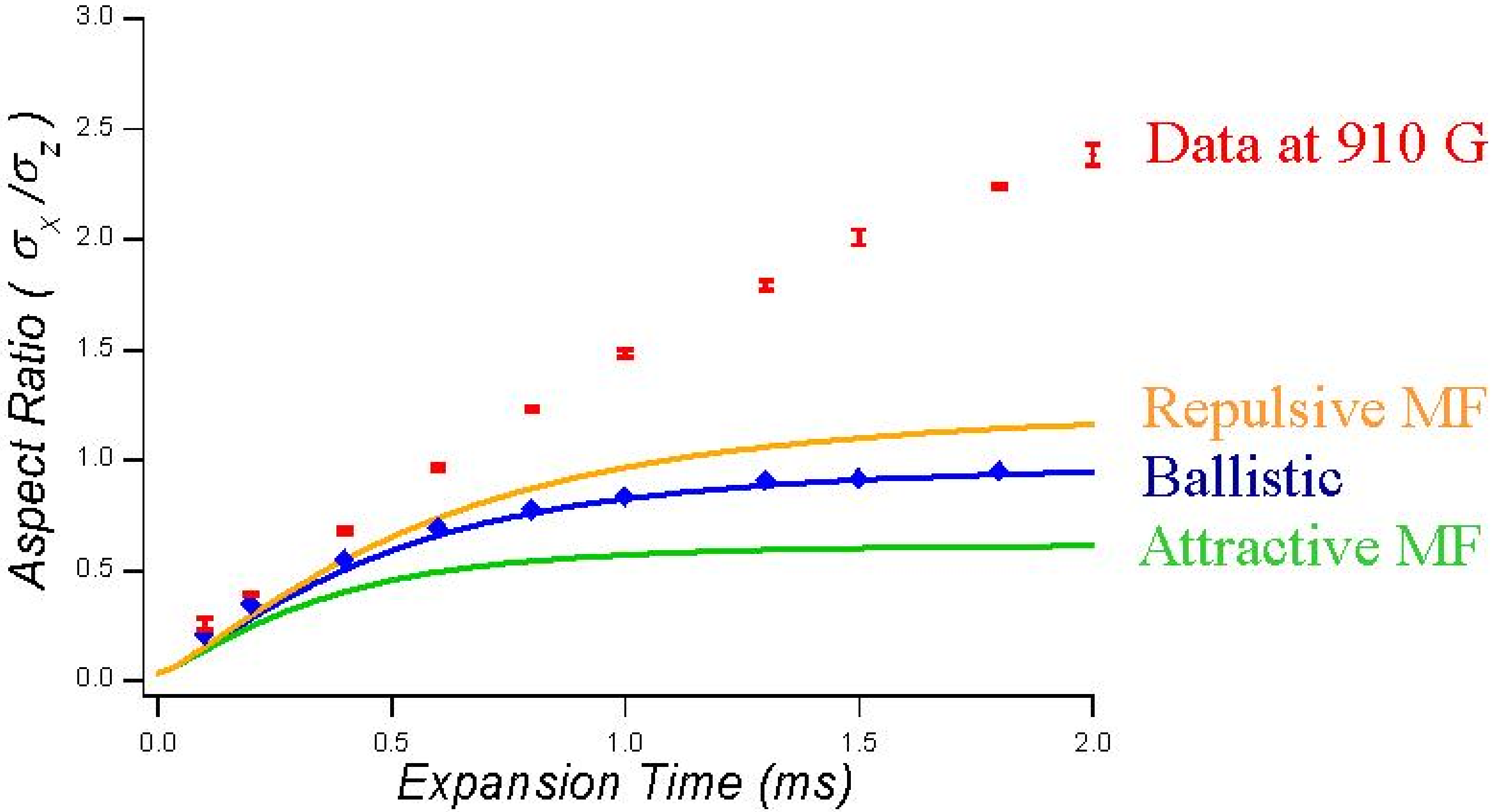


100 μs
200 μs
400 μs
600 μs
800 μs
1000 μs
1500 μs
2000 μs



# Hydrodynamic Expansion Data

Duke Physics
Atom Cooling and Trapping



# Superfluid Hydrodynamic Expansion

Duke Physics

Atom Cooling and Trapping

Continuity Equation

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{u}) = 0$$

Irrotational

$$\nabla \times \mathbf{u} = 0$$

Euler's equation for a Superfluid

$$M\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = -\nabla\mu(n)$$

Chemical potential with mean field $\quad \mu = (1+\beta_S)\,\varepsilon_F(x)$

$$\boxed{M\frac{\partial \mathbf{u}}{\partial t} = -\nabla\left((1+\beta_S)\,\varepsilon_F(x) + \tfrac{1}{2}M\,\mathbf{u}^2\right)}$$

# Expansion of the Interacting Gas

Duke Physics
Atom Cooling and Trapping

The form of the density distribution does not change:

$$n(x,t) = \frac{1}{b_x(t)} n_0\left(\frac{x}{b_x(t)}\right)$$

Good News:

The time-dependent scale factors obey simple differential equations:

$$\ddot{b}_i = \frac{\omega_i^2}{(b_x b_y b_z)^{2/3} b_i} \qquad b_i(0) = 1,\ \dot{b}_i(0) = 0$$

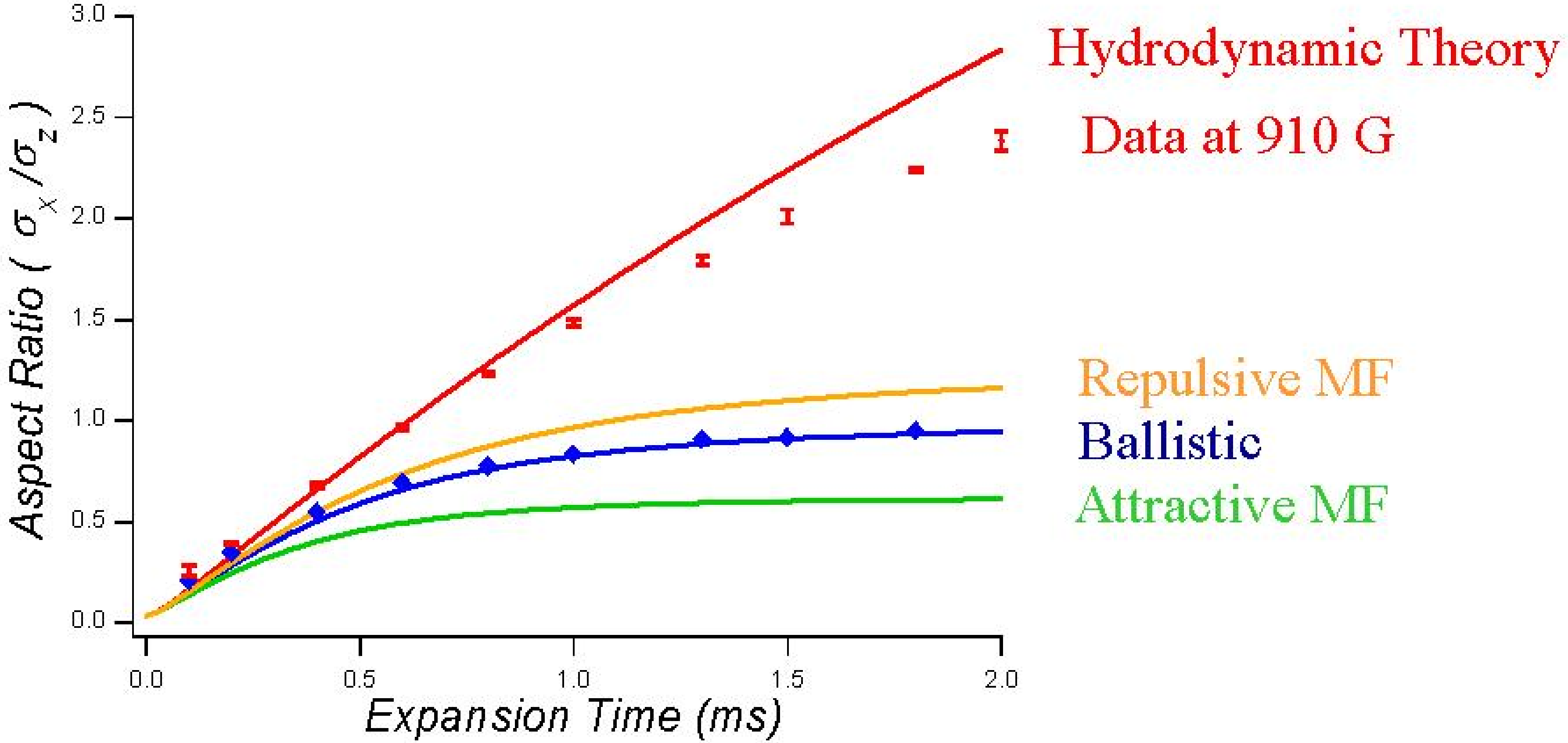
Bad News: Independent of $\beta$!


