Superfluid pairing in atomic Fermi gases

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Outline

1. Introduction
2. Weakly bound molecules at $a>0$. Elastic scattering.
3. Collisional relaxation. Long life-time. BEC.
4. mixtures of heavy and light fermions.
5. Conclusions

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2-component Fermi gas

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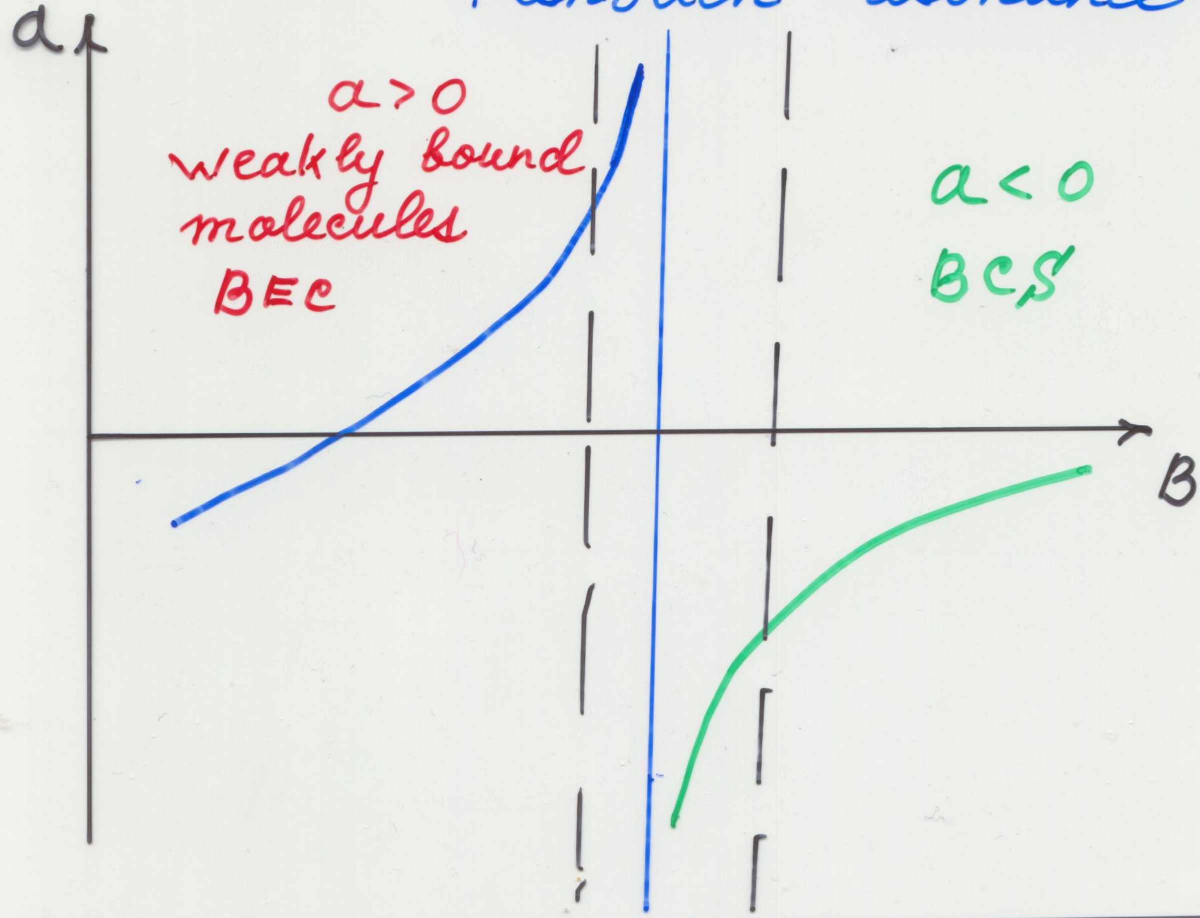
Dilute limit $n R_e^3 \ll 1$

