"Life is frittered away by details.

Simplify, simplify."

H. D. Thoreau

Properties of Fermi gases with resonant interactions

Georg Bruun (Niels Bohr Institute) (G.B. + C.J.P.,
PRL, in press..)

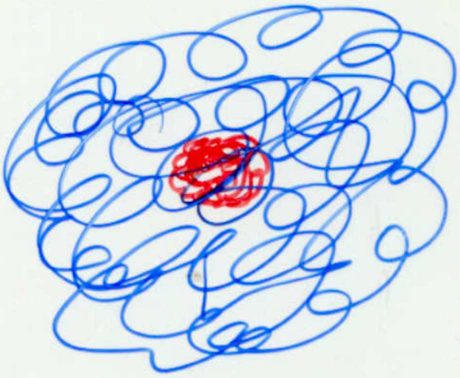
[Murray Holland, Henk Stoof - similar ideas]

- What does a Feshbach molecule look like?

Two channel example:

"Bare"
molecule

Molecule
away from resonance



Molecule close to
resonance

Even away from resonance, magnetic moment of molecule is different from "bare" moment.

Basic ingredients of low-energy theory:

- Energy of  2ν
- Coupling to atoms  g

$\underbrace{\qquad}_{\text{Important ingredients}}$

- Important energy scale

$$E_0 = \frac{g^4}{16\pi^2} \frac{m^3}{\hbar^6}$$

$\frac{E_0}{E_{Fermi}}$ is measure of maximum degree of correlation.

Universality.

- Scattering rates

Pauli blocking increases T matrix.

- Equivalence of models for scattering
  - Boson - fermion vs. pure fermion
  - Single channel vs. coupled channels

    (No magnetic tuning for single channel)

Feshbach resonance

Method of the virtual Landau.

- Identify low-energy degrees of freedom. 2 species of fermion, 1 molecule
- Parametrize g, 2ν, T_{bg} ← background

$$T = T_{bg} - g\frac{1}{2\nu}g \qquad (\omega = 0)$$

(Because we are working in a limited subspace, other contributions small.)

$$2\nu = -\Delta\mu\,(B - B_0)$$

↑ Magnetic moment of molecule − that of two atoms

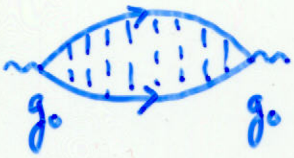
Energy of molecule (Zero total momentum)

Landau
$$\omega = 2\nu + \frac{g^2 m}{4\pi\hbar^2}\sqrt{-m\omega}$$
← only from low energy intermediate states

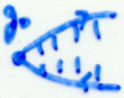
(Golden rule)

From bare Hamiltonian

$$\omega = 2\nu_0 + \Pi(\omega)$$



$g_{vac} =$



$$= 2\nu_0 + \Pi(\omega=0) + \frac{g_{vac}^2 m}{4\pi\hbar^2}\sqrt{-m\omega} + \frac{\partial\Delta\Pi}{\partial\omega}\omega$$

$$\omega = \underbrace{\tilde{Z}\left(2\nu_0 + \Pi(\omega=0)\right)}_{2\nu} + \frac{g_{vac}^2\tilde{Z} m}{4\pi\hbar^2}\sqrt{-m\omega}$$

$$\left(\tilde{Z} = \left(1 - \frac{\partial\Delta\Pi}{\partial\omega}\right)^{-1}\right) \qquad g^2 = g_{vac}^2\tilde{Z}$$

$\tilde{Z}$ - wave function renormalization factor

Probability of closed channel component in 

Magnetic moment difference $= \tilde{Z} \times$ bare result

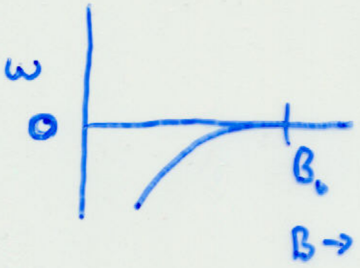
Simple physical interpretation

For actual molecule

Magnetic moment difference (ω < 0)

$$= \frac{\tilde{Z}}{1 + \frac{g^2 m^{3/2}}{4\pi\hbar^2}\frac{1}{|\omega|^{1/2}}}$$

$$= \underline{\underline{\frac{\tilde{Z}}{1 + \left(\frac{E_0}{|\omega|}\right)^{\frac{1}{2}}}}}$$



(c.f. Braaten et al., T. Koehler et al.)