*Kagan et al., Phys. Rev. A **55**, R18 (1997).

*Menotti et al., Phys. Rev. Lett. **89**, 250402 (2002).

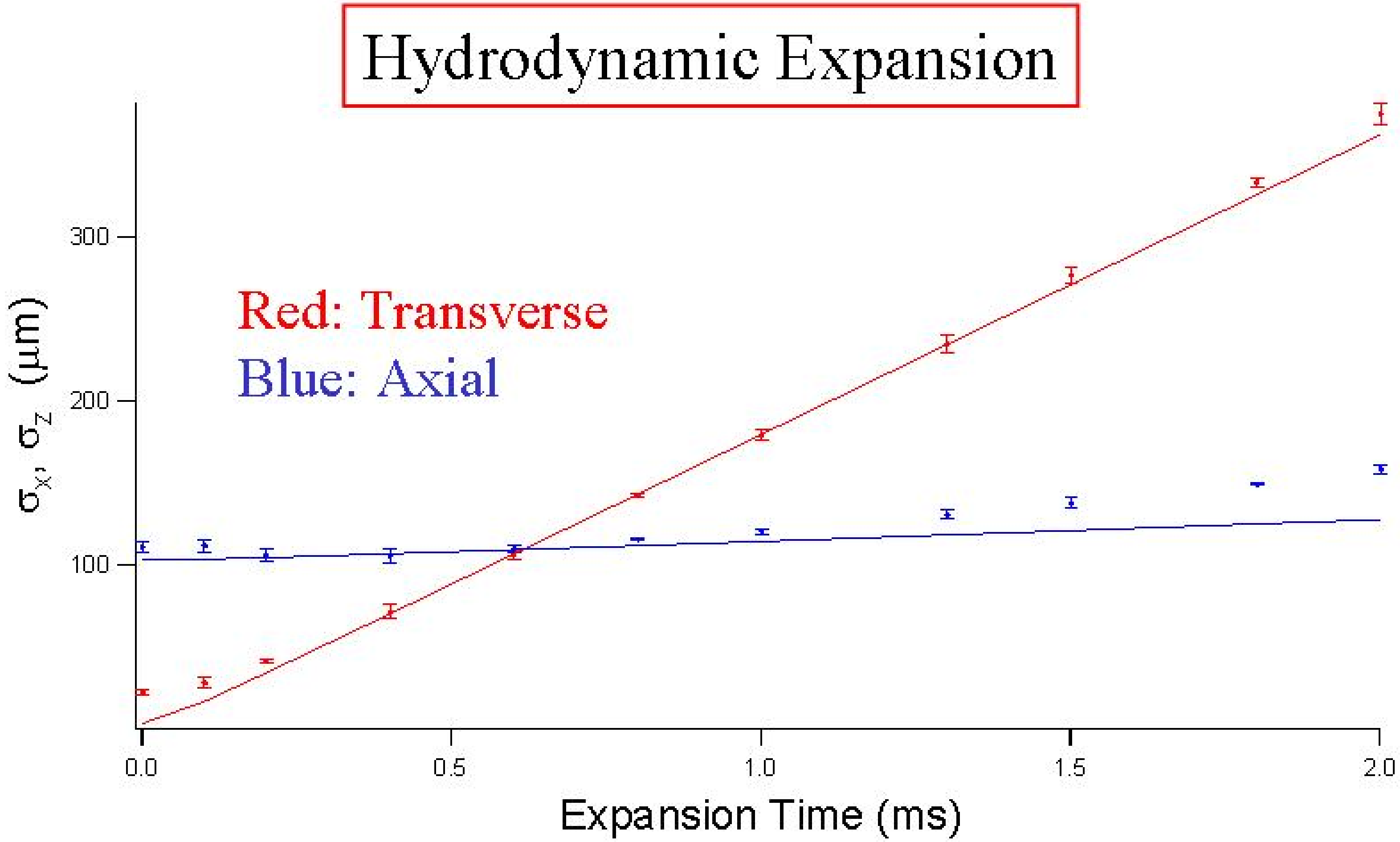
# Expansion Dynamics

Duke Physics
Atom Cooling and Trapping



# Transverse and Axial Widths vs Time

Duke Physics
Atom Cooling and Trapping



# Collisional Hydrodynamic Expansion?

Duke Physics
Atom Cooling and Trapping

Continuity Equation

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{u}) = 0$$

Irrotational

$$\nabla \times \mathbf{u} = 0$$

Euler's equation for a normal fluid

$$M\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = \frac{-\nabla P}{n}$$

Pressure $P(x) = \frac{2}{3}\frac{\langle E\rangle}{V} = \frac{2}{5}(1+\beta_N)\,\varepsilon_F(x)\,n(x)$

$$\boxed{M\frac{\partial \mathbf{u}}{\partial t} = -\nabla\left((1+\beta_N)\,\varepsilon_F(x) + \tfrac{1}{2}M\mathbf{u}^2\right)}$$

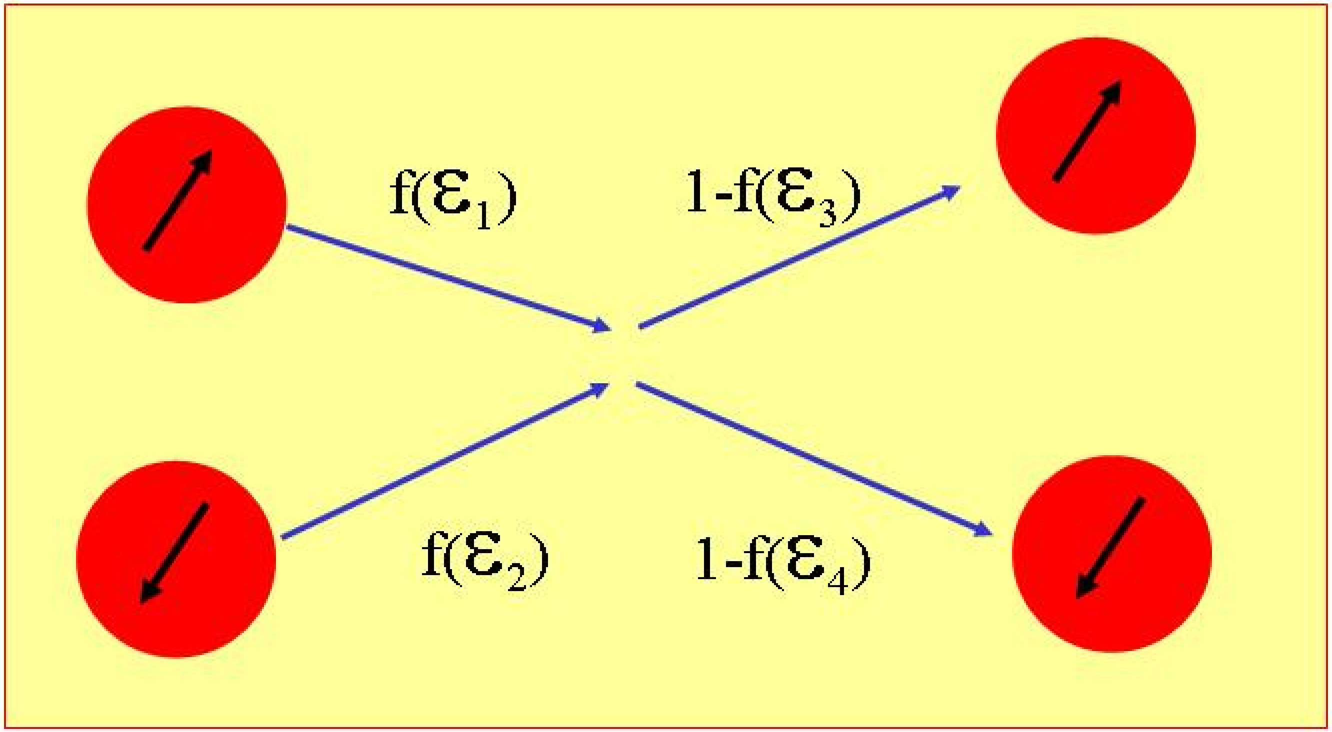
Same equation!