Ultracold limit $\Lambda_T \gg R_e$

S-wave scattering

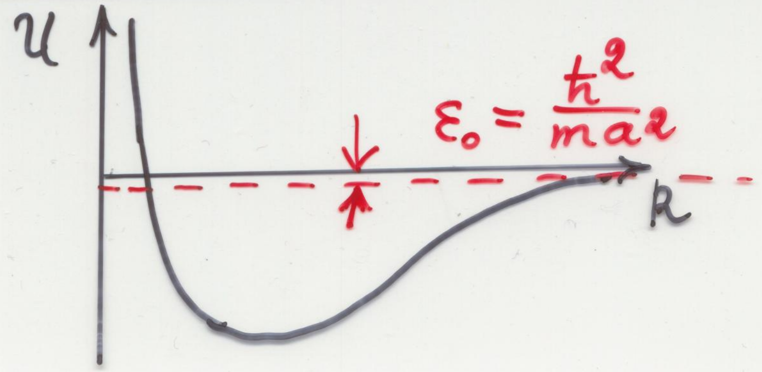
atom-atom scattering length a

Feshbach resonance



$a > 0$

$a \gg R_e$



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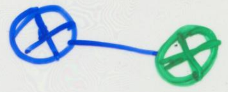
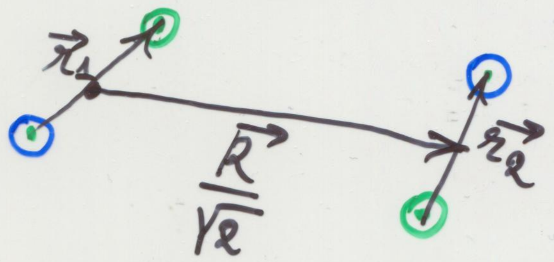
$a > 0$

$na^3 \ll 1 \; ; \quad T \ll \varepsilon_0$

Gas of weakly bound molecules (dimers; bosonic)



Elastic scattering



$\boxed{Ka \ll 1}$

$$-\frac{\hbar^2}{m}\left[\Delta_{r_1} + \Delta_{r_2} + \Delta_R\right]\psi +$$

$$+ \left[U(r_1) + U(r_2) + U(r_+) + U(r_-)\right]\psi$$

$$= -2\varepsilon_0 \psi ;$$

$$\vec{r}_\pm = \left(\vec{r}_1 + \vec{r}_2 \pm \sqrt{2}\,\vec{R}\right)/\sqrt{2} \; ;$$

(*) $$\psi(\vec{r}_1, \vec{r}_2, \vec{R}) \xrightarrow[r_1 \to 0]{} \underbrace{f(\vec{r}_2, \vec{R})}_{\text{3 variables (r_2, R and the angle between them)}} \left(\frac{1}{4\pi r_1} - \frac{1}{4\pi a}\right)$$

$a \gg R_e$

Integral equation for $f(\vec{r}_2, \vec{R})$, replacing U by the boundary condition (*)

Only one distance scale

a

$R \to \infty \quad \psi = \varphi_0(r_1)\varphi_0(r_2)\left(1 - \frac{\sqrt{2}\, a_{dd}}{R}\right);$

$\varphi_0 = (\sqrt{2\pi a}\, r)^{-1} \exp(-r/a)$

$R \to \infty$

$$f = \frac{2}{ra} \exp\left(-\frac{r}{a}\right)\left(1 - \frac{\sqrt{2}\, a_{dd}}{R}\right)$$

Solution of the integral equation for f

$$a_{dd} = 0.6\, a > 0$$

Quite different from the "old answer" $a_{dd} = 2a$.

