Kondo Physics and the BEC-BCS Crossover

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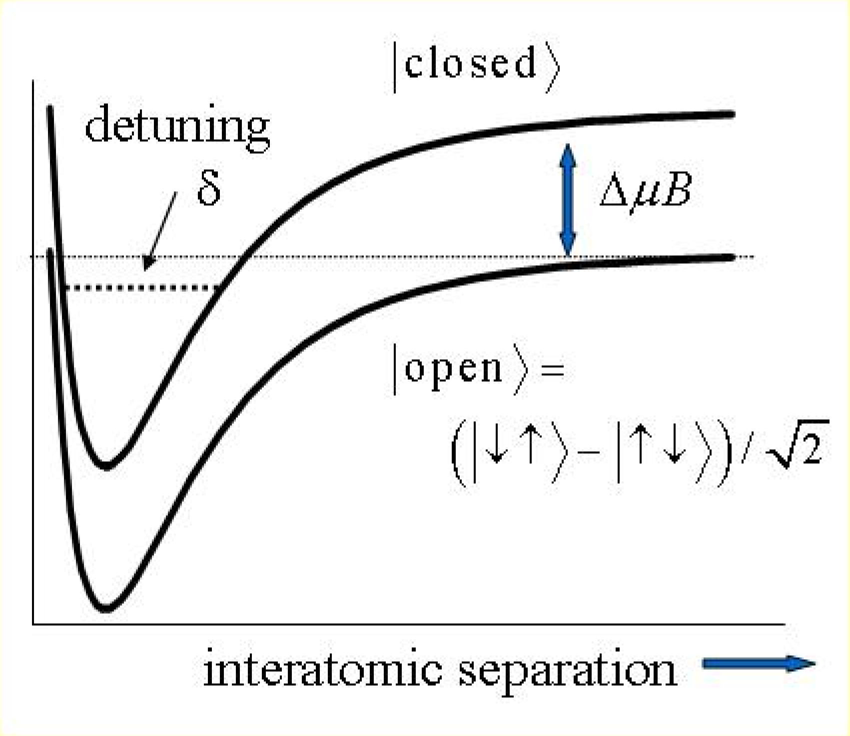
Introduction

- BEC-BCS crossover for high-temperature superconductors:
  - A. J. Leggett, J. Phys. C **41**, 7 (1980).
  - P. Nozières and S. Schmitt-Rink, J. Low Temp. Phys. **59**, 195 (1985).
  - C. A. R. Sáde Melo *et al.*, Phys. Rev. Lett. **71**, 3202 (1993).
- In atomic gases with a Feshbach resonance:
  - E. Timmermans *et al.*, Phys. Lett. A **285**, 228 (2001).
  - Y. Ohashi and A. Griffin, Phys. Rev. Lett. **89**, 140402 (2002).
  - J. N. Milstein *et al.*, Phys. Rev. A **66**, 043604 (2002).
- Our work:
  - R. A. Duine and H. T. C. Stoof, J. Opt. B: Quantum Semiclass. Opt. **5**, S212 (2003).
  - G. M. Falco *et al.*, Phys. Rev. Lett. **92** (2004); cond-mat/0304489.
  - G. M. Falco and H. T. C. Stoof, cond-mat/0402579.

Feshbach Resonance

- Open channel coupled to closed channel by hyperfine interaction
- Closed channel has (bare) bound state close to two-atom continuum threshold.
- Energy difference between two atoms and bare molecular state



$$\boxed{\delta(B) = \Delta\mu\left(B - B_0\right)}$$

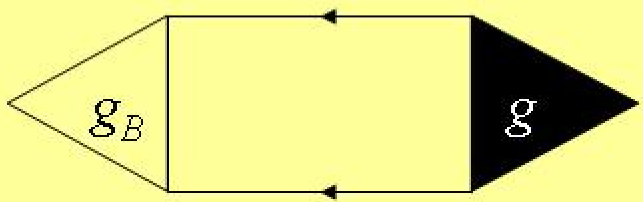
is experimentally tunable!

Effective Atom-Molecule Theory

- Atomic field operators coupled to molecular field operator as follows:

$$i\hbar\frac{\partial\hat{\psi}_{\sigma}(\vec{x},t)}{\partial t}=\left[-\frac{\hbar^2\nabla^2}{2m}+T^{2B}_{bg}\hat{\psi}^{\dagger}_{-\sigma}(\vec{x},t)\hat{\psi}_{-\sigma}(\vec{x},t)\right]\hat{\psi}_{\sigma}(\vec{x},t)+\sigma g\hat{\psi}^{\dagger}_{-\sigma}(\vec{x},t)\hat{\psi}_{m}(\vec{x},t)$$

$$i\hbar\frac{\partial\hat{\psi}_{m}(\vec{x},t)}{\partial t}=\left[-\frac{\hbar^2\nabla^2}{2(2m)}+\delta(B)-i\eta\sqrt{i\hbar\frac{\partial}{\partial t}+\frac{\hbar^2\nabla^2}{2(2m)}}\right]\hat{\psi}_{m}(\vec{x},t)+g\hat{\psi}_{\downarrow}(\vec{x},t)\hat{\psi}_{\uparrow}(\vec{x},t)$$

$\hbar\Sigma_m(\vec{0},\hbar\omega)=$ 

$\simeq -\boxed{g^2\frac{\sqrt{m^3}}{4\pi\hbar^3}}\, i\sqrt{\hbar\omega}$

$\boxed{\eta}$

Molecular Bound-State Energy

- Obtained from the poles of the propagator:

$$G_m^{(+)}(\vec{0},\omega) = \frac{\hbar}{\hbar\omega^+ - \delta(B) + i\eta\sqrt{\hbar\omega}}$$