→ Is collisional dynamics possible ?

# Can Pauli Blocking Suppress Collisions?

Duke Physics
Atom Cooling and Trapping

Collision cannot occur if the final state is occupied:




$$\Gamma_{\text{coll}} = \gamma \left( \frac{T}{T_F} \right)^2$$

$\Rightarrow 0$

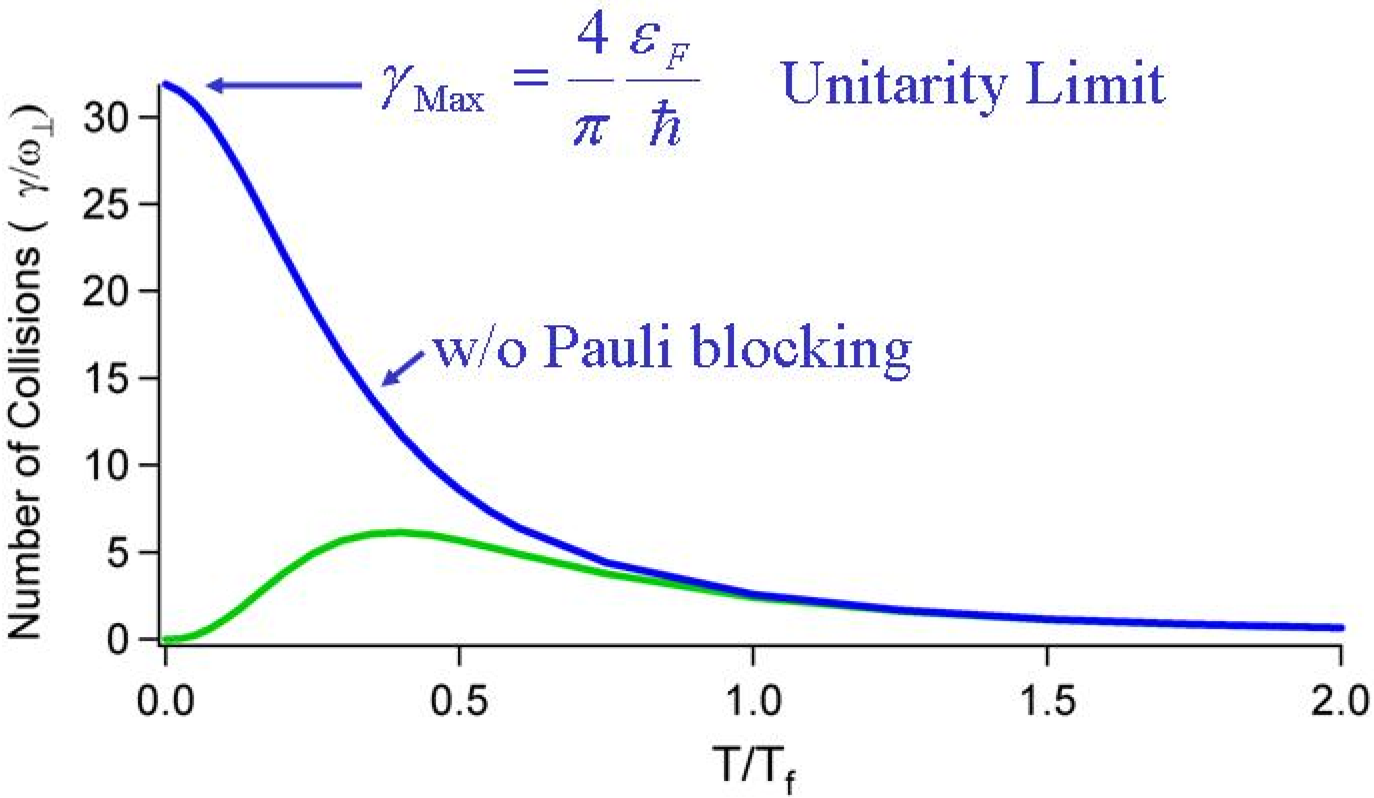
as $T \rightarrow 0$

$f(\varepsilon)$ is the occupation probability

# Pauli Blocking in a Harmonic Trap

Duke Physics
Atom Cooling and Trapping

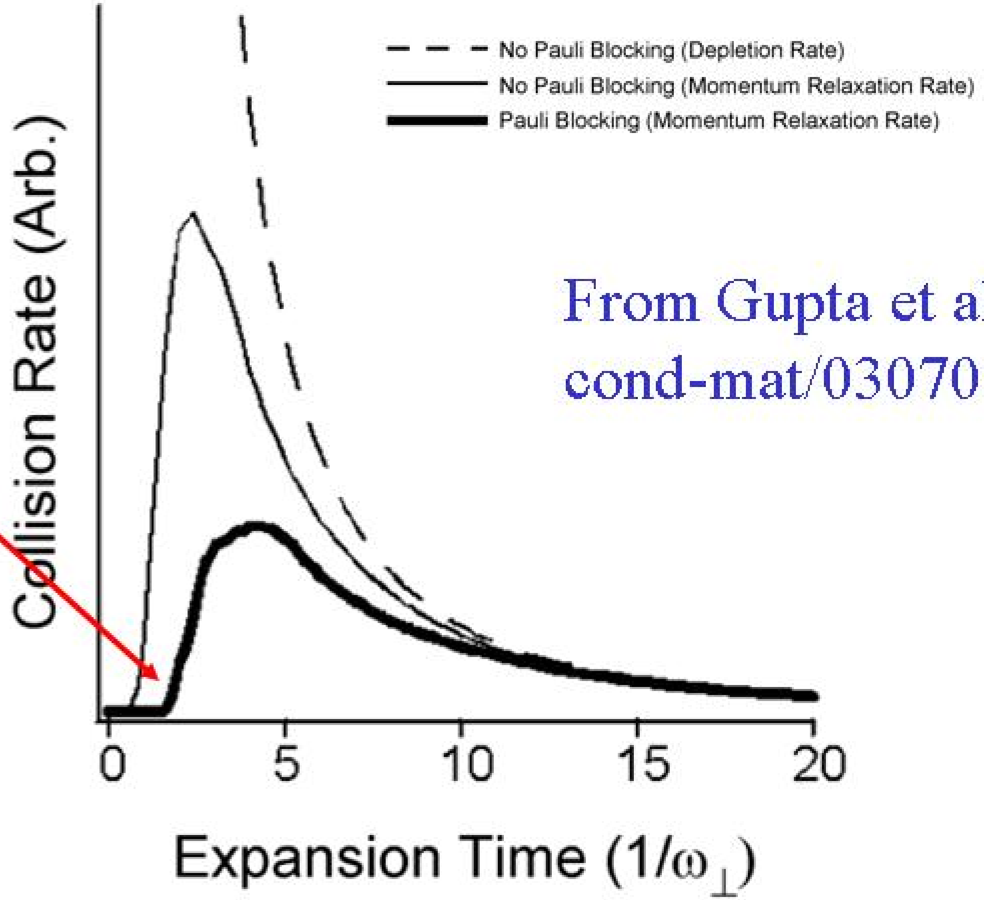
Depletion rate with and without Pauli Blocking:



# Step-wise Model of Pauli-blocking

Duke Physics
Atom Cooling and Trapping

Delay




From Gupta et al., arXiv: cond-mat/0307088

Model: Ballistic expansion for time $t_B$ followed by No Pauli Blocking

# Relaxation Approximation

Duke Physics

Atom Cooling and Trapping

Pedri et al., cond-mat/0305624 (2003); Guery-Odelin et al., PRA 60, 4851 (1999)

Ansatz: MB or Zero temp TF

$$\boxed{W(\mathbf{x},\mathbf{p},t) = \frac{1}{\Pi_j (b_j \theta_j^{1/2})} W_0\left(\tilde{x}_i, \frac{p_i - M u_i}{\sqrt{\theta_i}}\right)} \qquad \tilde{x}_i = \frac{x_i}{b_i}, \quad u_i = \tilde{x}_i \dot{b}_i$$

$$0 = \ddot{b}_i + \omega_i^2 b_i [1+\varepsilon(t)] - \frac{\theta_i}{b_i}\omega_i^2 + \frac{1}{M\langle \tilde{x}_i^2\rangle} \underbrace{\left[\frac{1}{b_i}\left\langle \tilde{x}_i \frac{\partial U_{\mathrm{MF}}(\tilde{\mathbf{x}},t)}{\partial \tilde{x}_i}\right\rangle - \frac{\theta_i}{b_i}\left\langle \tilde{x}_i \frac{\partial U_{\mathrm{MF}}(\tilde{\mathbf{x}},0)}{\partial \tilde{x}_i}\right\rangle\right]}_{\approx 0}$$

Trap Modulation

$\approx 0$ For hydrodynamic limit and $U_{\mathrm{MF}}(\mathbf{x}) = \frac{3}{5}\beta\varepsilon_{\mathrm{F}}(\mathbf{x})$

$$\dot{\theta}_i + 2\theta_i \frac{\dot{b}_i}{b_i} = -\frac{1}{\tau}(\theta_i - \overline{\theta}) \qquad \overline{\theta} = \frac{1}{3}\sum_j \theta_j$$

No Pauli Blocking:

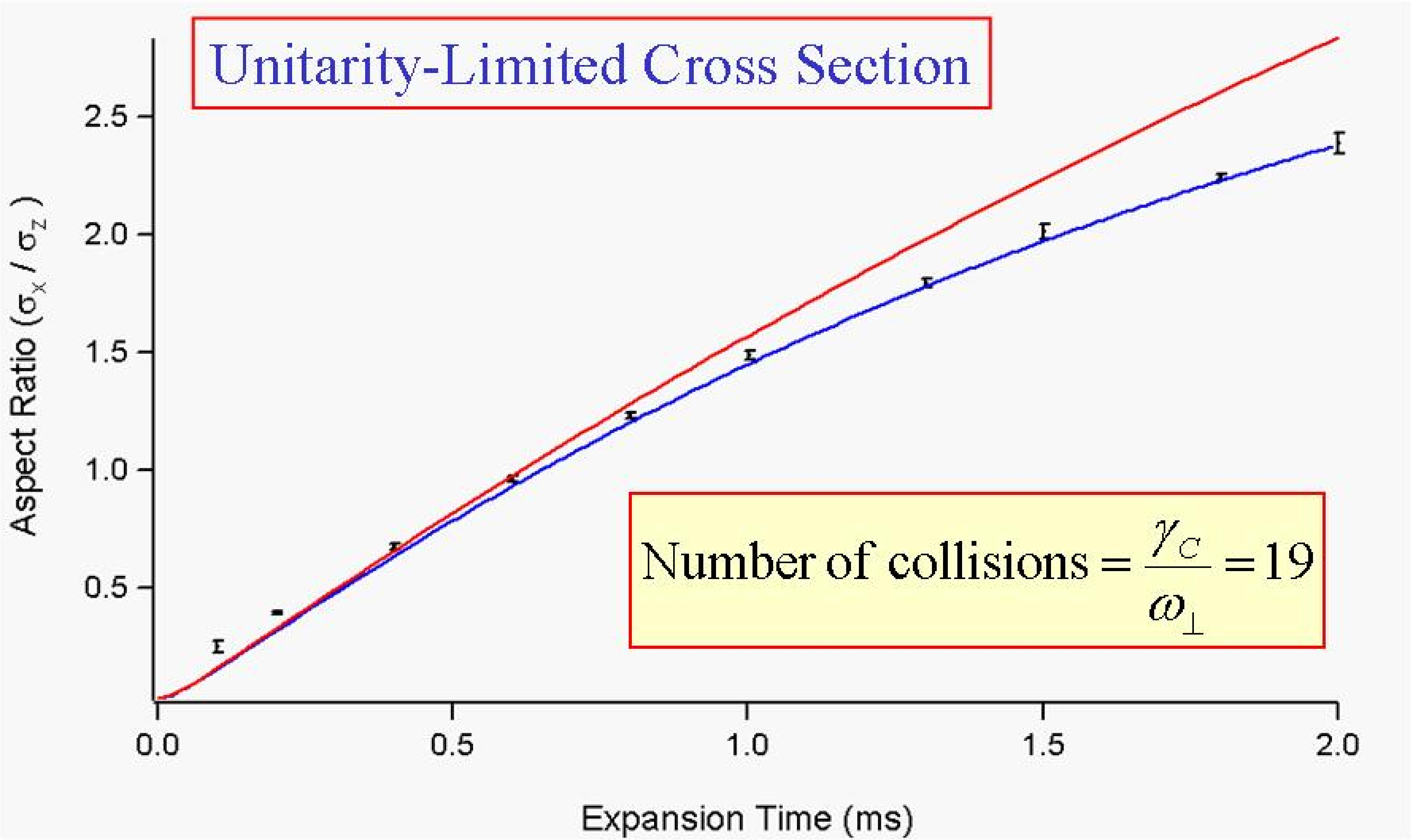
$$\boxed{\frac{1}{\tau} = \frac{1}{\Pi_j b_j \sqrt{\overline{\theta}}} \frac{8}{25}\left(\frac{4}{\pi}\frac{\varepsilon_{\mathrm{F}}}{\hbar}\right)}$$