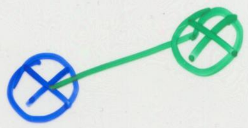
Experiments

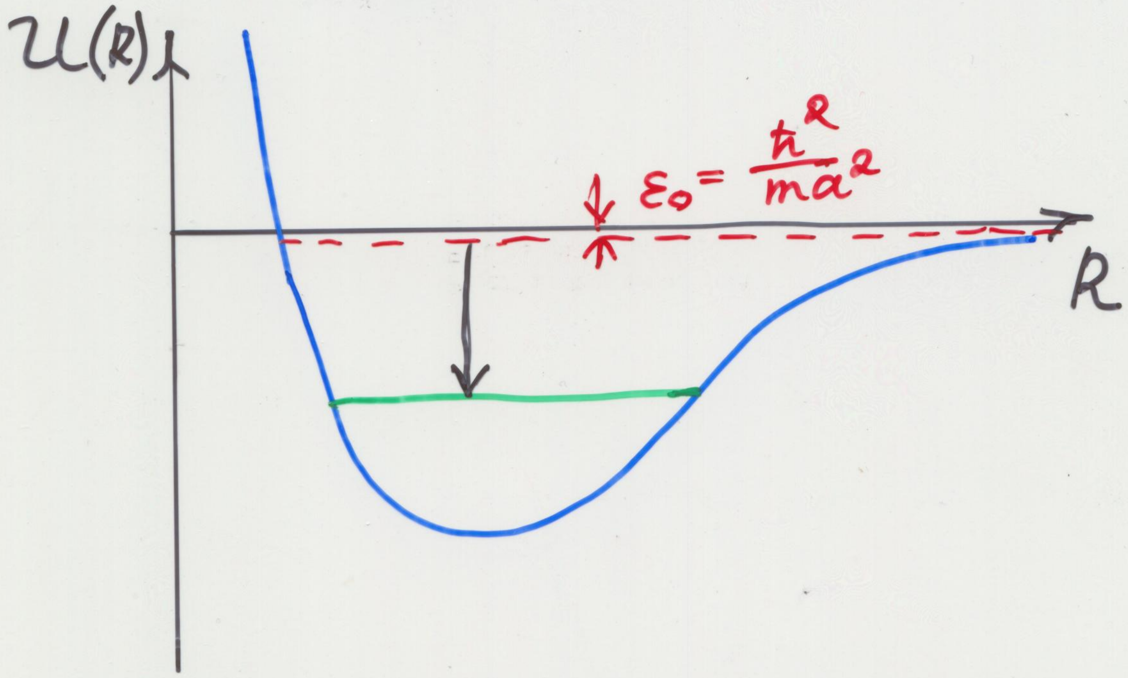
^{40}K JILA

^{6}Li Innsbruck, MIT, ENS

theory Petrov, Salomon, G.S.

5/ Collisional relaxation to deep bound states



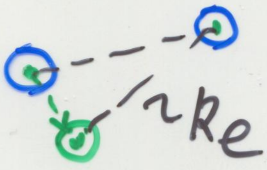


weakly bound dimer $\sim a$

deep bound state $\sim R_e \ll a$

$Ka \ll 1$

Atom-dimer relaxation



$$\alpha_{rel} \propto (K_{eff} R_e)^{2+?}$$

$$K_{eff} \sim \frac{1}{a} \Rightarrow \alpha_{rel} \propto \left(\frac{R_e}{a}\right)^{2+?}$$

Strong reduction of the relaxation due to fermionic nature of the atoms

6)

$$\alpha_{rel} = \frac{2\pi}{\hbar} \sum_f |\Psi_i V_{int} \Psi_f|^2 \delta(E_i - E_f)$$

Dependence on $a \Rightarrow \Psi_i$

The dependence of Ψ_i on a is the same for $\rho \sim R_e$ and for $R_e \ll \rho \ll a$. It follows from the solution of the Schrödinger equation with $\varepsilon_0 = \frac{\hbar^2}{ma^2} \to 0$:

$$\begin{cases} \Psi_i = A(a)\rho^\gamma \Phi_\gamma(\Omega) & ; \rho \ll a \\ \gamma = 0.16 \end{cases}$$

$\rightarrow$ zero-range approximation

Only one distance scal $\Rightarrow a$

$$\Psi_i \sim A(a)a^\gamma \cdot \left(\frac{\rho}{a}\right)^\gamma \Phi_\gamma(\Omega); \quad \rho \ll a$$

$$\Psi_i \sim \frac{1}{a^{3/2}} \cdot \frac{1}{\sqrt{2\pi}\left(\frac{r}{a}\right)} e^{-\frac{r}{a}} \left(1 - \frac{a_{ad}}{R}\right); \quad \rho \gg a$$

$$A(a)a^\gamma \sim a^{-3/2} \Rightarrow$$

$$\Rightarrow A(a) \sim a^{-3/2-\gamma}$$

$$\alpha_{rel} \sim A^2(a) \sim \boxed{\frac{1}{a^{3+2\gamma}}}$$

Similar calculation for the dimer–dimer relaxation

$$\alpha_{rel} = C \cdot \frac{\hbar}{m} \cdot R_e \cdot \left(\frac{R_e}{a}\right)^{s}$$

dimer-atom $s = 3.33$

dimer-dimer $s = 2.55$

^{40}K JILA ; ^{6}Li Innsbruck, MIT, ENS

theory Petrov, Salomon, Shlyapnikov

$\tau \sim$ seconds for $n \sim 10^{13}\,cm^{-3}$

Elastic collisions

$$\alpha_{el} = 8\pi\, v_T (0.6a)^2$$

$$\frac{\alpha_{el}}{\alpha_{rel}} \sim 10^3$$

Efficient evaporative cooling

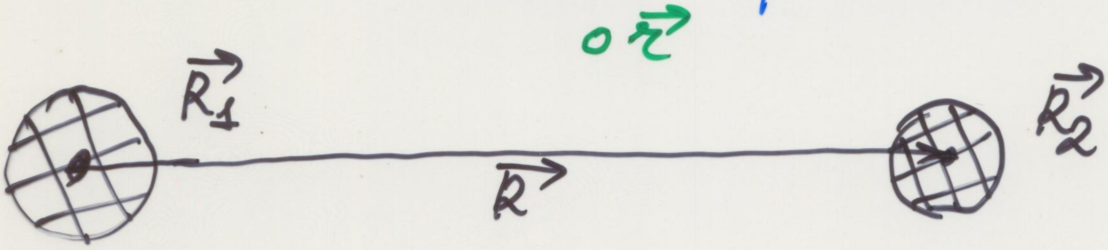
BEC

^{40}K JILA
^{6}Li Innsbruck, MIT, ENS

Mixture of Heavy and light fermions

Interaction between heavy and light fermions has the scattering length $a \to \infty$

