BEC

INFM

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University of Trento

COLLECTIVE OSCILLATIONS IN TRAPPED SUPERFLUID FERMI GASES

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HARMONIC TRAPS:
Test of hydrodynamics and of equation of state

ROTATING TRAPS:
Superfluid vs collisionless hydrodynamics

1D OPTICAL LATTICES:
Coherent tunneling vs umklapp processes

HARMONIC TRAPS:
Test of hydrodynamics and of equation of state

T=0 superfluidity $\Rightarrow$ irrotational hydrodynamics

Surface modes: unaffected by equation of state
Compression (m=0) modes sensitive to **equation of state** $\mu(n)$ of uniform matter

$\mu =$ chemical potential, $n =$ density

Explore behaviour of collective frequencies in the BCS-BEC crossover, near Feshbach resonance

(S. Stringari, cond-mat/0312614, Europhys. Lett. 65, 749 (2004))

Molecular BEC regime

a>0, a<<d

a=scattering length, d=interparticle distance

$T_c = 0.83\hbar\omega_{ho}N_m^{1/3}$ critical temperature

ω_{ho} geometrical trapping freq.

$N_m = N/2$ number of molecules

Eq. of state of molecular BEC: $\mu(n_m) = g_m n_m$

$g_m = \frac{4\pi\hbar^2 a_m}{2m}$ $\quad a_m = 0.6a$ molecular scattering length

(Petrov et al. 2003

see also Pieri, Strinati 2000)

Molecular BEC regime

Collective frequencies in harmonic traps **do not** depend on coupling constant (Thomas-Fermi regime $R >> a_{ho}$)

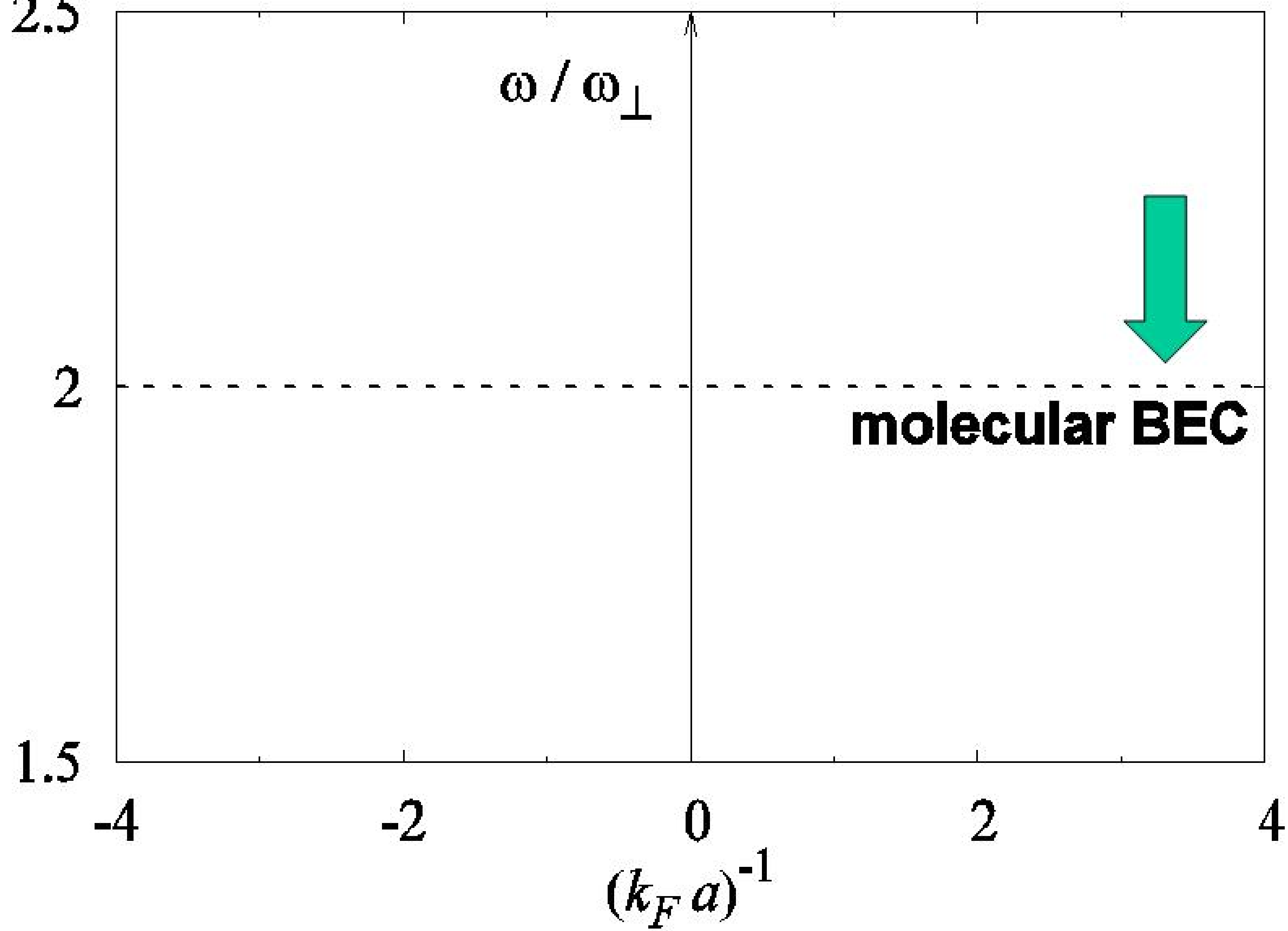
Example: cigar-like traps $(\omega_z << \omega_\perp)$

Radial compression mode $\boxed{\omega = 2\omega_\perp}$

Axial compression mode $\boxed{\omega = \sqrt{5/2}\omega_z}$

(well checked in BEC gases)

Radial compression mode



BEC + beyond mean field effects

If diluteness condition $n_m a_m^3 << 1$ is not satisfied
Bolgoliubov equation of state is changed:

$$\mu(n_m) = g_m n_m \left(1 + \frac{32}{3\sqrt{\pi}}\sqrt{n_m a_m^3}\right)$$

Huang, Yang, Lee, 1957

In harmonic traps molecular gas parameter is related to atomic parameters ($a_{ho} = \sqrt{\hbar / m\omega_{ho}}$) by

$$\sqrt{n_m a_m^3} = 0.13(k_F a)^{6/5} \quad \text{with} \quad k_F a = 1.7 N^{1/6} \frac{a}{a_{ho}}$$