Momentum relaxation rate

Turn off for a time $t_B$

# Collisional Expansion Dynamics

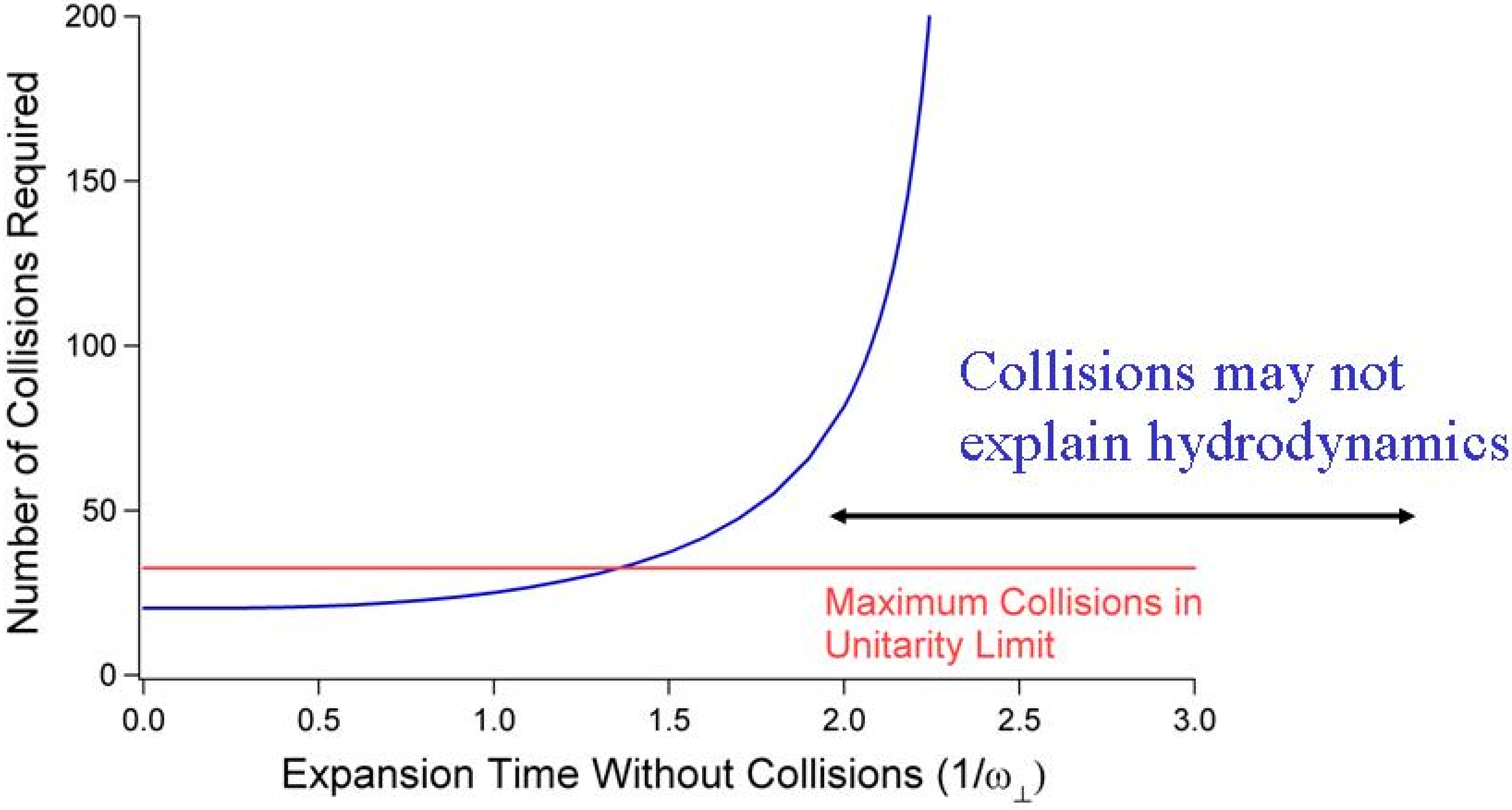
Duke Physics
Atom Cooling and Trapping



# Finite Time to Reinstate Collisions

Duke Physics
Atom Cooling and Trapping

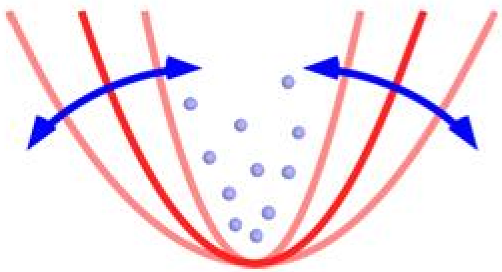
Number of collisions to achieve our aspect ratio $= \dfrac{\gamma_C}{\omega_\perp}$



# Hydrodynamics in a Trapped Fermi Gas

Duke Physics
Atom Cooling and Trapping

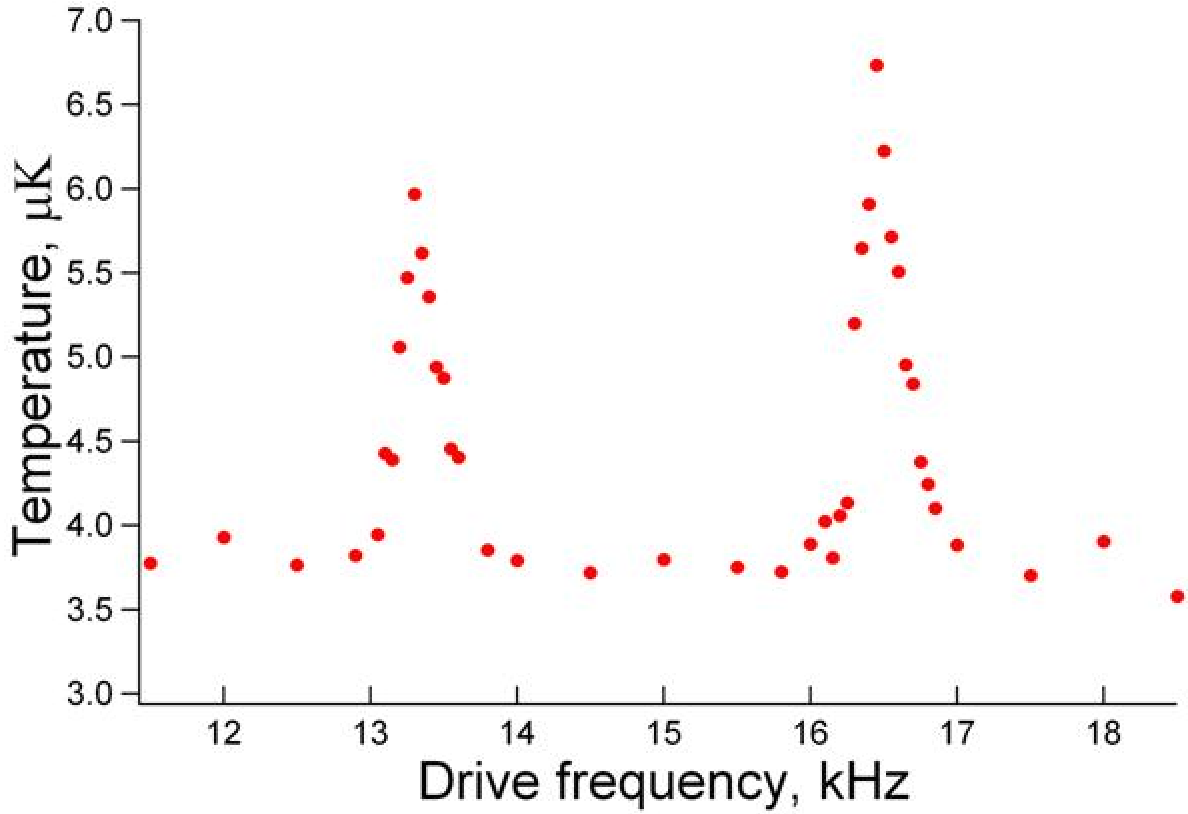
Parametric resonance



Modulate trap depth

Resonance at 2 ω

B = 300 G (few collisions)