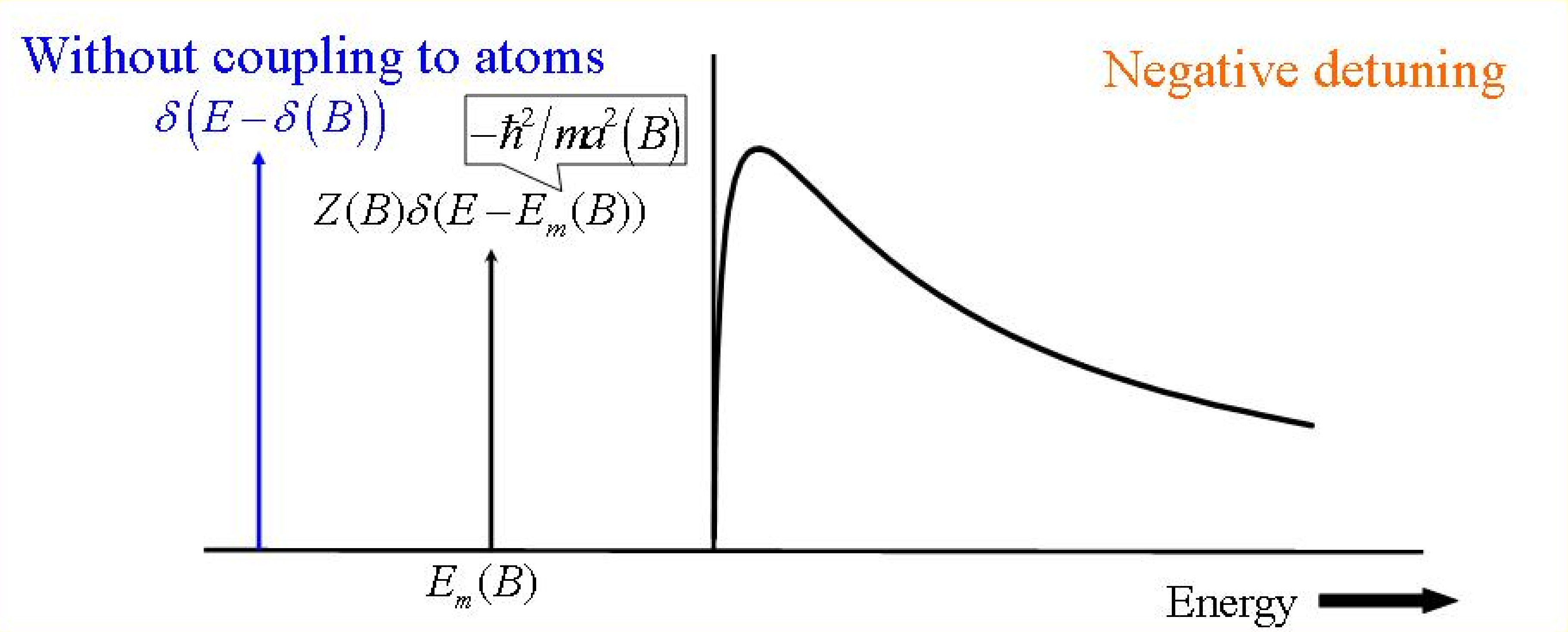
- This propagator has for negative detuning a pole at:

$$E_m(B) = \delta(B) + \frac{\eta^2}{2}\left[\sqrt{1 - \frac{4\delta(B)}{\eta^2}} - 1\right]$$

- Close to resonance:

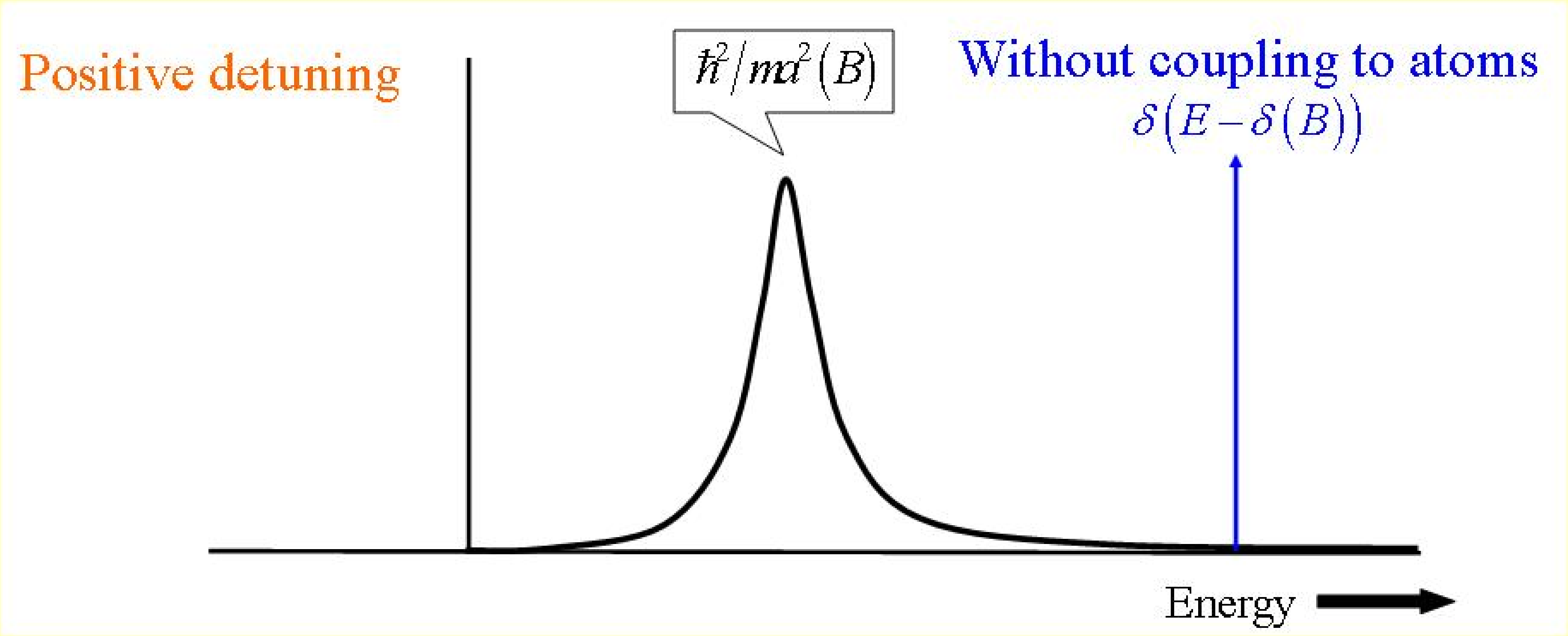
$$E_m(B) = \frac{\delta^2(B)}{\eta^2} = -\frac{\hbar^2}{ma^2(B)}$$

Molecular Density of States (I)



$$\left|\chi_m;\text{dressed}\right\rangle = \sqrt{Z(B)}\left|\chi_m;\text{closed}\right\rangle + \sum_{\vec{k}} C_{\vec{k}}\left|\vec{k},-\vec{k};\text{open}\right\rangle$$