BEC + beyond mean field effects in harmonic trap

Radial compression frequency becomes

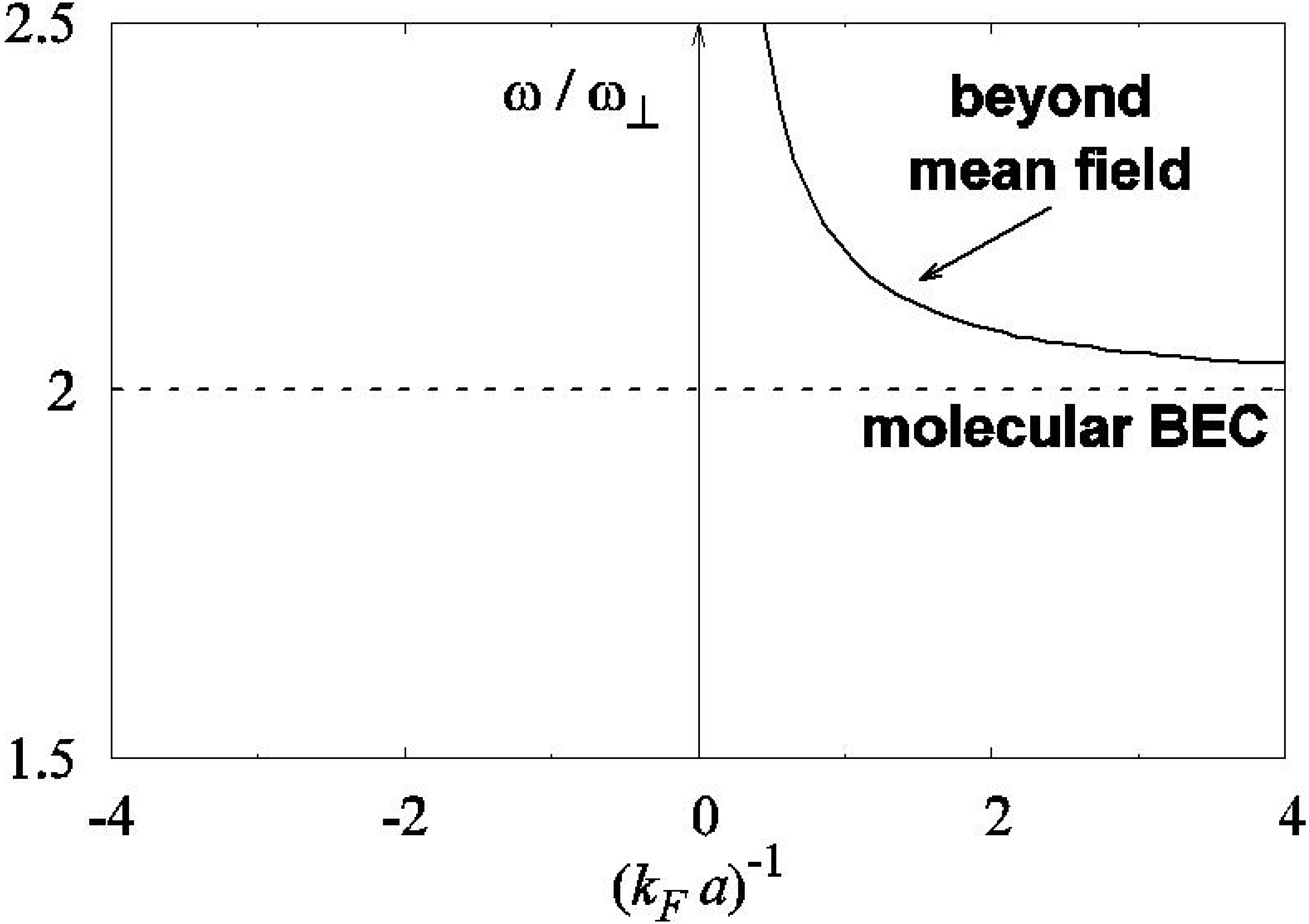
(Pitaevskii, Stringari 1998; Braaten, Pearson 1999)

$$\omega = 2\omega_{\perp}\left(1 + \frac{105\sqrt{\pi}}{256}\sqrt{n_m a_m^3}\right)$$

In terms of Fermi momentum:

$$\boxed{\omega = 2\omega_{\perp}\left(1 + 0.09(k_F a)^{6/5}\right)}$$

Radial compression mode



Unitarity limit in harmonic trap

When $k_F|a| >> 1$ the system exhibits universal behaviour, independent of sign of scattering length, (Heiselberg,2001):

$$\boxed{\mu(n) = (1+\beta)\frac{\hbar^2}{2m}(6\pi^2)^{2/3} n^{2/3}}$$

(holds at T=0 and for short range forces)

$\beta = -0.56$ (Carlson et al., 2002)

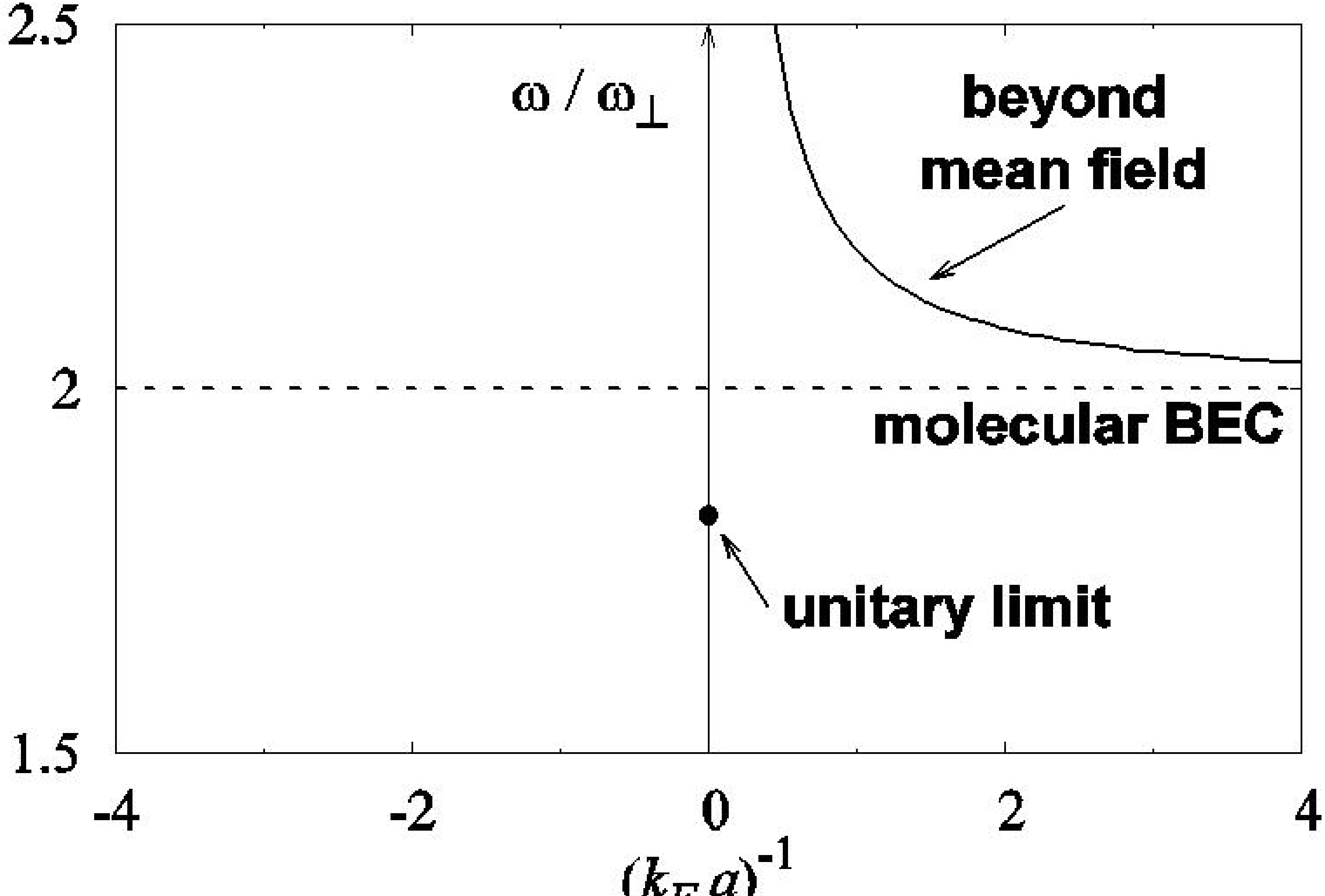
Radial mode:

$$\boxed{\omega = \sqrt{\frac{10}{3}}\omega_\perp}$$

Axial mode

$$\boxed{\omega = \sqrt{\frac{12}{5}}\omega_z}$$

Radial compression mode



BCS regime (dilute Fermi gas, a<0)

$$\mu(n) = \frac{\hbar^2}{2m}(6\pi^2)^{2/3} n^{2/3}$$

(neglected interaction corrections to eq. of state)
regime difficult to reach (very low critical Temp)