(Can be generalized to many channels.)

Example

$^{40}K \quad |\tfrac{9}{2}, -\tfrac{9}{2}\rangle + |\tfrac{9}{2}, -\tfrac{7}{2}\rangle$ in s wave

Only other s-wave channel is

$$|\tfrac{9}{2}, -\tfrac{9}{2}\rangle + |\tfrac{7}{2}, -\tfrac{7}{2}\rangle$$

$$(\Delta\mu)_{\text{bare}} = 1.78\,\mu_B \quad \text{at} \quad B \simeq 200\text{ G}$$

John Bohn $\quad \Delta\mu = 0.118\,\mu_B$ " " " .

Large difference. Reflects strong coupling between channels.

A.J. Leggett question. Two channels have single-particle state in common

Pauli blocking could affect closed channel

E. Kolomeitsev (Niels Bohr Inst.) + G. Brown

Energy scale $E_0 = \frac{g^4}{16\pi^2} \frac{m^3}{\hbar^6}$

$$k_F a \sim \left(\frac{m E_F}{\hbar^2}\right)^{\frac{1}{2}} \frac{g^2}{\nu} \frac{m}{\hbar^2} \sim \frac{(E_F E_0)^{\frac{1}{2}}}{\nu}$$

If $E_0 >> E_F$, get strong interaction even for ν well above E_F.

Universal behaviour

Basic assumption : a is only length scale in scattering

If $a >> \frac{1}{k_F}$ $\Rightarrow$ a irrelevant

Another length : effective range

$$\underset{\swarrow \text{ S-wave phase shift}}{k \cot \delta} = -\frac{1}{a} + \frac{1}{2} k^2 r_e$$

(Corresponds to $\underset{\frac{\hbar^2 k^2}{2m}}{\omega} - \nu$ in energy denominator

$$\frac{\hbar^2}{m r_e^2} \sim E_0$$

Require $E_0 >> E_F$ for universality.

(Weak coupling ($k_F a << 1$). Recover Galitskii results.)
G. Bruun cond-mat/0401497

Pauli blocking and scattering

Two effects — initial & final states (Usual suppression.)

— intermediate states in scattering

Gives increase in rate

T =  + . . . Ladder approximation

$$= \frac{V}{1 - V \circlearrowright}$$

$$|T|^2 = \frac{1}{\left|\frac{1}{V} - \mathrm{Re} \circlearrowright\right|^2 + |\mathrm{Im} \circlearrowright|^2} \leq \frac{1}{|\mathrm{Im} \circlearrowright|^2}$$

↗ Density of states

(cf Scattering of particle $|T| < \frac{1}{\left(\frac{mk}{2\pi\hbar^2}\right)^2} \sim \bar{\lambda}^2$)

Effective density of states in medium

$$\sim \sum_{1,2} \underbrace{\left[(1-f_1)(1-f_2) \quad - f_1 f_2\right]}_{1 - f_1 - f_2} \delta(\omega - \varepsilon_1 - \varepsilon_2)$$
$$\delta(\underline{k} - \underline{p}_1 - \underline{p}_2)$$

(Familiar from BCS)

$|1 - f_1 - f_2| \leq 1$ $\quad\therefore$ | Density of states | reduced. $|T|^2$ increases.