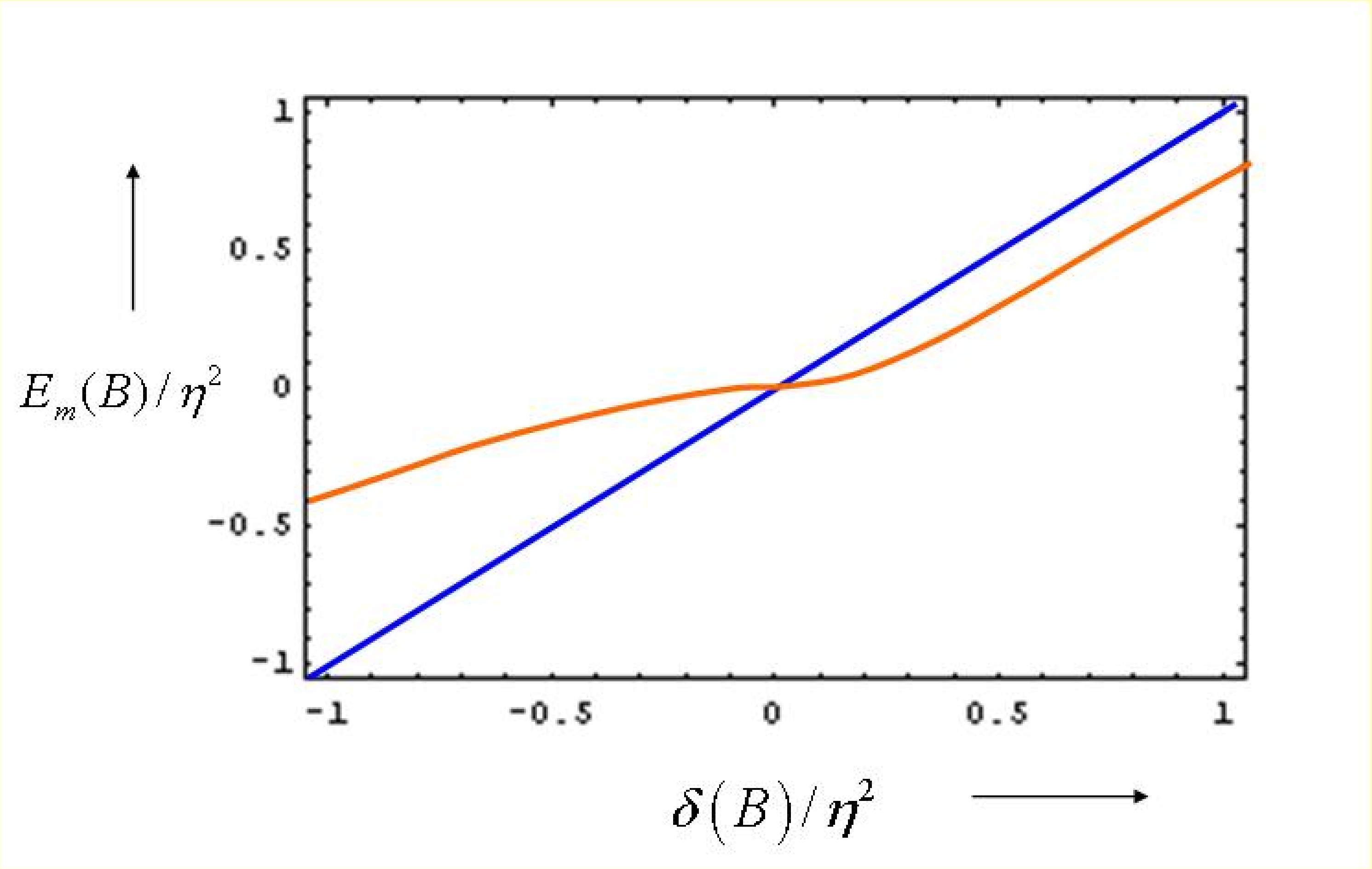
Molecular Density of States (II)



$$E_m(B)=\frac{1}{3}\left(\delta(B)-\frac{\eta^2}{2}+\sqrt{\frac{\eta^2}{4}-\eta^2\delta(B)+4\delta^2(B)}\right)$$

Molecular Energy Level

- Neglecting finite lifetime effects:



Coherent Atom-Molecule Oscillations

- Coherent atom-molecule oscillations observed in the two-pulse experiments by Donley *et al.* [Nature **417**, 531 (2002)] are understood as Josephson oscillations between atomic and molecular condensates.
- Mean-field theory:

$$i\hbar\frac{\partial\phi_a}{\partial t}=\left[-\frac{\hbar^2\nabla^2}{2m}+T_{bg}^{2B}\left|\phi_a\right|^2\right]\phi_a+2g\phi_a^*\phi_m$$

$$i\hbar\frac{\partial\phi_m}{\partial t}=\left[-\frac{\hbar^2\nabla^2}{4m}+\delta(B(t))-g^2\frac{\sqrt{m^3}}{2\pi\hbar^3}i\sqrt{i\hbar\frac{\partial}{\partial t}+\frac{\hbar^2\nabla^2}{4m}-2\hbar\Sigma^{HF}}\right]\phi_m+g\phi_a^2$$

- Oscillation frequency is found by linearization. This takes into account the fractional derivative exactly.

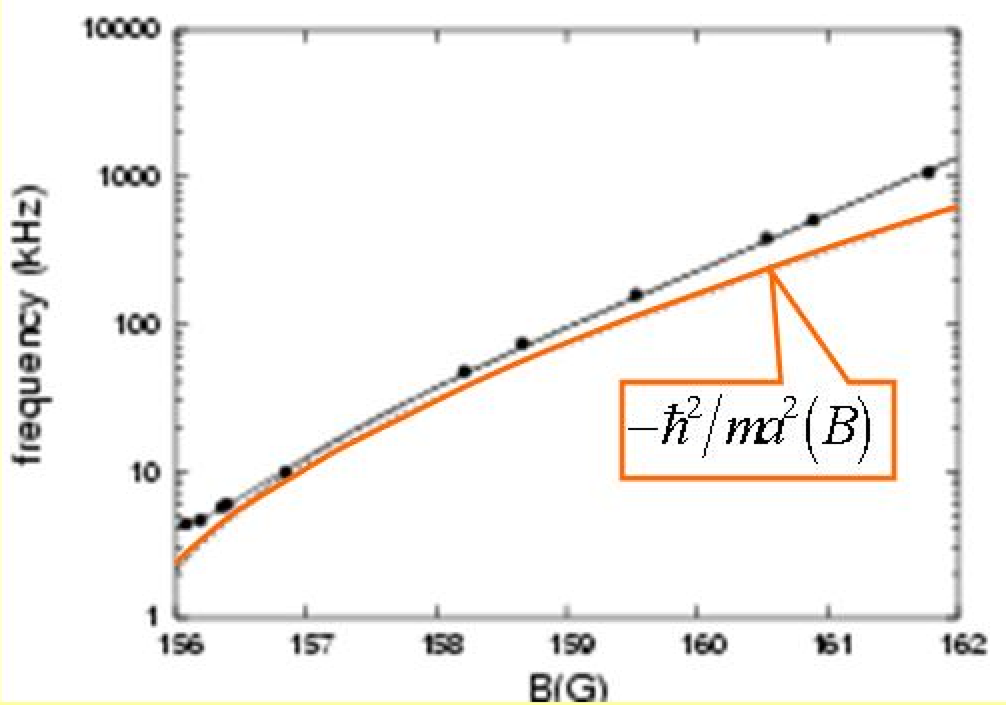
Mean-field energy barrier due to atomic condensate.