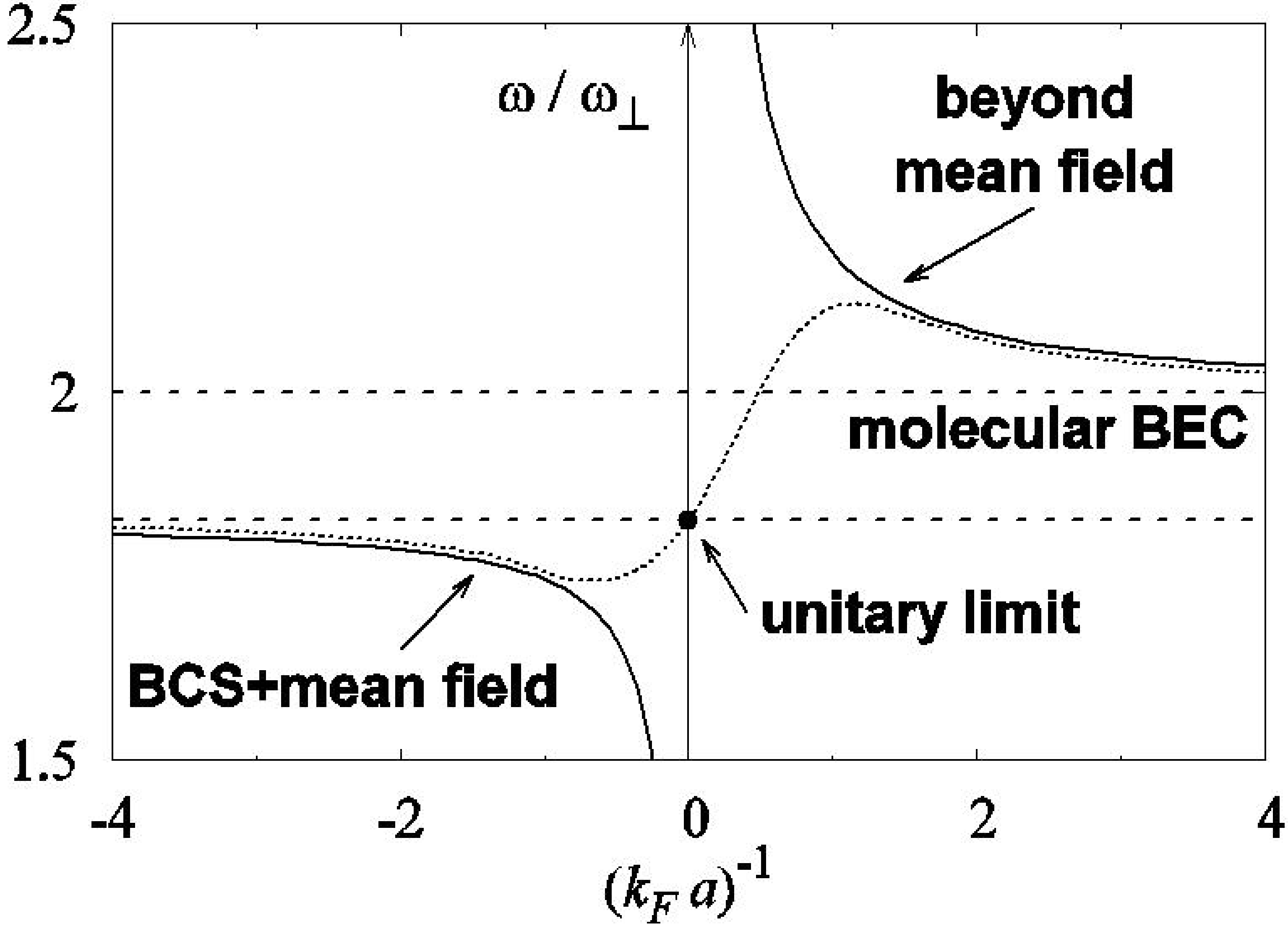
Same predictions for collective frequencies as in unitarity limit

BCS + mean field (radial mode):

$$\boxed{\omega = \sqrt{\frac{10}{3}}\omega_\perp (1 + 0.04 k_F a)}$$

Radial compression mode

(S. Stringari, cond-mat/0312614, Europhys. Lett. 65, 749 (2004))



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ROTATIONAL EFFECTS

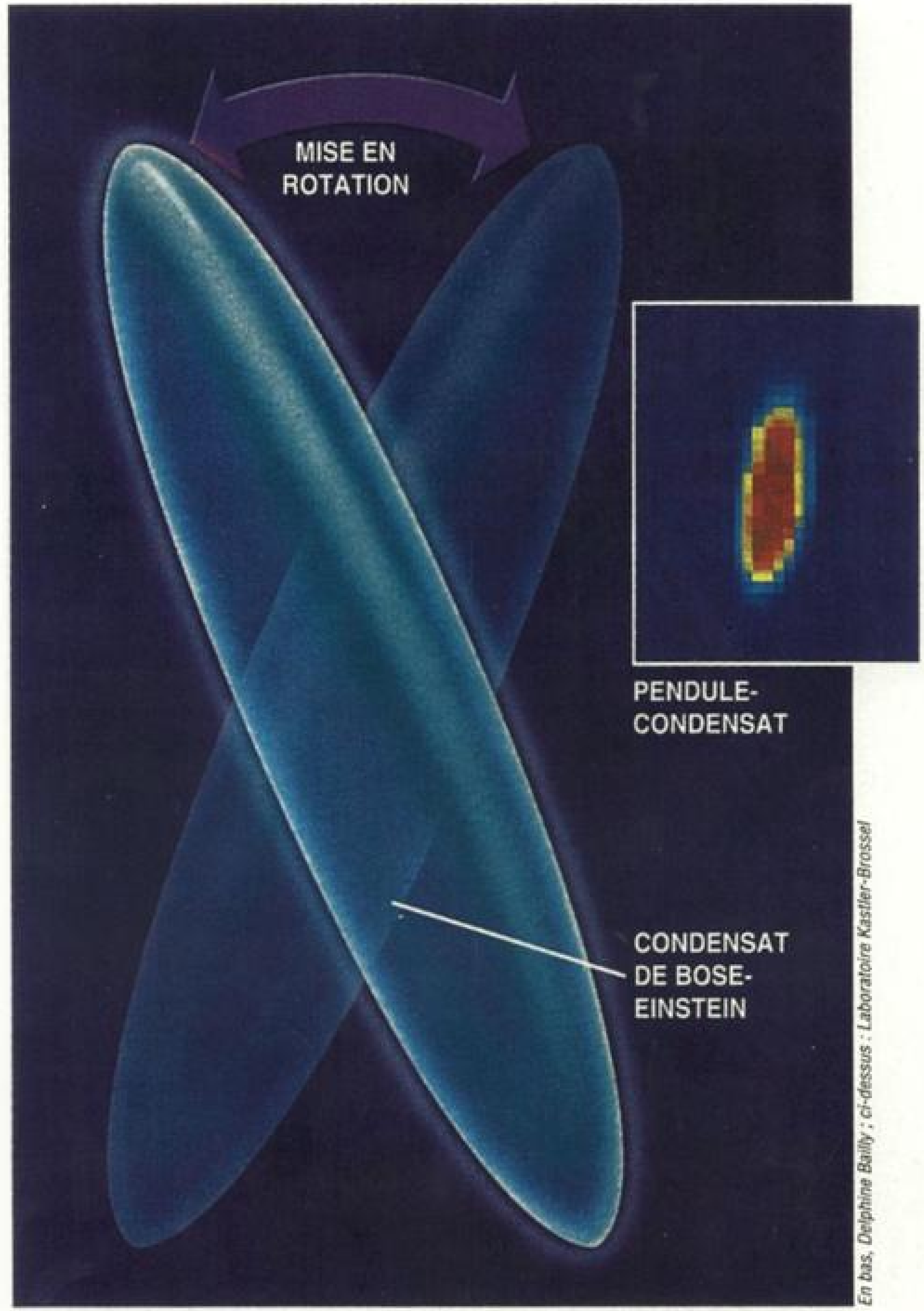
Probe transverse response
Crucial test of superfluidity

