Workshop on ultracold Fermi gases

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Single-particle excitations and collective modes in a trapped Fermi gas near a Feshbach resonance

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(1) Introduction

A brief review on our previous work

(2) single-particle excitations and collective modes in the BCS-BEC crossover of a superfluid Fermi gas in a trap

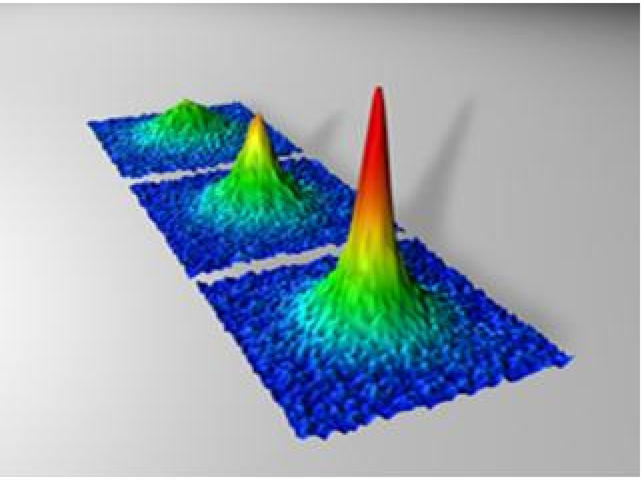
Superfluid density of states

Goldstone mode (uniform gas)

dipole, quadrupole, monopole modes

(3) summary

New Fermion Superfluidity in ^{40}K Fermi gas



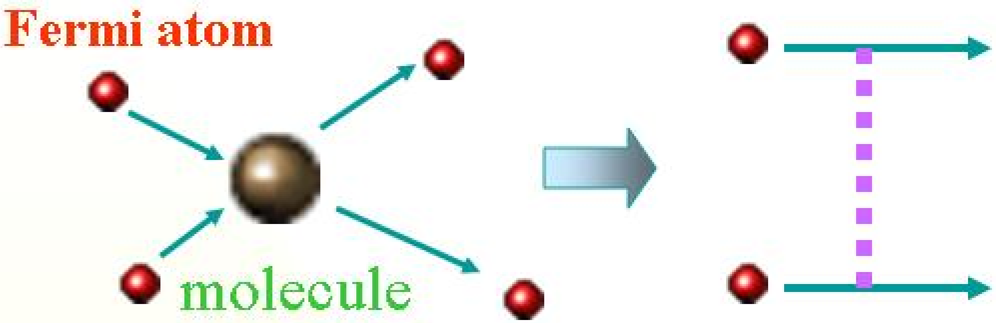
C. A. Regal, M. Greiner, and D. S. Jin (2004)

$$T_c / T_F \sim 0.07 - 0.15$$

Key words to understand this new superfluidity

- **Feshbach resonance**
- **BCS-BEC crossover**

Feshbach resonance



$$V_{eff} = -g^2 \frac{1}{2\nu - 2\mu}$$

tunable by magnetic field

characteristic energy of atoms

(Timmermans (2001), Holland (2001))

tunable **pairing interaction**

↓

BCS-BEC crossover

We have studied the BCS-BEC crossover in a trapped Fermi gas with a Feshbach resonance (F.R.), extending the strong-coupling theory developed by Nozieres and Schmitt-Rink (NSR, 1985).