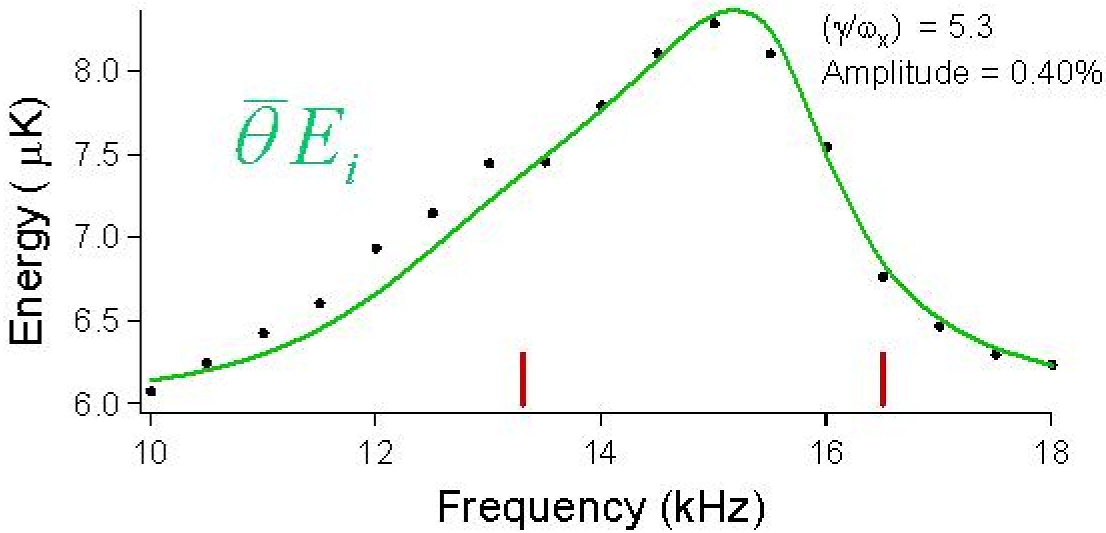


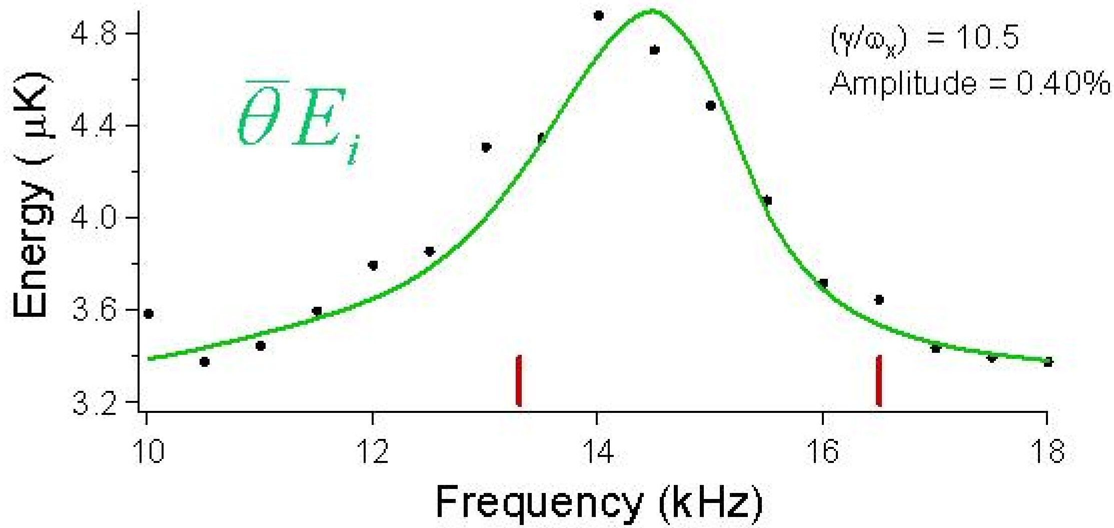
# Hydrodynamics in a Trapped Fermi Gas

Duke Physics
Atom Cooling and Trapping

B = 910 G
(Strong Interactions)

$T = 0.6\ T_F$



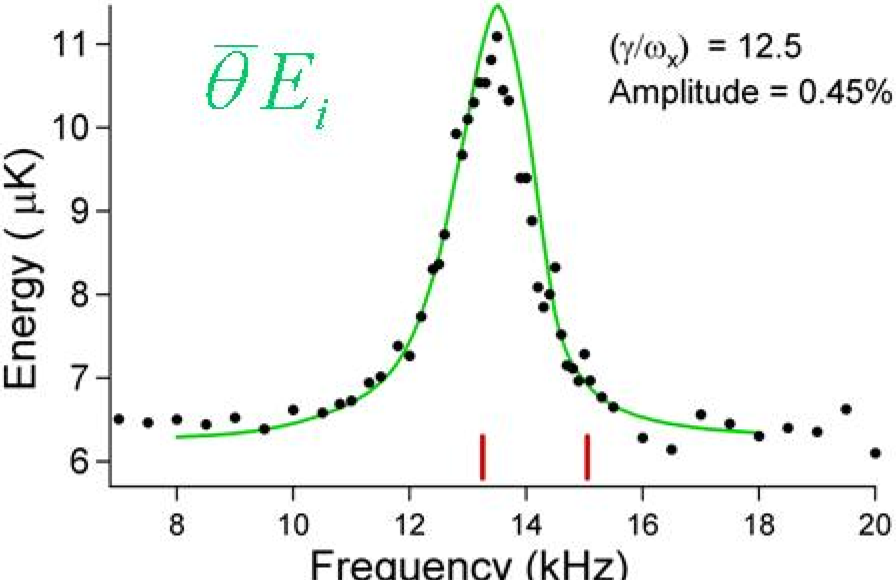



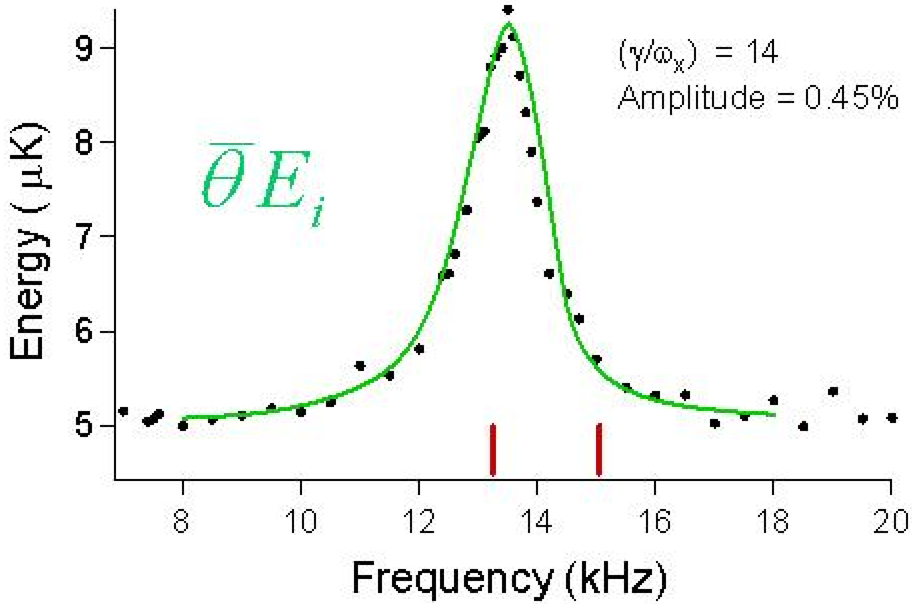
# Hydrodynamics in a Trapped Fermi Gas

Duke Physics
Atom Cooling and Trapping

B = 910 G
(Strong Interactions)

$T = 0.6\ T_F$

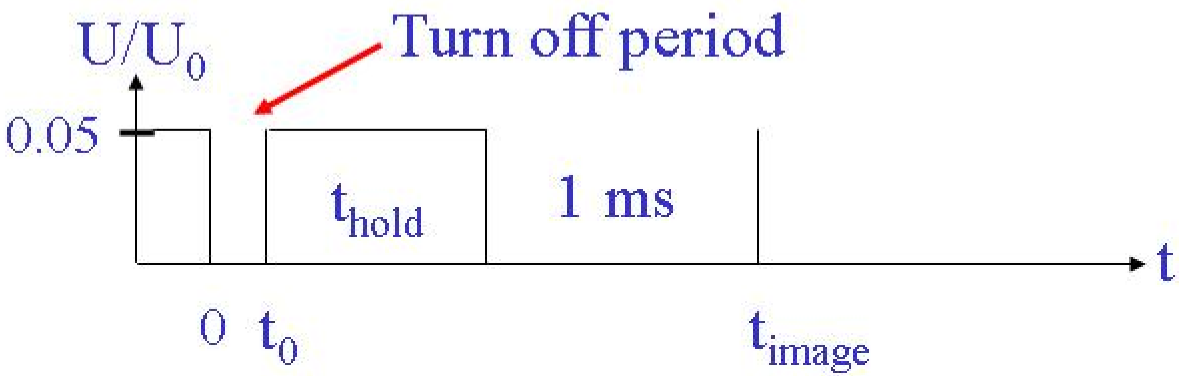



$T = 0.5\ T_F$

# Collective Modes in a Trapped Fermi Gas

Duke Physics
Atom Cooling and Trapping




$$t_0 = \frac{1}{16}\frac{2\pi}{\omega} \quad \Rightarrow \quad \Delta E = \tfrac{1}{2} M\omega^2 \mathrm{v}^2 t_0^2 = \tfrac{1}{2}(\omega t_0)^2 E = \frac{\pi^2}{128} E$$