Josephson Oscillations

Josephson frequency:

$$\hbar\omega_J = \sqrt{(coupling)^2 + \left(E_m(B)\right)^2}$$

$$\boxed{16Z(B)g^2n_a}$$



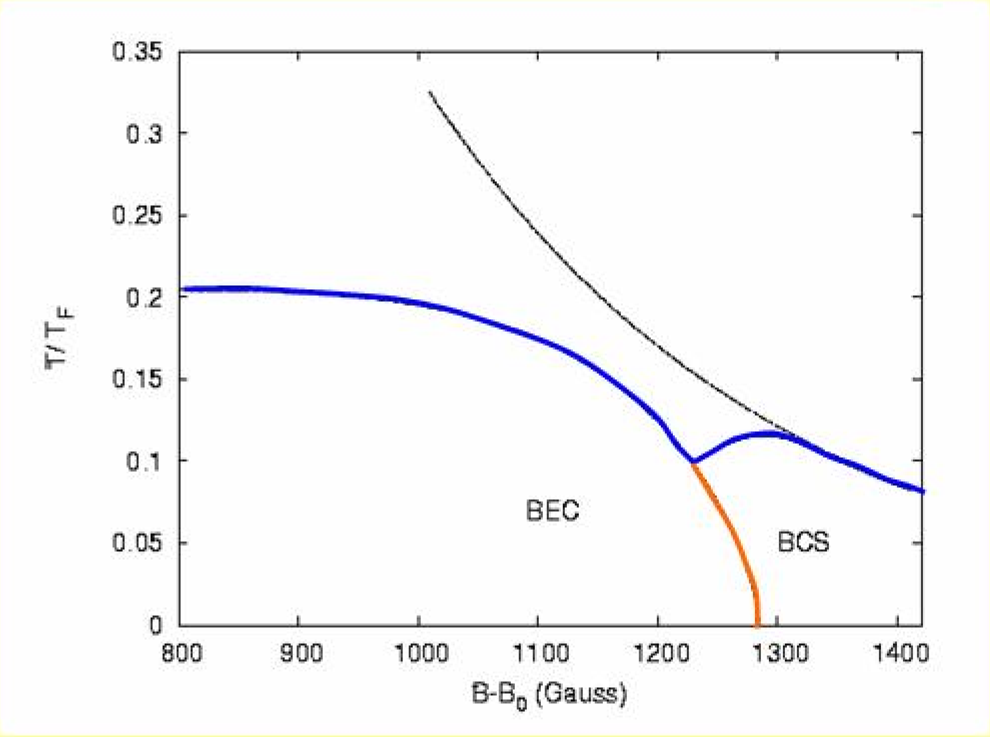
- Excellent agreement with experiments by N.R. Claussen *et al.* [Phys. Rev. A **67**, 060701 (2003)].
- For large detuning reduce to two-body result (we have to include energy dependence of effective interactions).
- Condensate density n_a=2 10^{12} cm^{-3}.

BEC-BCS Crossover (I)

- For a Feshbach resonance the crossover is determined by the pair wave function

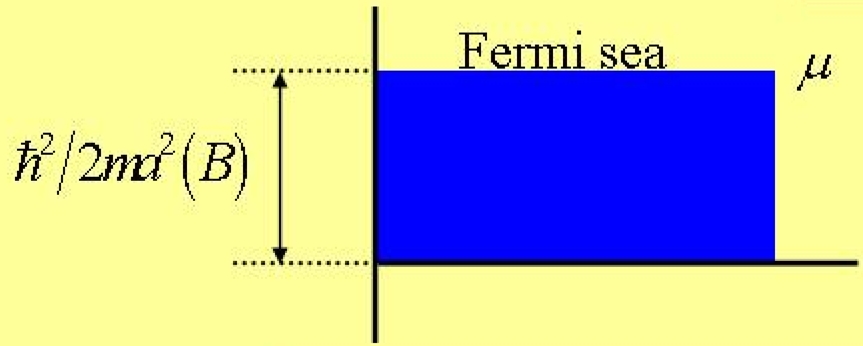
$$\boxed{\sqrt{Z(\delta)}\chi_m(\vec{x},\vec{x}')\left|\text{closed}\right\rangle + \sqrt{1-Z(\delta)}\chi_{aa}(\vec{x},\vec{x}';\delta)\left|\text{open}\right\rangle}$$

- For potassium-40 experiment by Regal *et al.* this leads to:



BEC-BCS Crossover (II)

- For the potassium-40 experiment of Regal *et al.* [Phys. Rev. Lett. **92**, 040403 (2004)] we have $\boxed{\eta^2 \simeq 2\mu}$ and therefore



- This gives at T=0

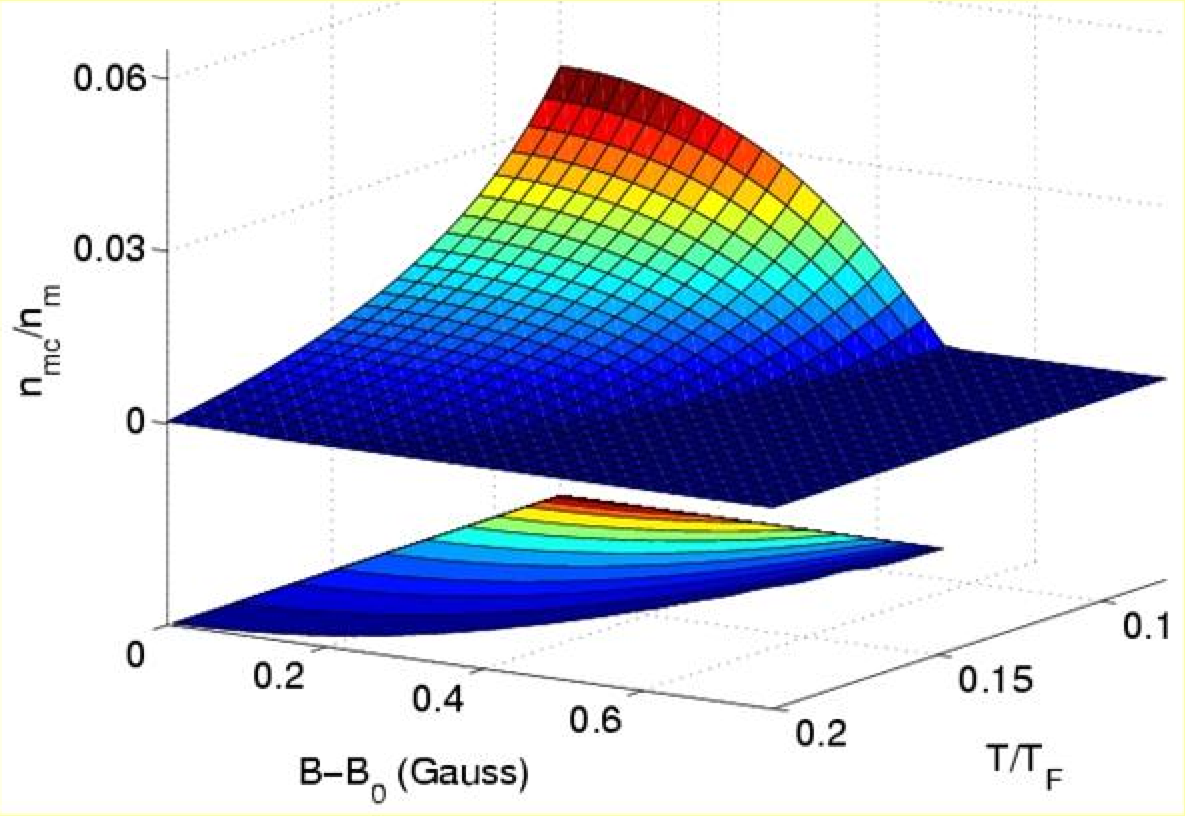
$$\boxed{n_{mc} \simeq \frac{n}{2}\left(1-\frac{1}{(k_F\,|\,a(B)\,|)^3}\right)}$$