Irrotationality

↓

- Quenching of moment of inertia at low angular velocity (**scissors mode**)
- Quantization of circulation (**quantized vortices**)

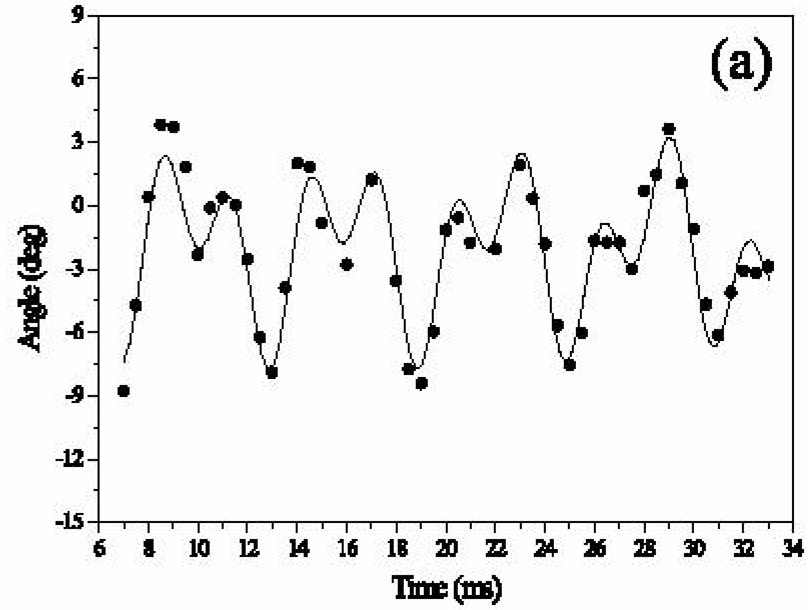
Scissors mode



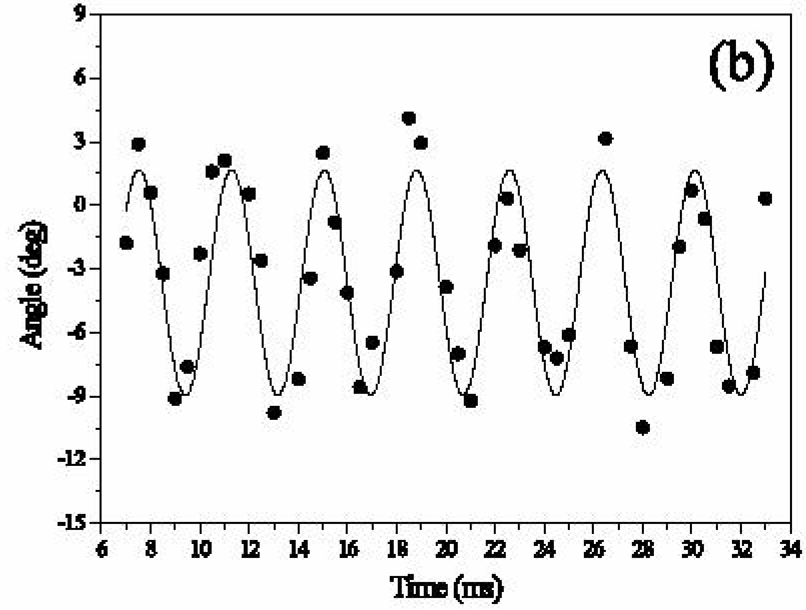
Scissors at Oxford

Marago'et al, PRL 84, 2056 (2000)

above Tc
rigid value of
moment of inertia



below Tc
irrot. value of
moment of inertia



IS THE OBSERVATION OF THE SCISSORS MODE AT FREQUENCY

$$\omega^2 = \omega_x^2 + \omega_y^2$$

DIRECT PROOF OF SUPERFLUIDITY?

- **YES** IF NORMAL GAS IS COLLISIONLESS (OXFORD EXP)
- **NO** IF NORMAL GAS IS COLLISIONAL **AND** NON ROTATING
- **YES** IF NORMAL GAS IS COLLISIONAL **AND** ROTATING

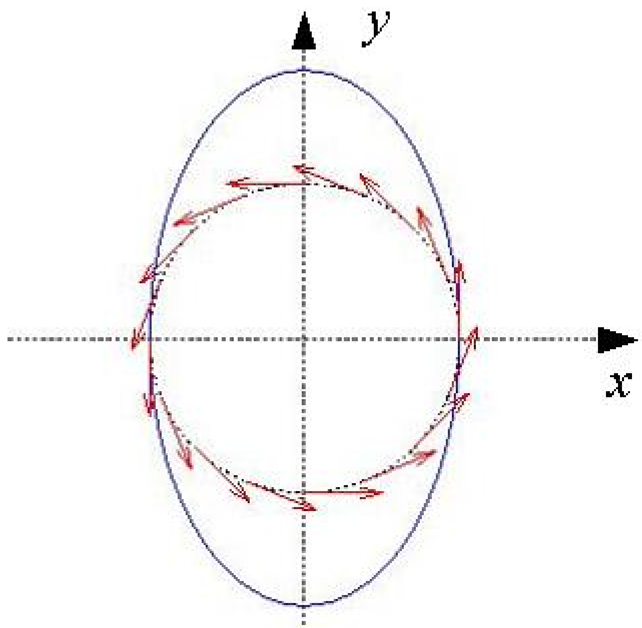
INVESTIGATE COLLECTIVE MODES IN THE PRESENCE OF ROTATING TRAPS!!