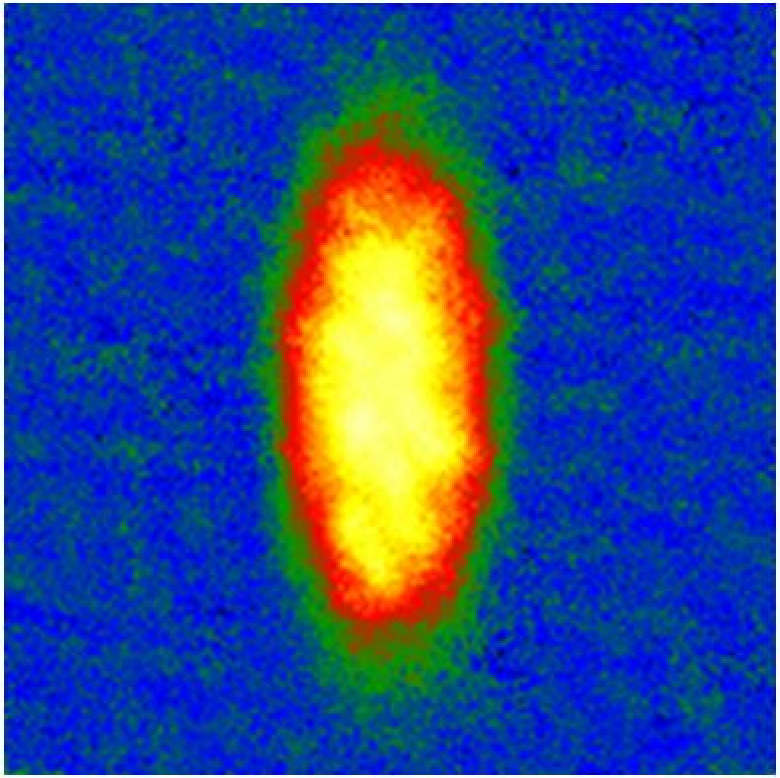
$$E = \tfrac{3}{4}\varepsilon_F \left[1 + \frac{2\pi^2}{3}\left(\frac{T}{T_F}\right)^2\right] \qquad \frac{T_i}{T_F} \cong 0.1 - 0.15 \quad \Rightarrow \quad \boxed{\frac{\Delta T}{T_F} \cong 0.05}$$