- and thus

$$\boxed{\frac{T}{T_F} \simeq 2\pi\left\{\frac{1}{6\pi^2\varsigma(3/2)}\left(1-\frac{1}{(k_F\,|\,a(B)\,|)^3}\right)\right\}^{2/3}}$$

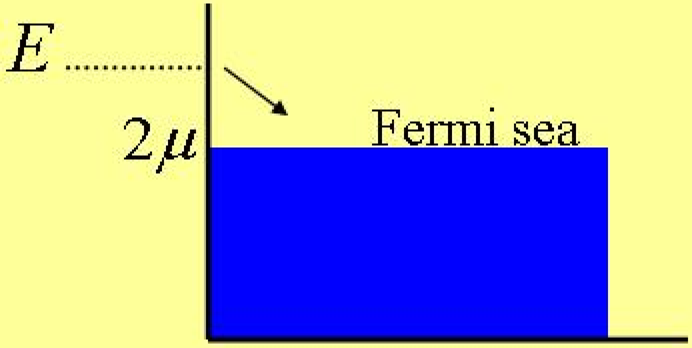
BEC-BCS Crossover (III)

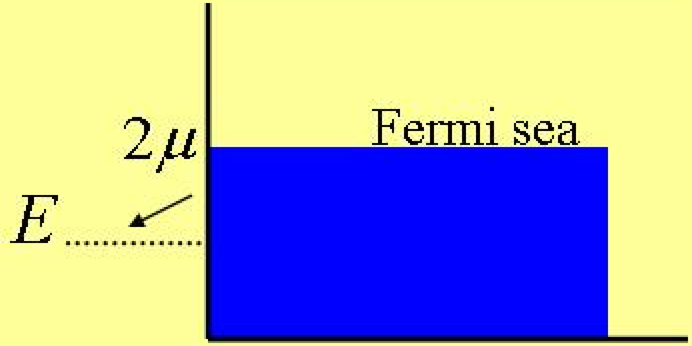
- To calculate the molecular condensate density, we use the Bogoliubov approximation. We ultimately find:

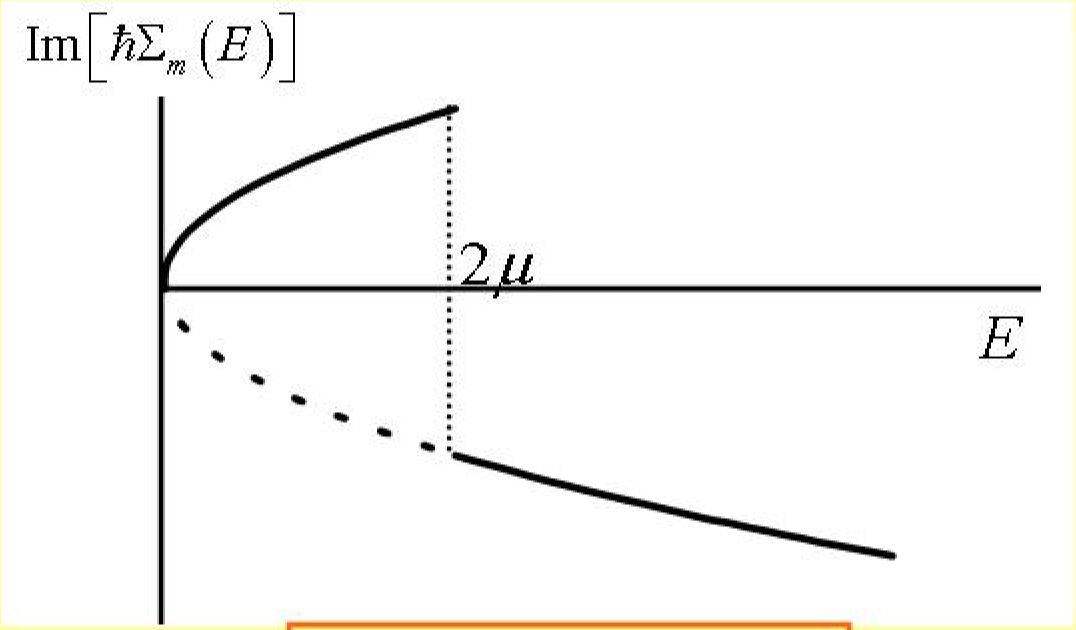


Molecular Self-energy Revisited

- Effects of Fermi-sphere on molecules important: molecule decays only if its energy is larger than Fermi energy