Rotating Anisotropic Trap

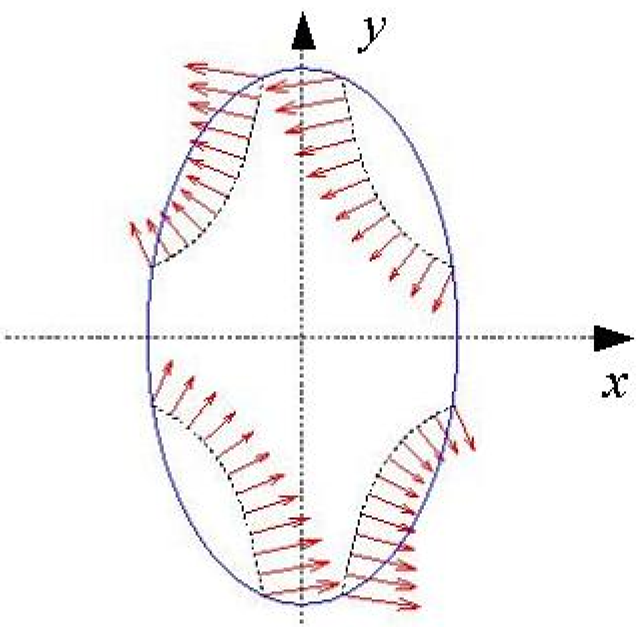
$$V_{\text{ext}} = \frac{M}{2}\left(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2\right) = \frac{M}{2}\left[\omega_\perp^2 (1+\varepsilon) x^2 + \omega_\perp^2 (1-\varepsilon) y^2 + \omega_z^2 z^2\right]$$

Rigid rotation $\mathbf{v}_0 = \boldsymbol{\Omega} \wedge \mathbf{r}$:

Irrotational flow $\mathbf{v}_0 = \alpha \nabla (xy)$:



normal



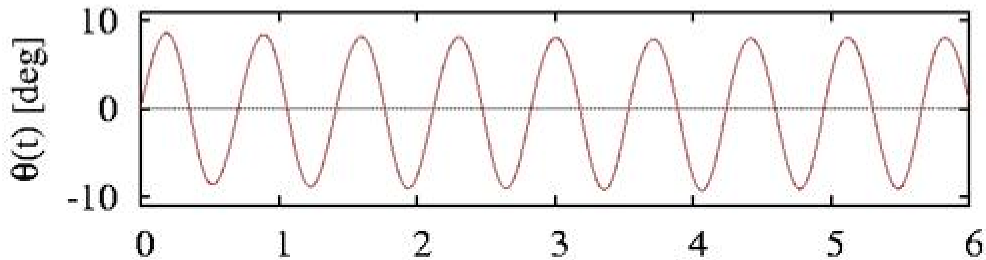
superfluid

TESTS OF **SUPERFLUIDITY**
(able to distinguish between superfluid and collisional hydrodynamics)

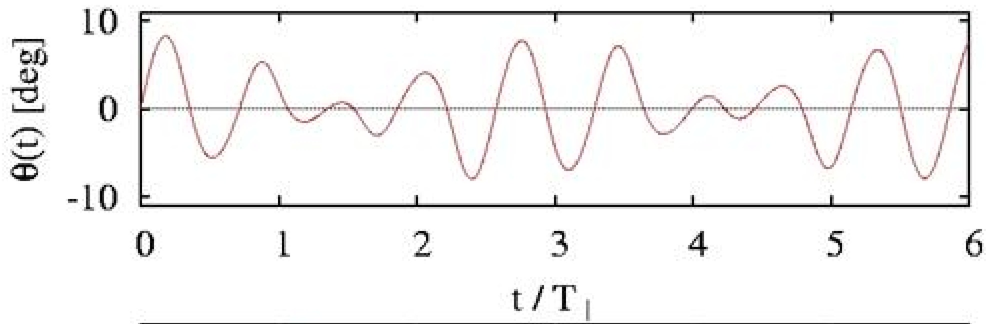
(Cozzini, Stringari, PRL 91, 070401 (2003))

- **SCISSORS MODE** IN ROTATING GAS
- **PRECESSION OF QUADRUPOLE OSCILLATION** IN ROTATING GAS

scissors mode (stopping rotation of the trap)



Superfluid (T=0)

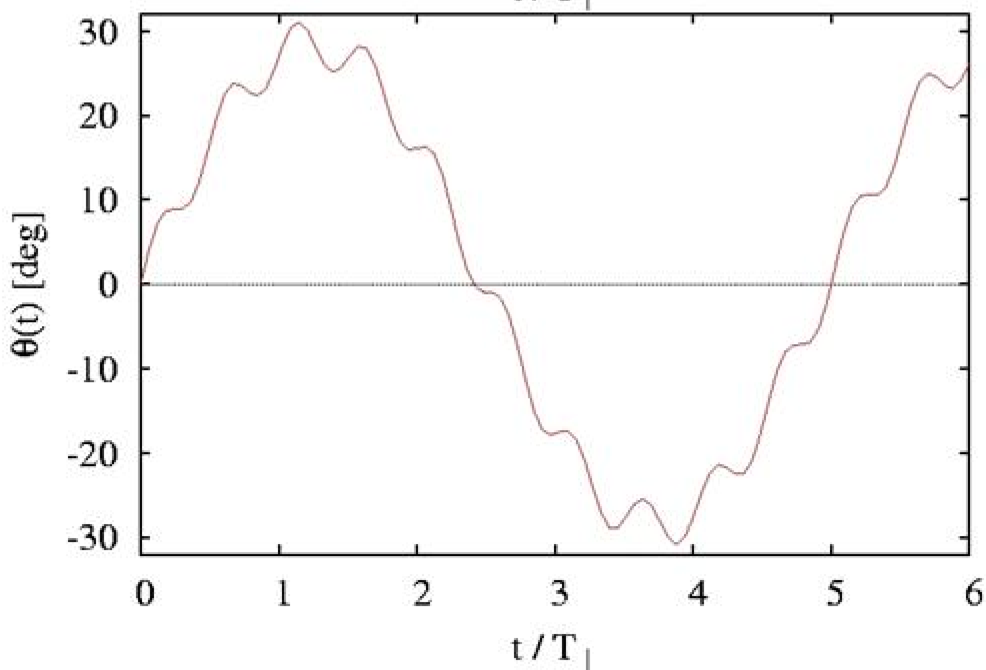


Normal (collisional)

$(\Omega >> \varepsilon^2 \omega_\perp)$

$\Omega = 0.2\omega_\perp$

$\varepsilon = 0.2$

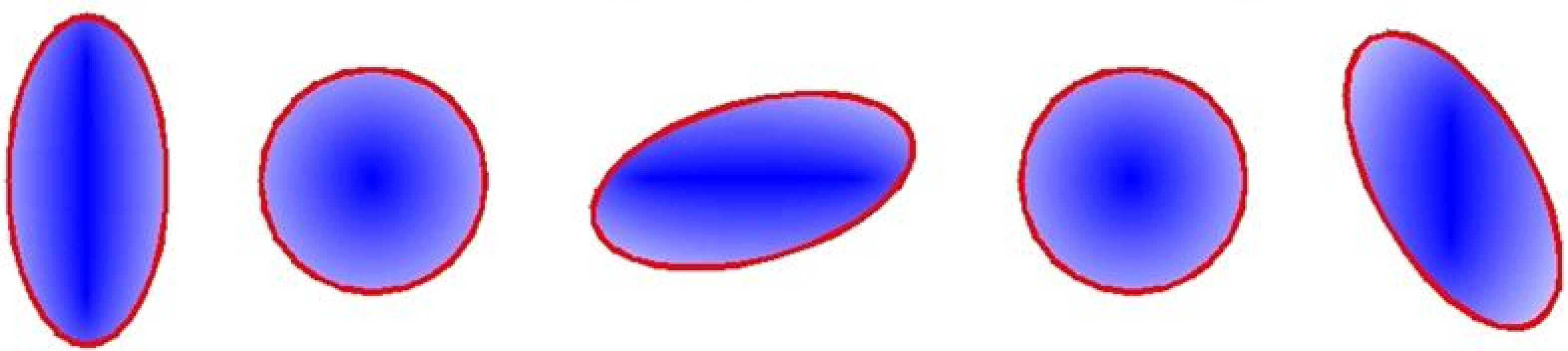


Normal (collisionless)