# Collective Modes in a Trapped Fermi Gas



B = 870 G

$$\frac{T_i}{T_F} = 0.1 - 0.15$$



# Collective Modes in a Trapped Fermi Gas

Duke Physics

Atom Cooling and Trapping

## Hydrodynamic:

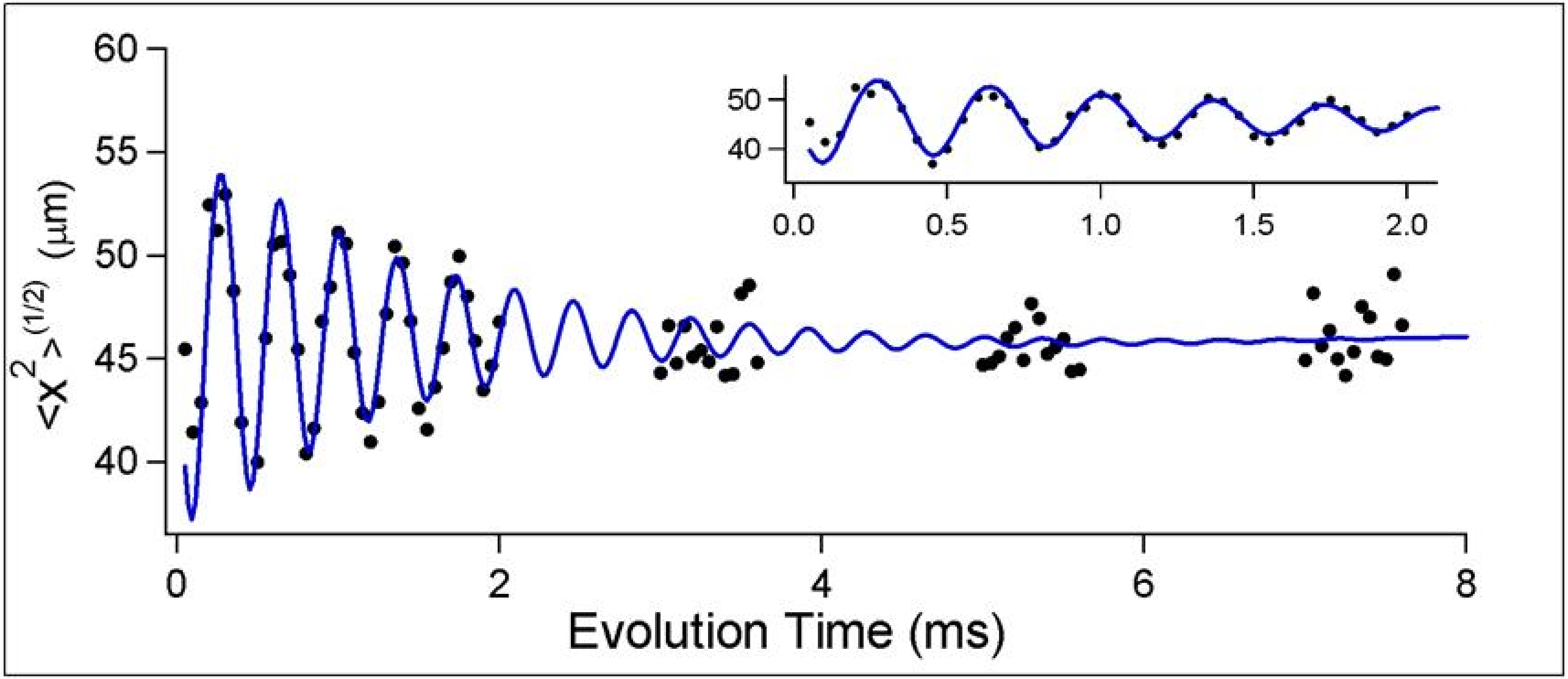
Fit Parameters

$1/(\omega_x \tau_o) = 4.8$ $\quad (\langle x^2 \rangle_o)^{(1/2)} = 4.1\ \mu m$

$\gamma/\omega_x = 14$ $\quad \nu_x = 1520\ Hz$

$$B = 870\ G$$

$$\frac{T_i}{T_F} = 0.3$$



# Collective Modes in a Trapped Fermi Gas

Duke Physics
Atom Cooling and Trapping

## Hydrodynamic:

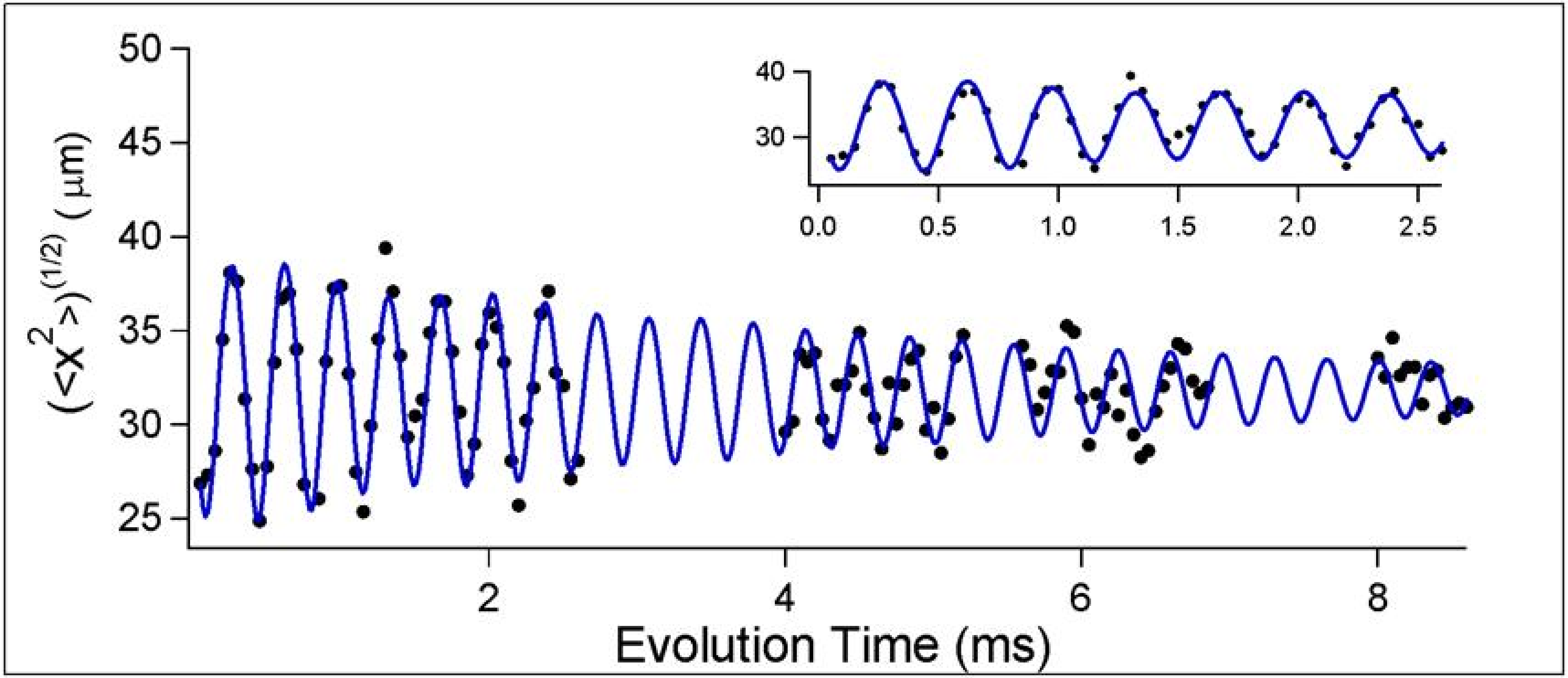
Fit Parameters

$1/(\omega_x \tau_o) = 20$ $\qquad (\langle x^2\rangle_o)^{(1/2)} = 2.6\ \mu m$

$\gamma/\omega_x = 63$ $\qquad \nu_x = 1600\ Hz$

$B = 870\ G$

$$\frac{T_i}{T_F} = 0.1 - 0.15$$



# Collective Mode Frequencies



Measured Parametric Resonance Frequencies:

$$\omega_x = 2\pi \times 1610 \text{ Hz}$$

$$\omega_y = 2\pi \times 1510 \text{ Hz}$$

$$\omega_\perp = \sqrt{\omega_x \omega_y} = 2\pi \times 1550 \text{ Hz}$$

Predicted Frequency for Hydrodynamic Fermi Gas:

$$\omega_{\text{Fermi}} = \sqrt{\frac{10}{3}}\,\omega_\perp = 2\pi \times 2.83 \text{ kHz}$$

Measured Oscillation Frequency for Sinusoidal Fit:

$$\omega_{\text{Meas}} = 2\pi \times 2.84 \text{ kHz}$$

# Summary

- All-Optical Production of Degenerate Fermi Gas
  - Efficient evaporation near Feshbach resonance
  - Very low $T/T_F$

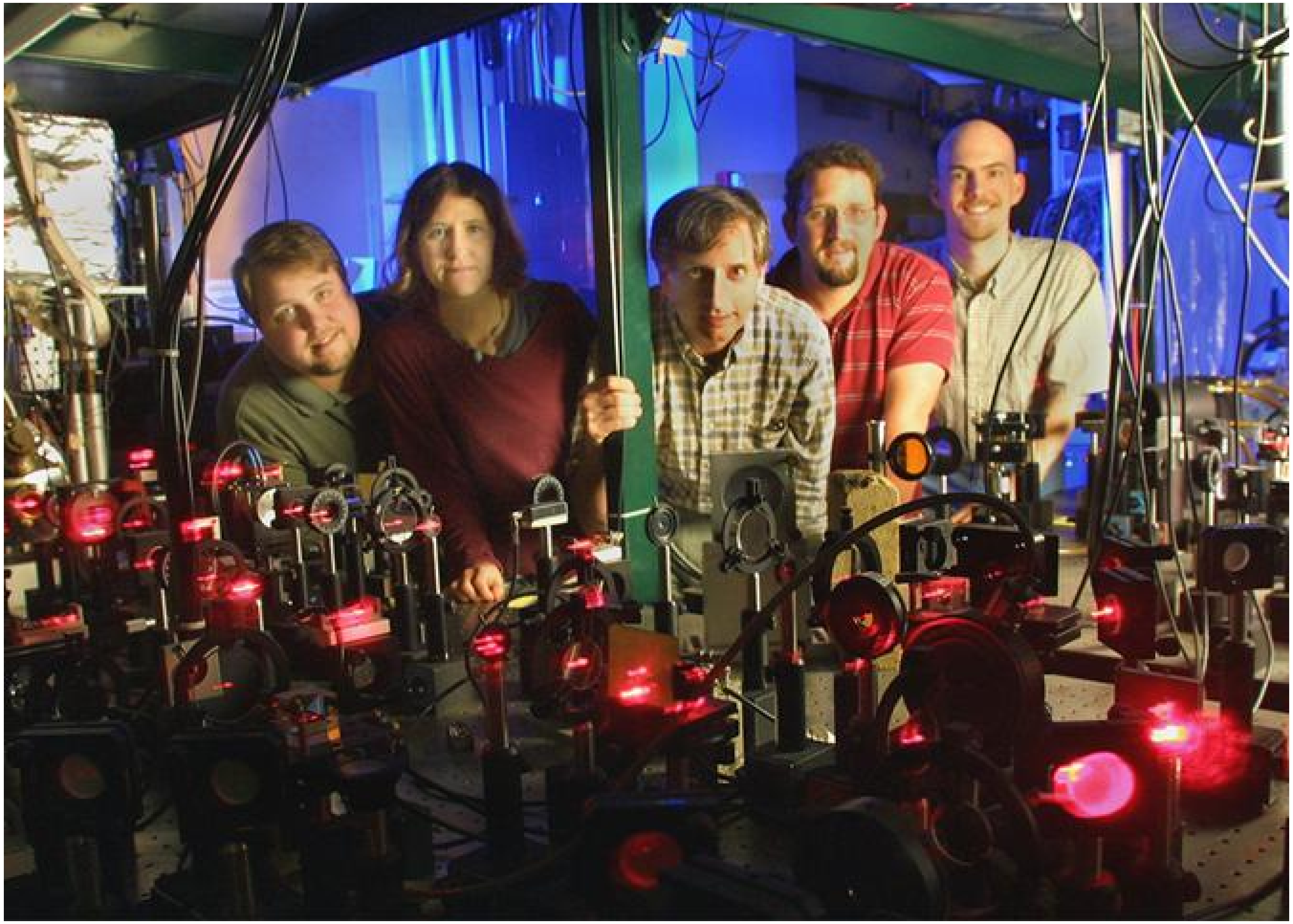
## Hydrodynamics of a Strongly-Interacting Fermi Gas

- Observation of Anisotropic Expansion
  - Relaxation approximation model – no Pauli blocking
  - Step-wise model: ballistic drift time $t_B$ + no Pauli blocking
  - For low T, $t_B > 1.5 / \omega_\perp$, collisions may not explain hydrodynamics

- Trapped Atom Hydrodynamics
  - Collisionally-damped hydrodynamic modes at high T
  - Weakly-damped hydrodynamic modes at low T
  - Hydrodynamic Fermi gas at T = 0.1: Fermi Superfluid?

# The Team





Mike Gehm
Staci Hemmer
John Thomas
Ken O'Hara
Stephen Granade

Joe Kinast
Andrey Turlapov