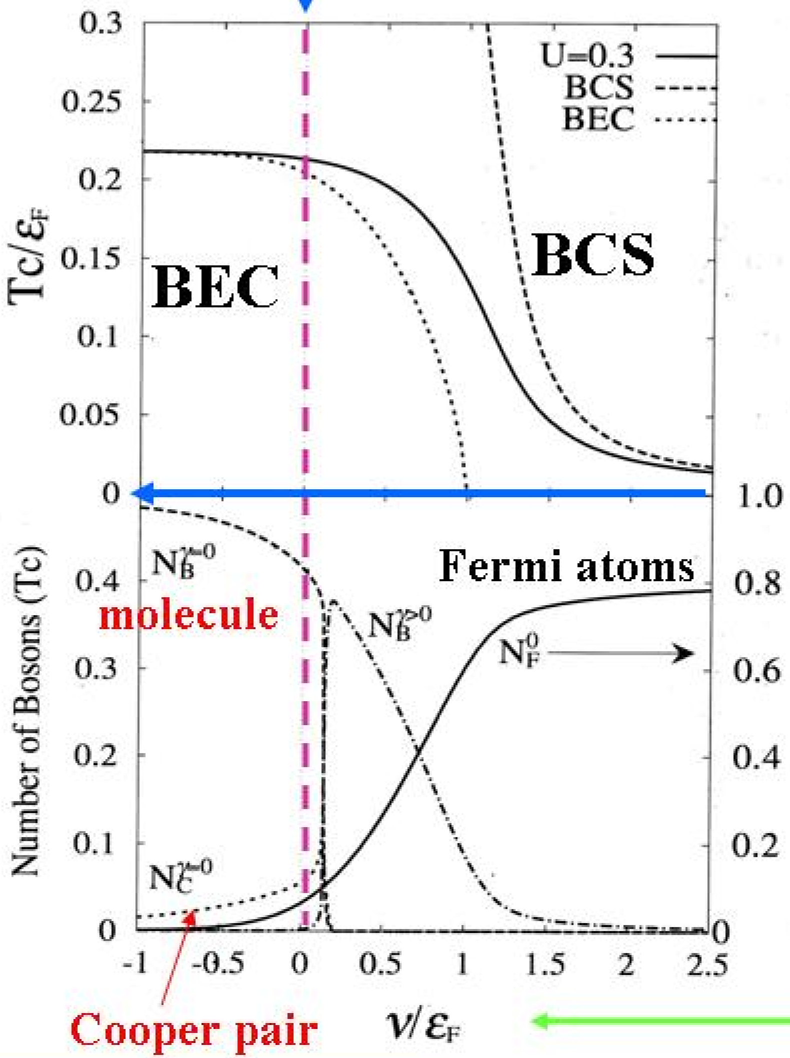
Y. Ohashi and A. Griffin PRL 89 (2002) 130402.
PRA 67 (2003) 033603, 063512.

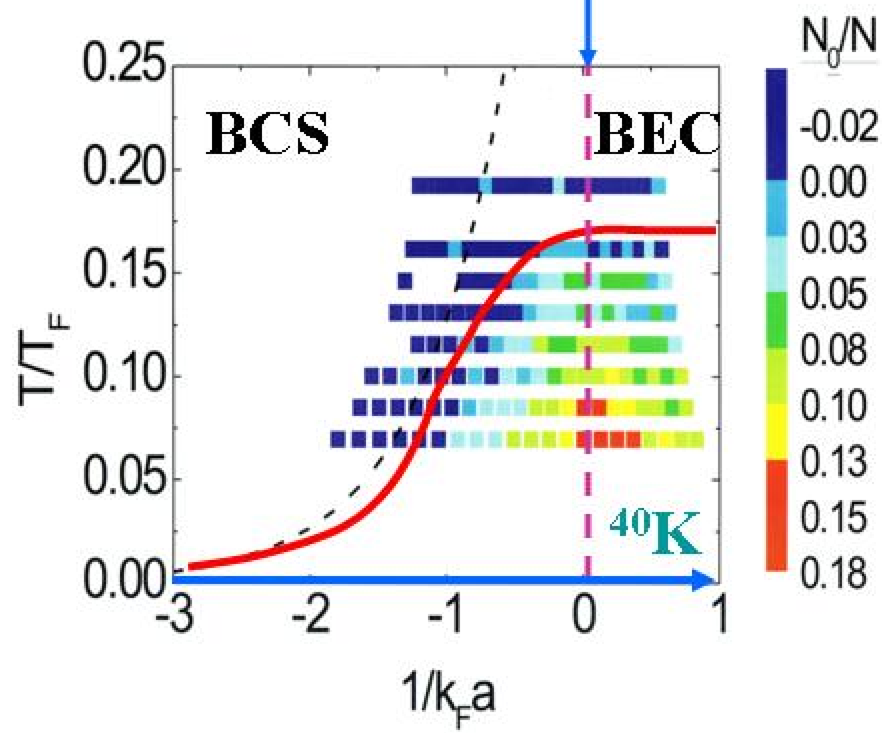
weak-coupling → strong-coupling

	BCS	*crossover regime*	*BEC*
F.R.	**Cooper pair**	*composite* **bosons** (**CP+FR molecule**)	**Feshbach molecule**
BCS-BEC crossover in superconductivity theory	**Cooper pair**	**Cooper pair**	**Cooper pair**