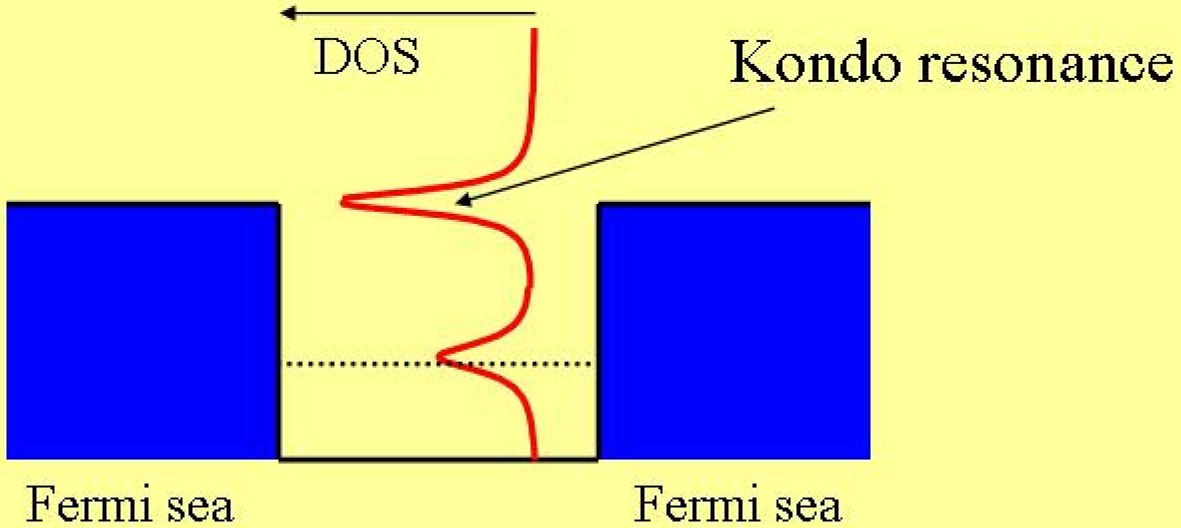






Real part follows from Kramers-Kronig

Kondo Physics in Quantum Dots



Quantum dot $\Longleftrightarrow$ Molecule (Boson)

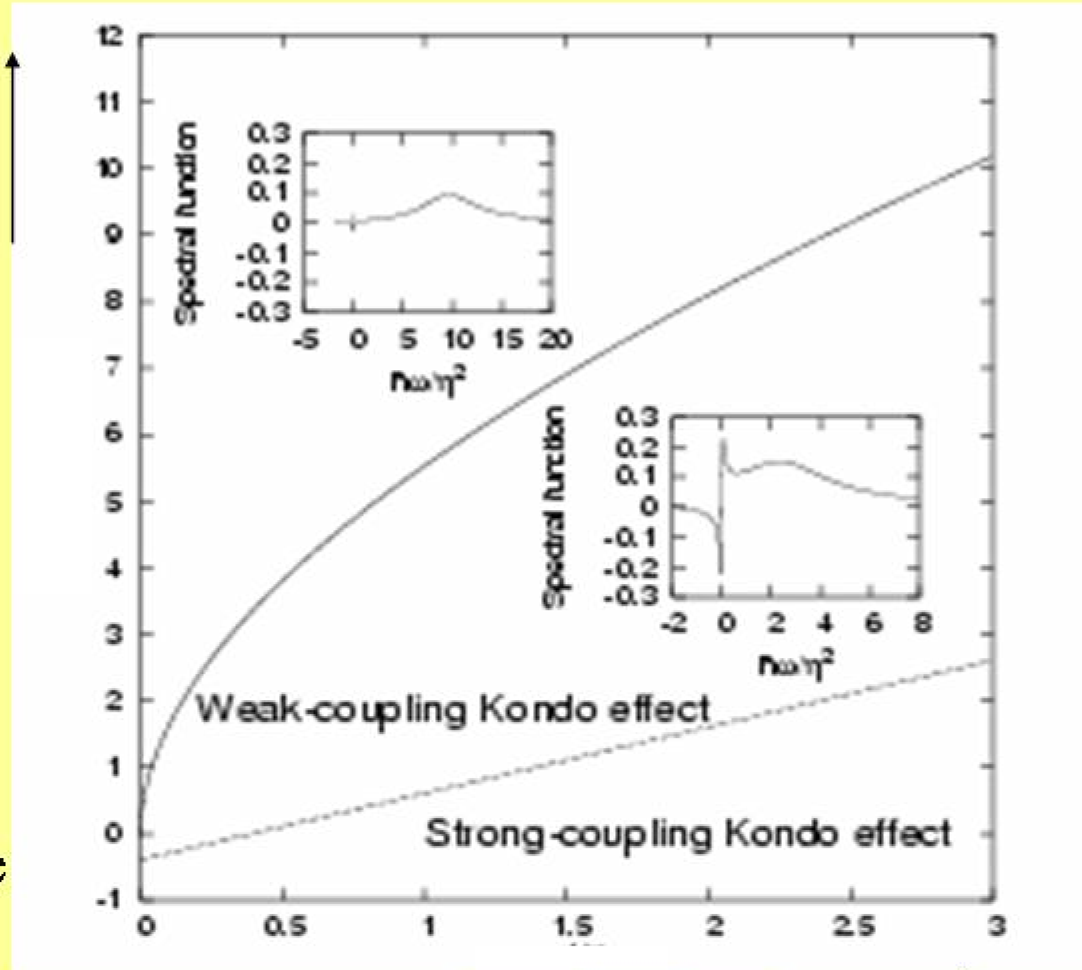
Conduction electrons $\Longleftrightarrow$ Atomic Fermi gas

D. Goldhaber-Gordon *et al.*, Nature **391**, 156 (1998).

S.M. Cronenwett *et al.*, Science **281**, 540 (1998).