Precession of quadrupole oscillation after switching off deformation of rotating trap

superfluid

normal gas (collisional HD)



$$\dot{\theta} = \frac{l_z}{2m\omega^2{}_z\langle r^2{}_\perp\rangle} = \frac{\Omega}{2}$$

QUANTIZED VORTICES

- **Quantum of circulation** in a Fermi superfluid is **h/2**
- **Density change** on the vortex line **less pronounced** than in a BEC (see Bulgac, 2003 in unitarity limit)
- **Splitting of quadrupole frequencies** useful test of vortices

Zambelli, Stringari 1998
Bruun, Viverit 2001

$$\omega_+ - \omega_- = \frac{2}{M}\frac{\langle l_z \rangle}{\langle r_\perp^2 \rangle}$$

Measurement of angular momentum in BEC gas (Chevy et al., PRL 85, 2223 (2000))

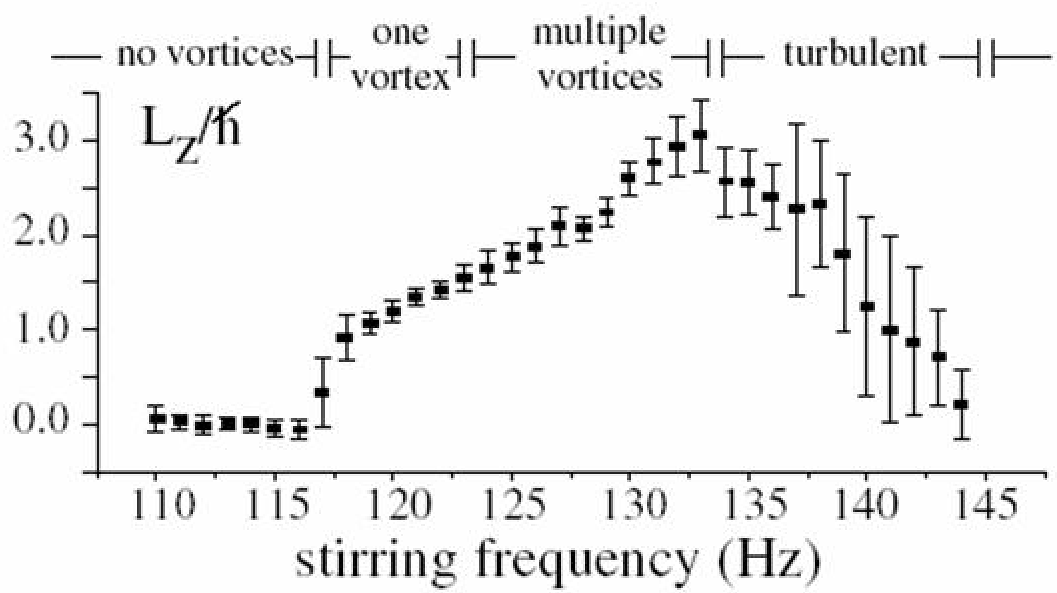


FIG. 2. Variation of the angular momentum deduced from (1) as a function of the stirring frequency Ω for $\omega_\perp/2\pi = 175$ Hz and $2.5\ (\pm 0.6) \times 10^5$ atoms.

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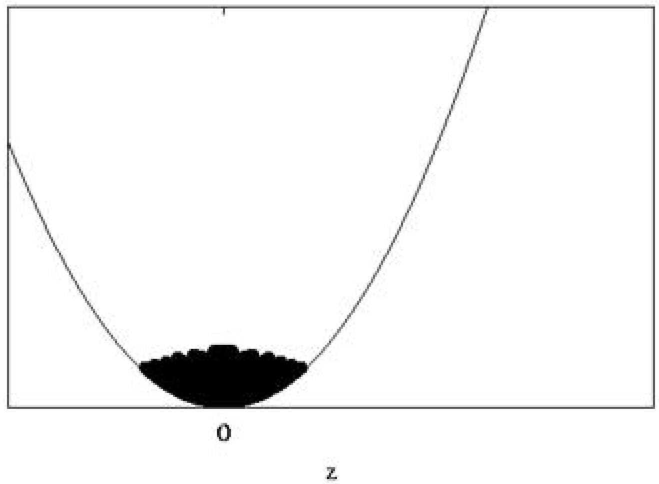
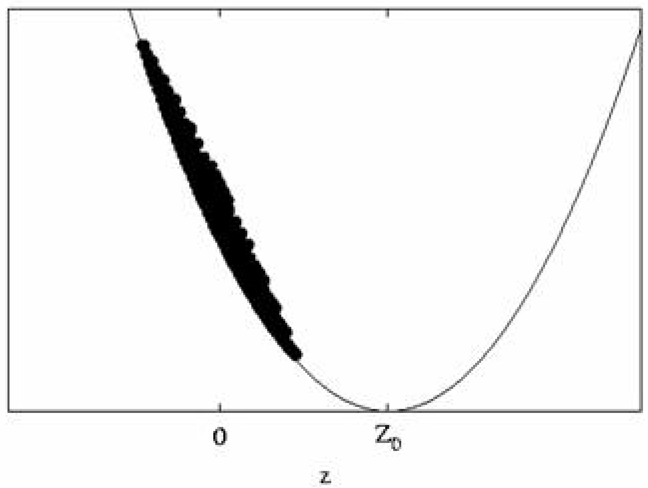
Center of mass oscillation in harmonic trap

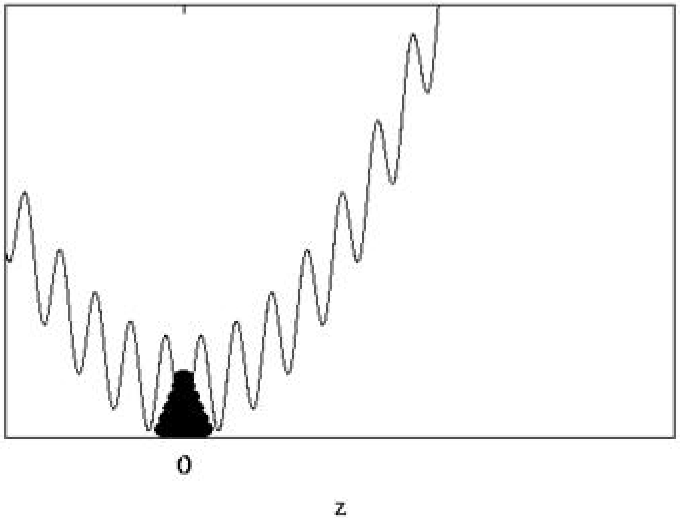


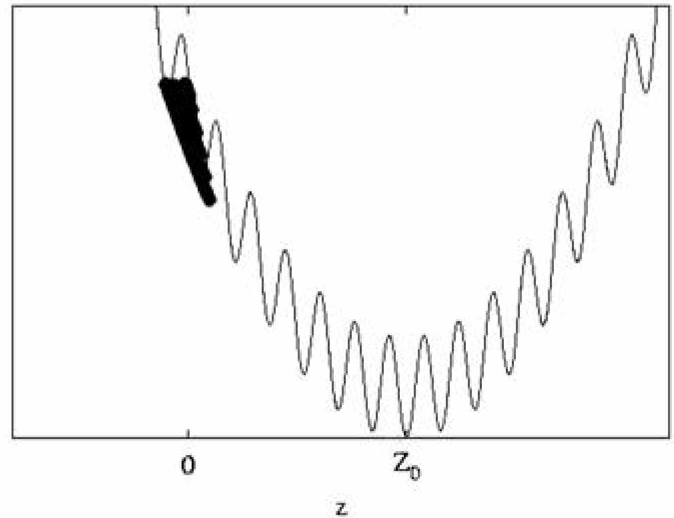


Frequency: $\omega = \omega_z$

Independent of interactions, temperature, quantum statistics (Kohn's theorem)

What happens in the presence of 1D lattice ?