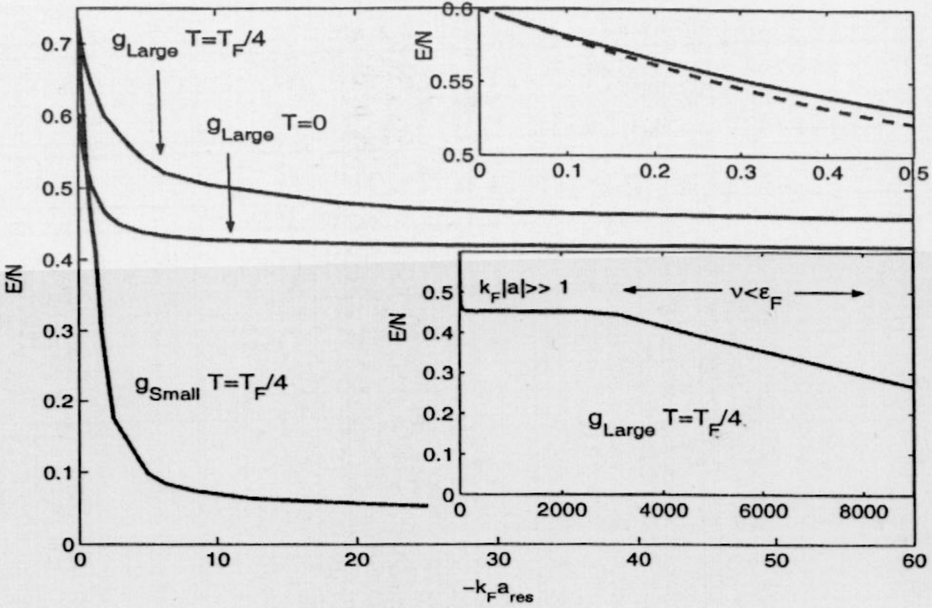


FIG. 1: The energy per particle E/N in units of ϵ_F as a function of $-k_F a_{res}$ for a narrow and a broad resonance.

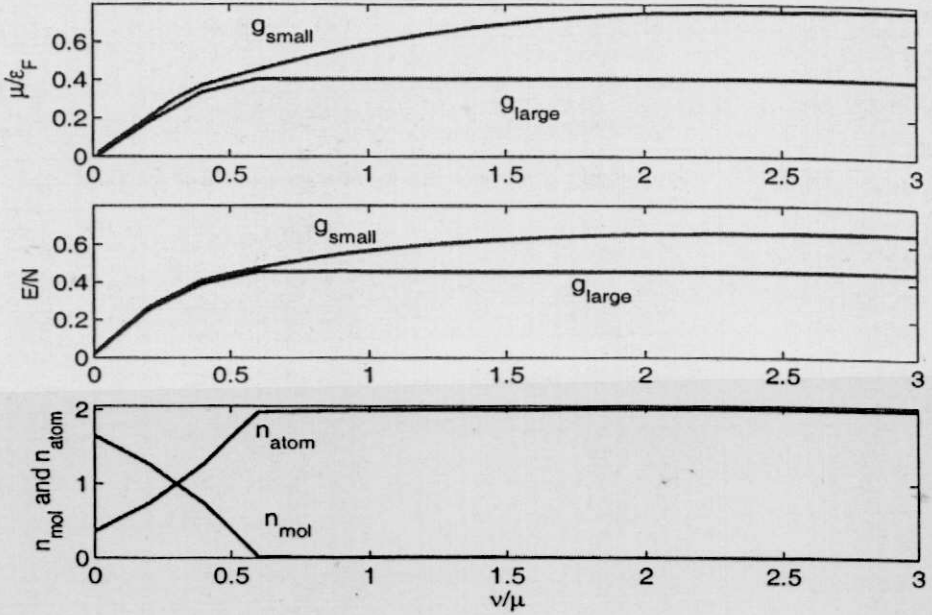


FIG. 2: The chemical potential μ, the energy per particle E/N in units of ϵ_F and the density of atoms and molecules in units of $n_\sigma = n/2 = (2m\epsilon_F)^{3/2}/(6\pi^2)$ as a function of ν for a narrow and a broad resonance.