3-atom problem



Born-Oppenheimer approximation

$$\psi = f(\vec{R})\,\tilde{\psi}(\vec{r}, \vec{R_1}, \vec{R_2})$$

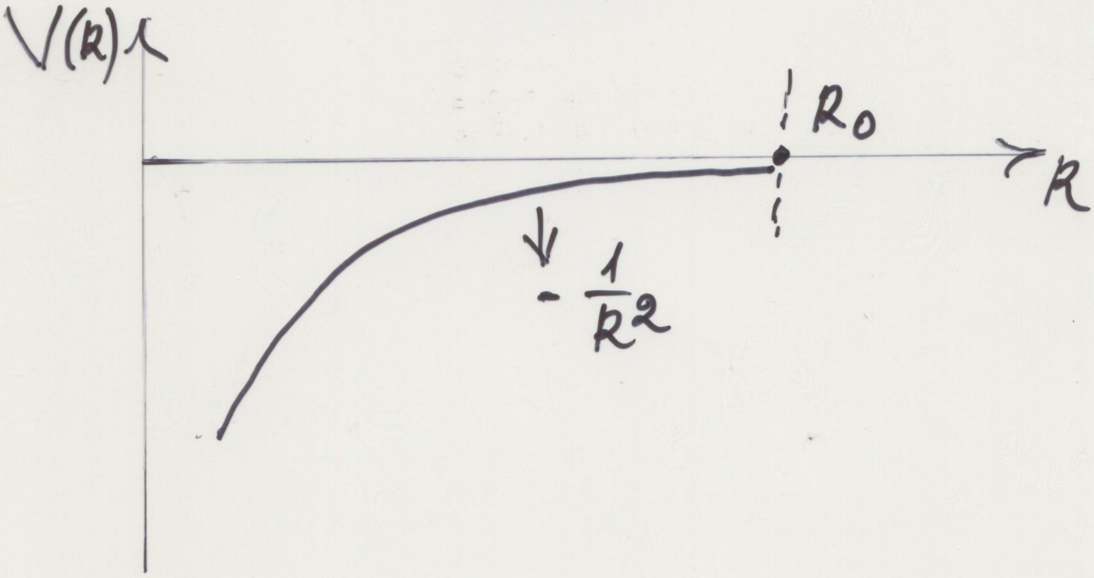
$$\tilde{\psi} = C\left\{ \frac{e^{-\lambda|\vec{R_1}-\vec{r}|}}{|\vec{R_1}-\vec{r}|} + \frac{e^{-\lambda|\vec{R_2}-\vec{r}|}}{|\vec{R_2}-\vec{r}|} \right\}$$

$$\vec{r} \to \vec{R_1} \Rightarrow \tilde{\psi} \to \frac{1}{|\vec{R_1}-\vec{r}|};$$

$$-\lambda + \frac{e^{-\lambda R}}{R} = 0 \Rightarrow \lambda = \frac{0.567}{R}$$

Only 1 bound state (long) of the light fermion with the pair of heavy ones

$$E = -\frac{\hbar^2 \lambda^2(R)}{2m} = -0.161 \frac{\hbar^2}{mR^2} \equiv V(R)$$



BCS pairing between heavy fermions in the p-wave channel

$\otimes$ $\vec{k}$ $\qquad$ $\otimes$ $-\vec{k}$ $\qquad$ (relative momentum k_F)

$$T_c \sim E_F \exp\left\{-\frac{1}{\lambda_1}\right\}$$

$$\lambda_1 = |f_1| \, \nu(E_F)$$

$$|f_1| \sim \frac{1}{k_F} \; ; \; \nu(E_F) \sim k_F$$

Born approximation

$$f_1 = -\frac{2\pi^2}{3} \cdot \frac{\hbar^2}{m k_F};$$

$$\nu(E_F) = \frac{M k_F}{2\pi^2 \hbar^2}$$

$$\lambda_1 = 0.0536 \frac{M}{m} \Rightarrow \frac{1}{2.8} \text{ for K-Li}$$

$$T_c \sim E_F \exp\left\{-\frac{1}{\lambda_1}\right\} \sim 0.06\, E_F$$

$V(R)$ trancated at R_0

$$T_c \sim E_F \cdot 0.025$$

Second Born correction to the scattering amplitude + Gor'kov - Melik-Barhudarov corrections $\Rightarrow$ increase of T_c

$$T_c \approx (0.05 \div 0.1)\, E_F$$

$$n|a^3| \gg 1$$

$\frac{T_c}{E_F}$ is independent of the density

Lobo, Baranov, Petrov, G.S.

Conclusions

1. K and Li Fermi gases have already provided us with remarkable physics.

2. Mixtures can do even more!