DIFFERENT REGIMES

DIFFERENT ANSWERS

COLLISIONLESS NON SUPERFLUID
(single spin species)

L. Pezze' et al. cond-mat/0401643
(Florence-Trento collaboration see poster **2.13**)

- Due to non quadratic dispersion law particles with different quasi-momentum have different effective mass

- Dephasing of single particle orbits

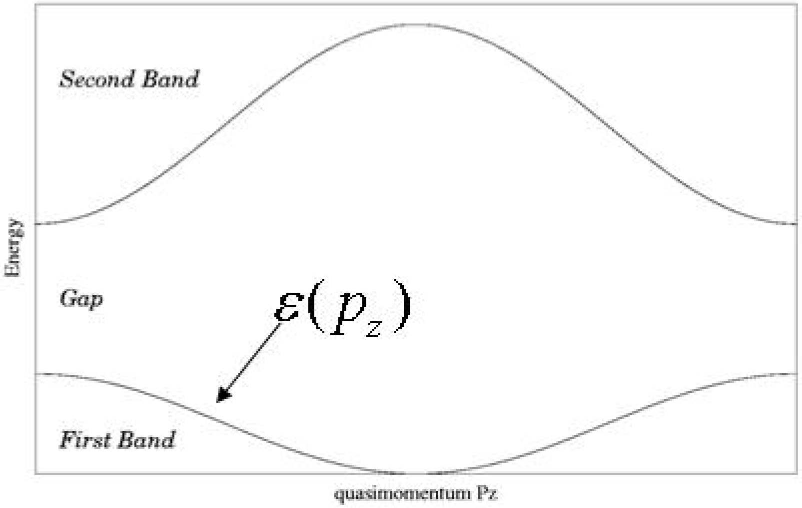
- Damping of oscillation

COLLISIONLESS NON SUPERFLUID
(single spin species)

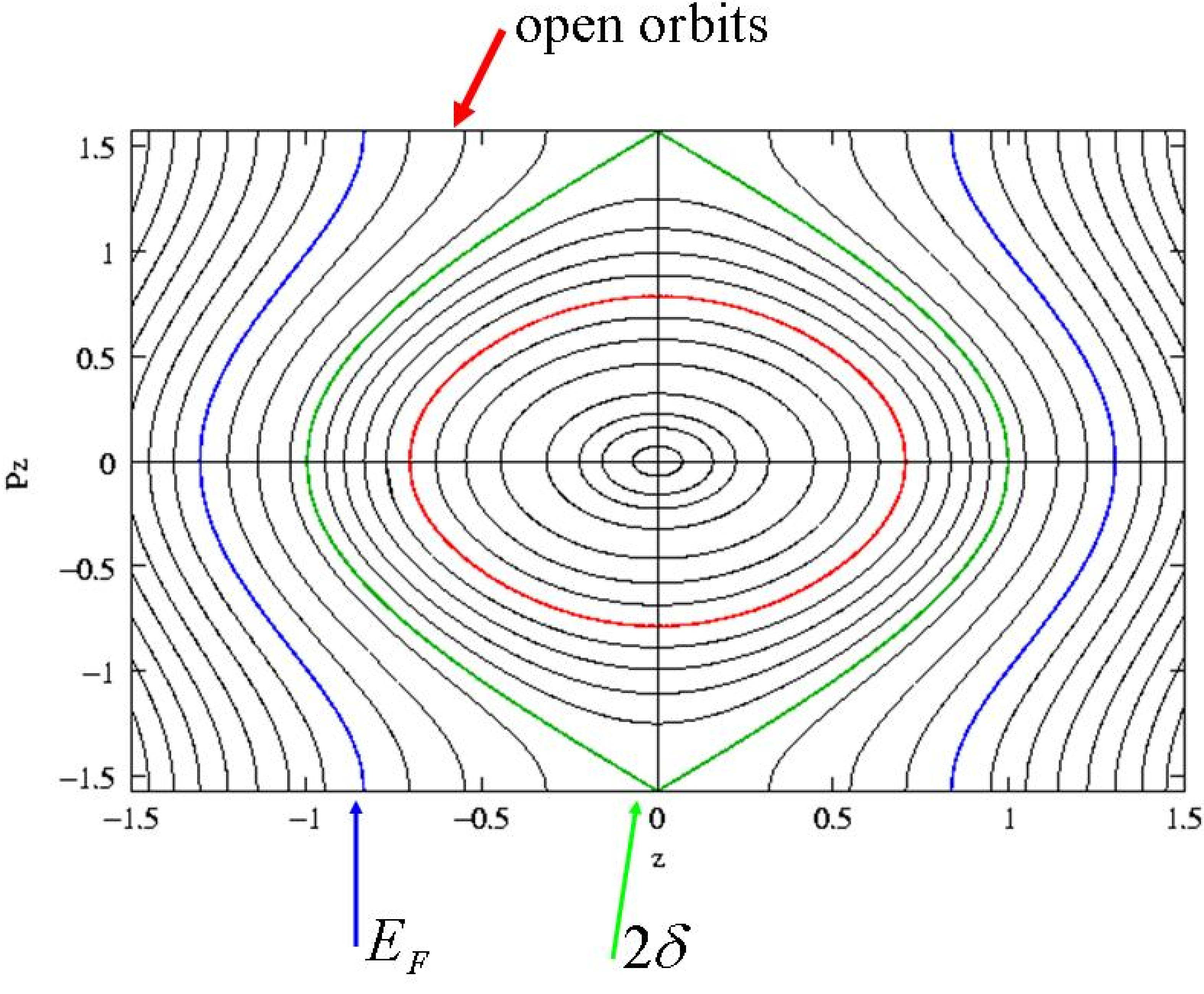
- If Fermi energy is larger than band width