Molecular Spectral Function

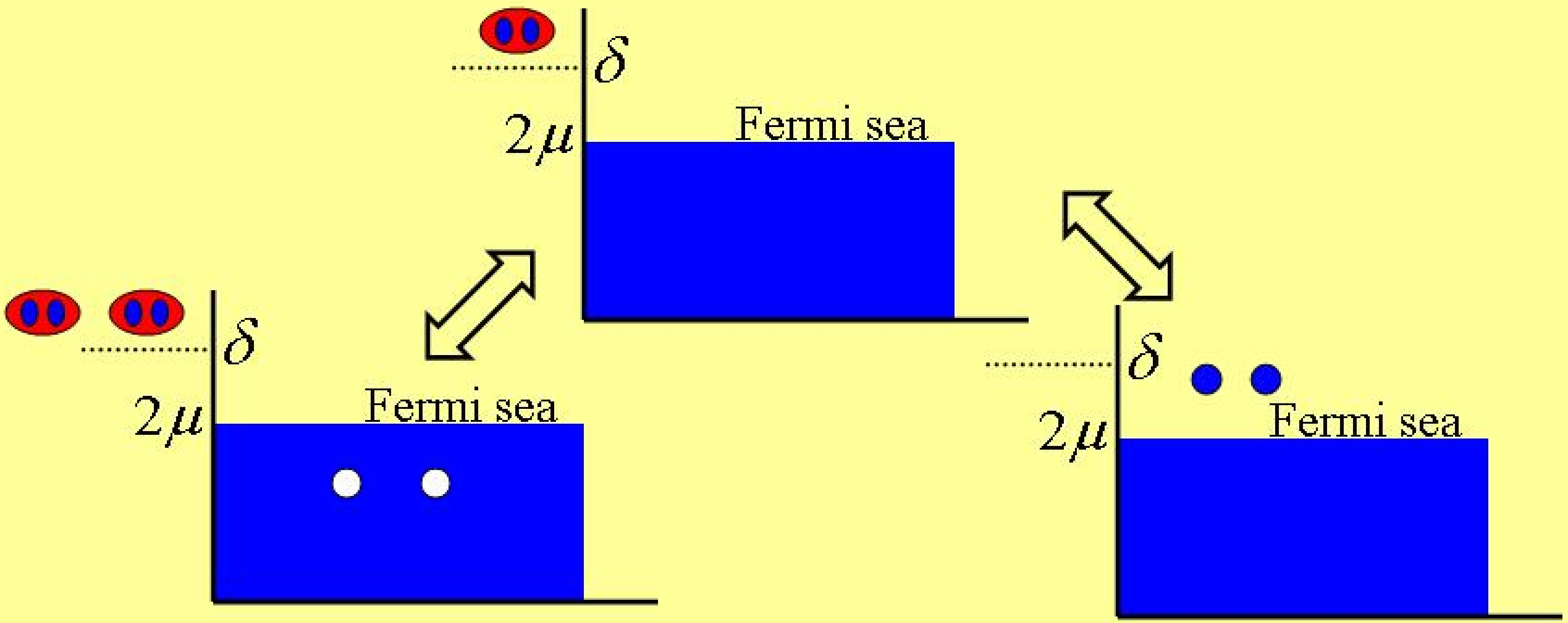
$\delta(B)/\eta\sqrt{2\mu}$



$\sqrt{2\mu}/\eta$

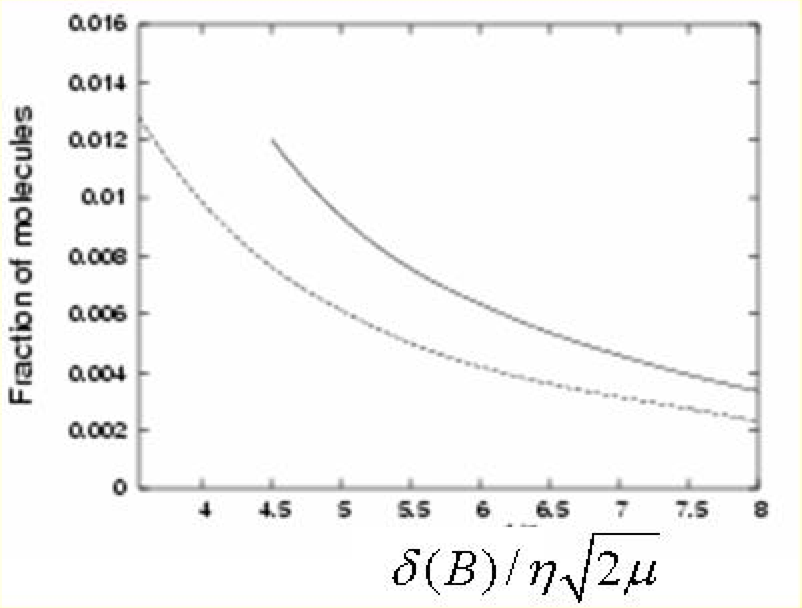
- Large detuning: no visible Kondo resonance.
- Small detuning: Kondo resonance.
- Molecule is a boson. This results in negative spectral weight.

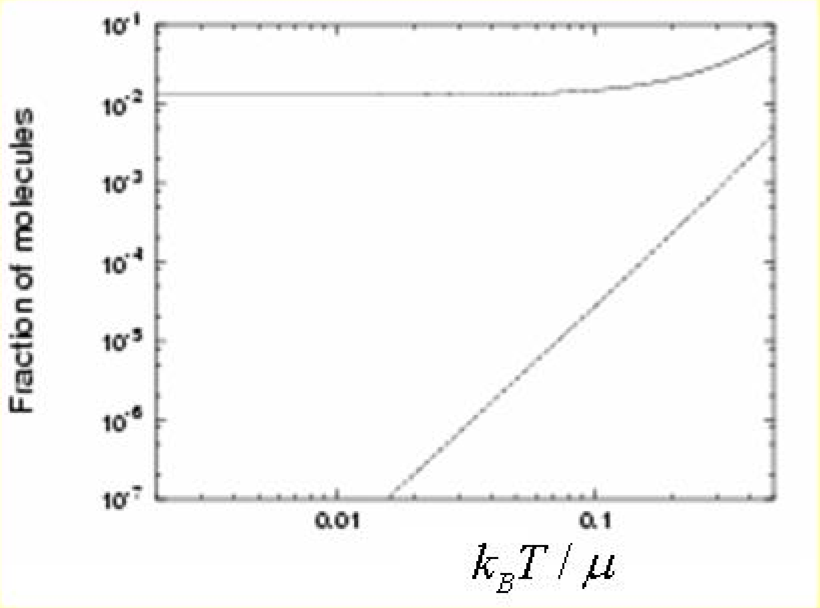
Kondo-Resonant State



$$\left|\Psi_K\right\rangle = \sqrt{Z}\, b_{\vec{0}}^{\dagger}\left|\Phi_F\right\rangle + \sum_{\vec{k}>\vec{k}_F} u_{\vec{k}}\, c_{\vec{k},\uparrow}^{\dagger} c_{-\vec{k},\downarrow}^{\dagger}\left|\Phi_F\right\rangle + \sum_{\vec{k}<\vec{k}_F} v_{\vec{k}}\, b_{\vec{0}}^{\dagger} b_{\vec{0}}^{\dagger} c_{\vec{k},\uparrow} c_{-\vec{k},\downarrow}\left|\Phi_F\right\rangle$$

Number of Molecules





- Even molecules at zero temperature.
- Significant enhancement in number of bosons. This can be observed experimentally by photodissociation as shown by C. Chin *et al*. [PRL **90**, 033201 (2003)].

Outlook

- Determine BEC-BCS crossover in an optical trap.
- Kondo temperature is larger than the critical temperature of the BCS transition. Consider BCS-BEC crossover including Kondo physics and finite lifetime effects of the molecules.
- Include particle-hole excitations around Fermi surface, i.e., strong-coupling limit.
- Formation of cold fermionic atoms by applying a magnetic field sweep to a molecular Bose-Einstein condensate.
- Is spectroscopic detection of BCS transition possible?