Eagles (1969), Leggett (1980), NSR(1985)

BCS-BEC crossover





(Regal, Greiner, and Jin (2004))

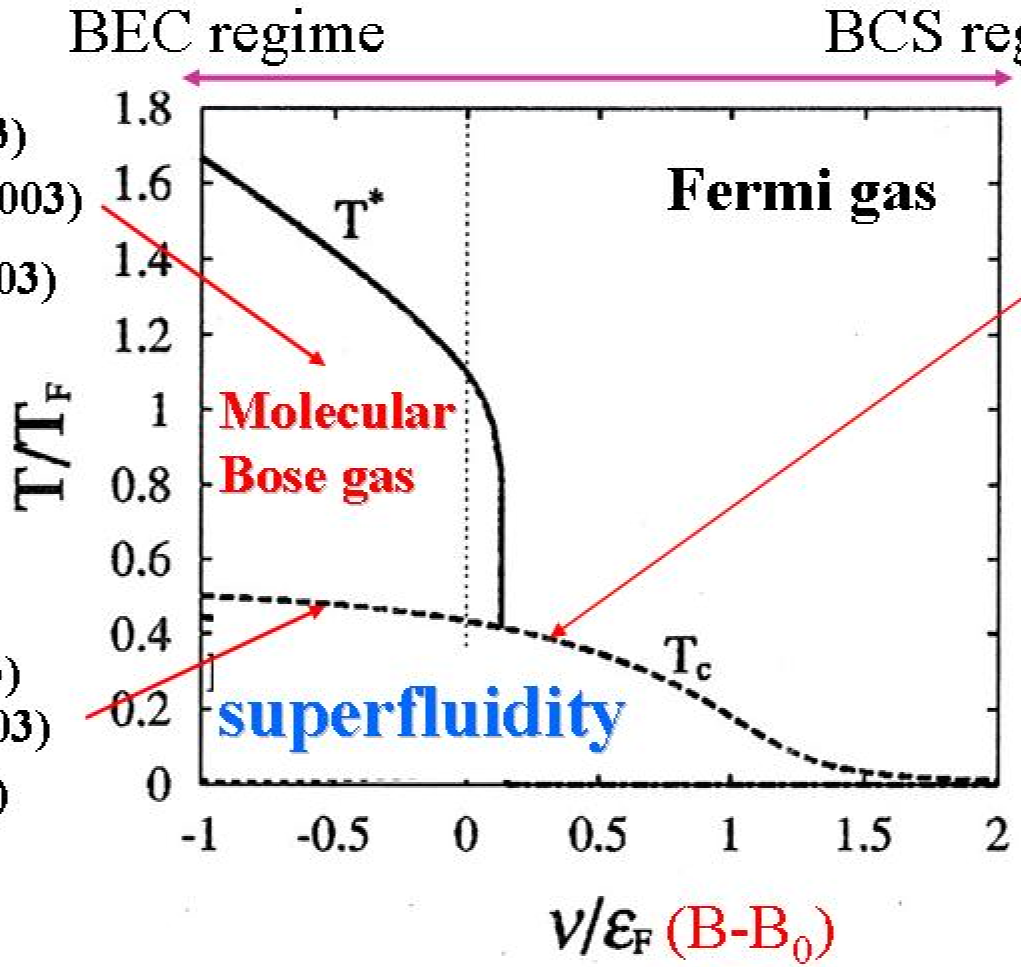
$$V_{eff}^{two-body} = -g^2 \frac{1}{2\nu} = \frac{4\pi a}{m}$$

(Ohashi and Griffin (2002))

long-lived molecules associated with the F. R.