Zero temperature

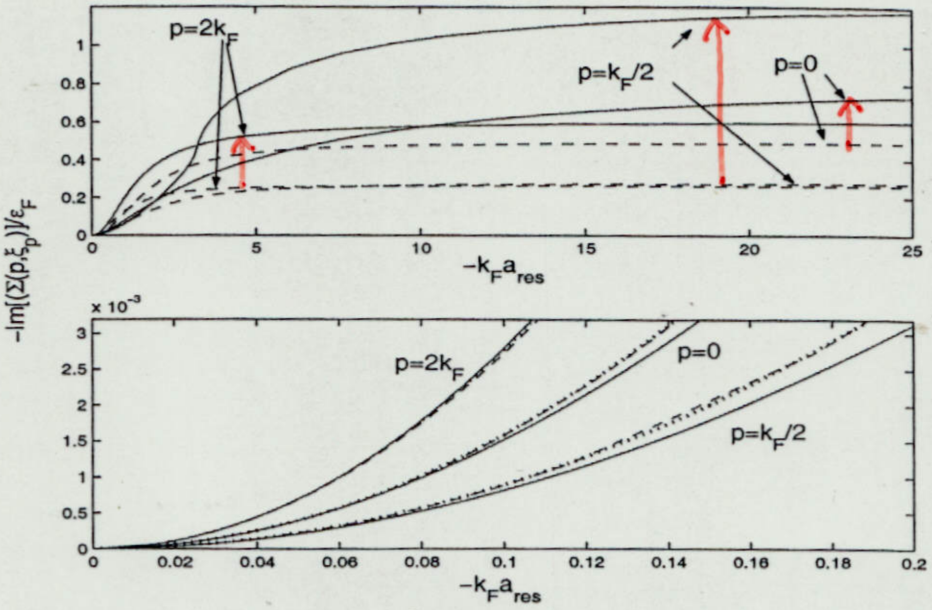


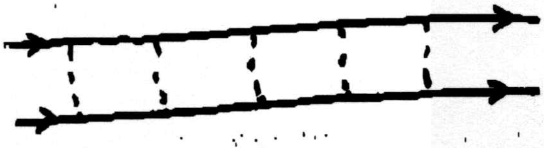
FIG. 1: $-\mathrm{Im}[\Sigma(p,\xi_p)]/\epsilon_{\mathrm{F}}$ as a function of $k_{\mathrm{F}}|a_{\mathrm{res}}|$ for various momenta. The solid curves are numerical results based on Eqs. (7) and (12), the dashed curves are the results if the effect of the medium on the scattering amplitude is neglected, and the dotted curves in Fig.(b) are the Galitskii results [Eq.(11)]. All these results become identical for $k_{\mathrm{F}}|a_{\mathrm{res}}| \ll 1$.

Does unitarity impose bounds on rates of processes in medium?

Need to consider higher order effects.

Equivalence for models for scattering

Two-body scattering

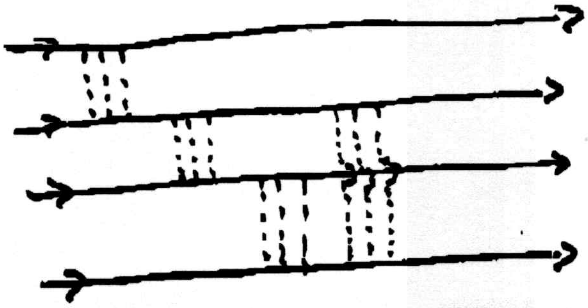


If resonant, will be pole term

Structure not sensitive to nature of intermediate states
(one-channel, multi channel)

(Cannot tune resonance by magnetic field if single channel.)

4-body scattering



2-particle scattering blocks can be factored out

(Does not rely on molecules being "good" bosons.)

c.f. M. Combescot + O. Betbeder-Matibet.

More complicated in medium.