Microscopic Atom-Molecule Hamiltonian

$$\hat{H} = \int d\vec{x}\hat{\psi}_a^\dagger(\vec{x})\left\{-\frac{\hbar^2\nabla^2}{2m} + \int d\vec{x}'\hat{\psi}_a^\dagger(\vec{x}')V_{\downarrow\downarrow}(\vec{x}-\vec{x}')\hat{\psi}_a(\vec{x}')\right\}\hat{\psi}_a(\vec{x})$$

$$+\int d\vec{x}\hat{\psi}_m^\dagger(\vec{x})\left[-\frac{\hbar^2\nabla^2}{2(2m)} + \delta(B)\right]\hat{\psi}_m(\vec{x})$$

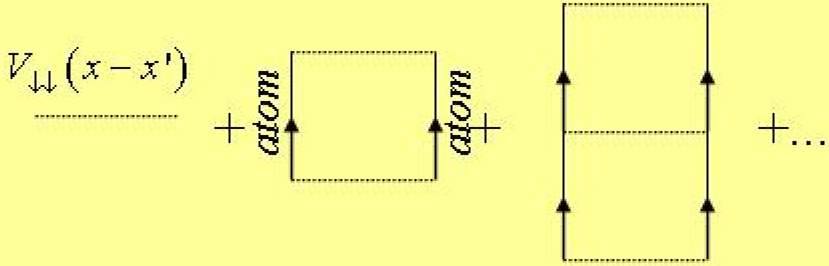
$$+\int d\vec{x}\int d\vec{x}' g_B^*(\vec{x}-\vec{x}')\hat{\psi}_m^\dagger\left(\frac{\vec{x}+\vec{x}'}{2}\right)\hat{\psi}_a(\vec{x})\hat{\psi}_a(\vec{x}') + h.c.$$

Microscopic (bare) atom-molecule coupling: $g_B(\vec{x}) = \frac{V_{\uparrow\downarrow}(\vec{x})}{\sqrt{2}}\chi_m(\vec{x})$

For applications with a dilute atomic gas, we now have to take into account all two-atom collisions. This means that we have to sum all ladder (Feynman) diagrams.

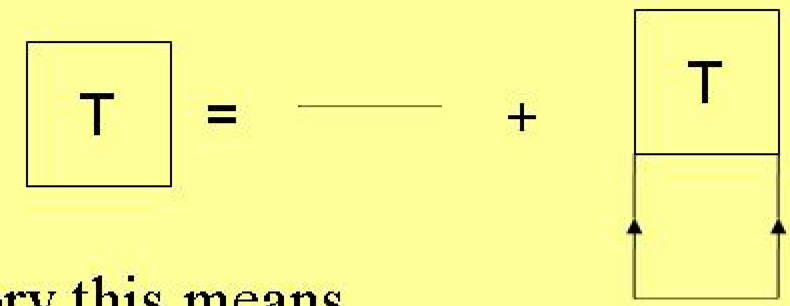
Ladder Summations (I)

- Renormalization of atom-atom interaction:



Geometric series

Series is summed by Lippmann-Schwinger equation for T-matrix



For the low-energy effective theory this means that the atom-atom interaction renormalizes to:

$$V_{\downarrow\downarrow}(\vec{x}-\vec{x}\,') \rightarrow \frac{4\pi a_{bg}\hbar^2}{m}\delta(\vec{x}-\vec{x}\,') = T_{bg}^{2B}\delta(\vec{x}-\vec{x}\,')$$

Energy dependence

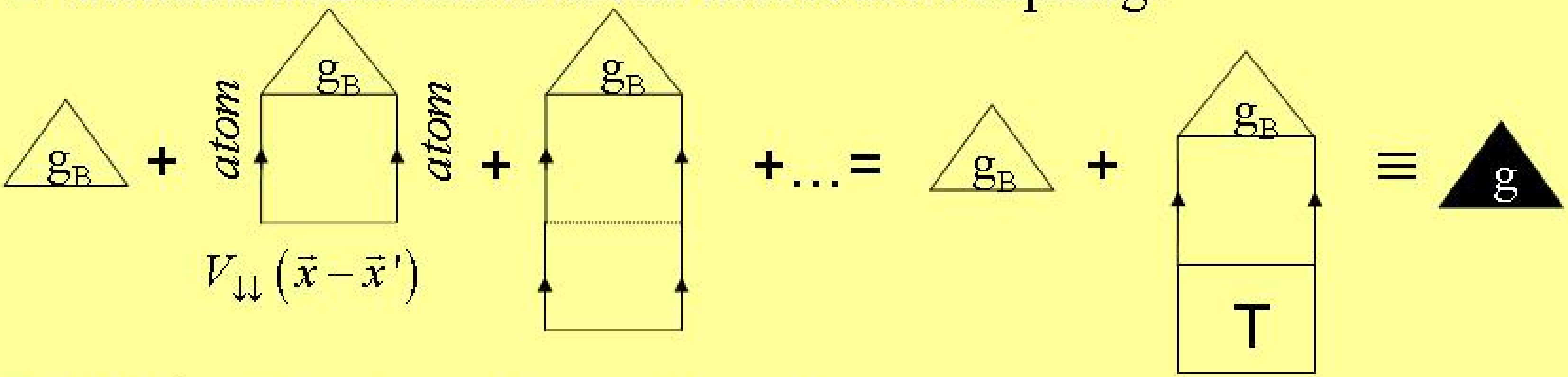
The interatomic interactions renormalize to the T-matrix. Because the binding energy is large far off resonance we also need its energy dependence. For low momenta and energies the T-matrix is given by

$$\boxed{T^{2B}(\hbar\omega) = \frac{4\pi a_{bg}\hbar^2}{m}\left[\frac{1}{1 + i a_{bg}\sqrt{\frac{m\omega}{\hbar}} - \frac{a_{bg} r_{bg} m\omega}{2\hbar}}\right]}$$

With scattering length a_{bg} and effective range r_{bg}.

Ladder Summations (II)

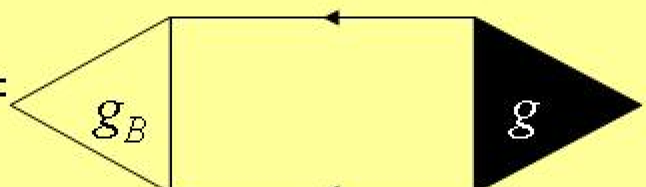
- Renormalization of atom-molecule coupling:



- We have $g_B(x-x') \to g\delta(x-x')$
- Molecular self-energy:

Proportional to g^2, due to optical theorem.

$\hbar\Sigma_m(\hbar\omega) =$



$\approx -g^2 \frac{\sqrt{m^3}}{2\pi\hbar^3} i\sqrt{\hbar\omega}$

Usual square-root behavior for decay into continuum.

Hartree-Fock Self-energy

- Mean-field effects of condensate on noncondensed atoms

$$\boxed{\hbar\Sigma^{HF} = 2n_a\left[T_{bg}^{2B} + \frac{2g^2}{\hbar\Sigma^{HF} + \mu - \delta(B) - g^2 \frac{m^{3/2}}{2\pi\hbar^3}\sqrt{\hbar\Sigma^{HF} - \mu}}\right]}$$

- Far off resonance:

$$\boxed{\hbar\Sigma^{HF} = \frac{8\pi a(B)\hbar^2 n_a}{m}}$$