$$T_F > 2\delta$$

$$\varepsilon(p_z) = 2\delta \sin^2\left(\frac{p_z d}{2\hbar}\right)$$

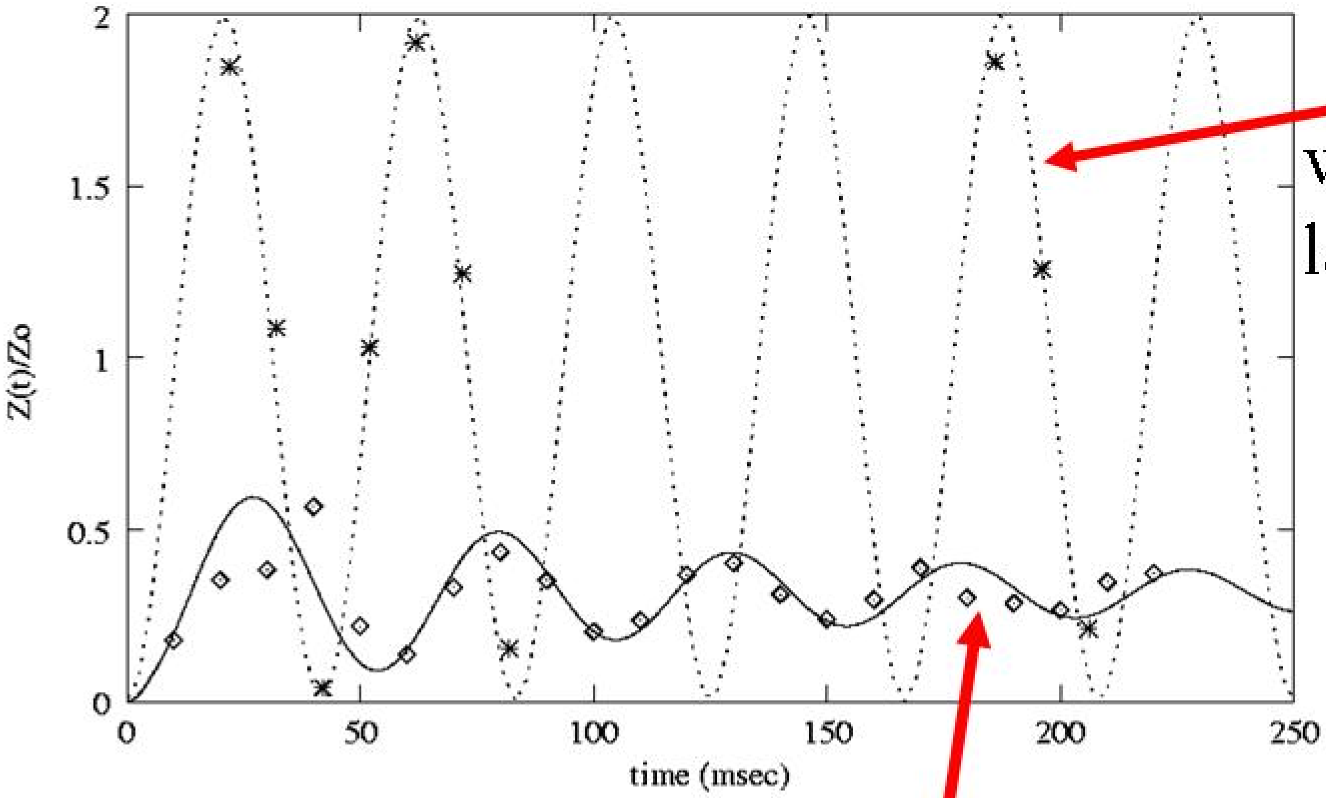


- single particle orbits at the Fermi surface are open (the gas does not oscillate around minimum of potential)
- **off-set of the oscillation** (localization)



open orbits
1.5
1
0.5
0
−0.5
−1
−1.5
pz
−1.5
−1
−0.5
0
0.5
1
1.5
z
E_F
2δ

Damping and **offset** of center of mass oscillation theory vs experiment (Lens)



without lattice

with lattice

COLLISIONAL NON SUPERFLUID
(two spin species, HD regime)

(G. Orso et al. cond-mat/0401643 poster n. **2.11**)

Equations for center of mass oscillation in hydrodynamic regime (local equilibrium ansatz)

$$f(r,p,t) = f_0((\varepsilon(r,p)+u(t)p_z)$$

Boltzmann eqs. yield:

$$\boxed{\begin{aligned} &\frac{\partial}{\partial t}Z = \frac{P_z}{m^*} \\ &\frac{\partial}{\partial t}P_z + m\omega_z^2 Z = -\frac{P_z}{\tau_{uk}} \end{aligned}}$$

Center of mass relaxation time

$$\frac{1}{\tau_{uk}} = -\frac{1}{kT}\frac{\int p_{1z}(p_{1z}+p_{2z}-p_{3z}-p_{4z})Ddp_1dp_2dp_3dp_4dr}{\int p_z^2 \partial f_0 / \partial\varepsilon(p_z)dpdr}$$

Numerator vanishes if momentum is conserved
Relaxation time determined by **umklapp collisions**

$$\boxed{p_1 + p_2 = p_3 + p_4 + 2\hbar q_B}$$

$$\boxed{q_B = \frac{\pi}{d}}$$

Bragg momentum

Ratio between umklapp and normal collisional rate depends on band filling factor $T_F / 2\delta$

If $T_F << 2\delta$ umklapp collisions are rare and center of mass oscillates undamped

If $T_F >> 2\delta$ umklapp and normal rates are comparable and center of mass oscilllation is overdamped in HD regime

Conclusion: if $T_F > 2\delta$

a normal Fermi gas in 1D optical lattice **cannot** exhibit center of mass oscillation:

In the collisionless regime due to **localization** (LENS experiment)

In the collisional regime **overdamping** due to umklapp collisions

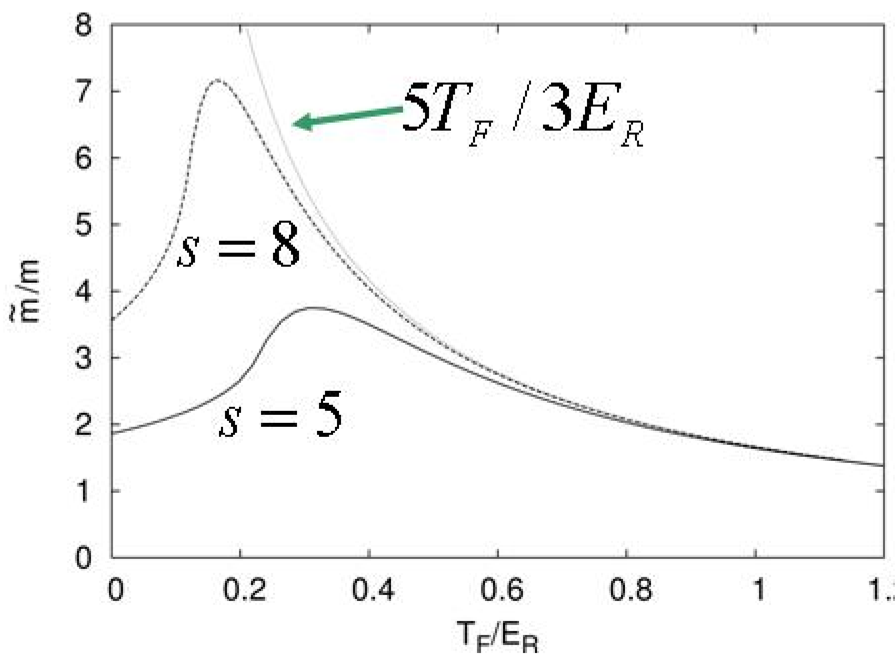
In the Fermi **superfluid** phase one expects **undamped Josephson-like oscillation** of center of mass (Wouters et al. Cond-mat/0312154, see poster n. **2.27**) similar to the case of bosons (LENS, 2001)

In BCS regime one predicts

$$\omega = \sqrt{\frac{m}{m^*}}\omega_z$$

with

$$m^* = -\frac{2}{Nh^3}\int p_z^2 \frac{\partial f_0}{\partial \varepsilon(p_z)} dp dr$$



MAIN CONCLUSIONS

- **Compression modes sensitive to equation of state** (non trivial predictions in **unitarity limit**)
- Oscillations with **rotating and periodic** traps permit to distinguish between **superfluid** and **normal** **regimes.**

OPEN PROBLEMS

- **Equation of state** along the BCS-BEC crossover
- Collective oscillations at **finite T.**