Regal et al. (2003)
Strecker et al. (2003)
Jochim et al. (2003)

^{40}K ^{6}Li



Regal et.al. (2004)

^{40}K

Greiner et al. (2003)
Zwierlein et al. (2003)
Jochim et al. (2003)

^{40}K ^{6}Li

(Ohashi and Griffin (2003))

In this talk, we discuss single-particle and collective excitations in the BCS-BEC crossover regime of this newly discovered fermion superfluidity at T=0.

(1) We extend the strong-coupling theory developed by Eagles (1969) and Leggett (1980) to include the Feshbach resonance and a trap potential .

(2) We do not use LDA, but take into account correct eigen-functions of a harmonic potential.

(3) Collective modes are obtained from the poles of the density correlation function, which we calculate using HFB-GRPA in the crossover regime.

Our theory is consistent with the Goldstone's theorem and Kohn's theorem in the whole range of BCS-BEC crossover.

Some related work

(This is not the complete list of recent work!)

F.R. mechanism superfluidity in a Fermi atom gas (coupled F-B (CFB) model)

Timmermans et al. (2001) (mean-field theory)
Holland et. al. (2001) (mean-field theory)

possibility of BCS-BEC crossover using F.R. (CFB model) in a Fermi atom gas

Ohashi and Griffin (2002) (strong-coulping (extended NSR) theory beyond MFA)
Milstein et al. (2002) (strong-coulping (extended NSR) theory beyond MFA)
Bruun (2004)

density profile, order parameter, and DOS in a trapped superfluid Fermi gas

Bruun et al. (1998) (BCS model with no F.R. in weak-coupling regime)
Ohashi and Griffin (2003) (CFB model, LDA at Tc and above in the crossover regime)
Perali et al. (2003) (BCS model with no F.R., LDA at T=0 in the crossover regime)
Ohashi and Griffin (2004) (CFB model, DOS in the crossover regime)

collective modes in superfluid Fermi atom gas

Minguzzi et al. , Bruun et al. (1999) (hydrodynamics)
Baranov et al. (2001) (BCS model with no F. R., uniform gas, weak-coupling))
Bruun et al. (2001) (BCS model with no F.R, trapped gas, T=0, GRPA, weak-coupling)
Ohashi and Griffin (2003) (CFB, uniform gas, T>0, GRPA, crossover regime)
Stringari (2003) (trapped gas, hydrodynamics, crossover regime)
Ohashi and Grifin (2004) (CFB, trapped gas, T=0, GRPA, crossover regime)

Coupled Fermion-Boson model in a trap

Superconductivity: Lee, Ranninger,
Fermi atom gas: Timmermans, Holland, Ohashi, and their collaborators

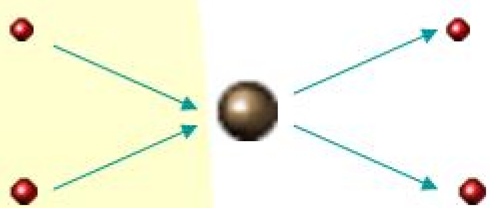
two hyperfine states = $\downarrow \; \uparrow$

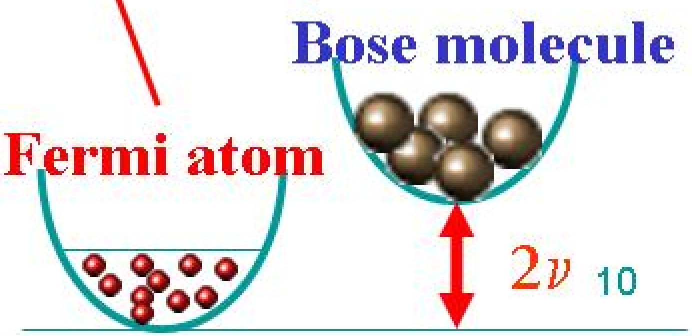
$$H = \sum_{\sigma} \int dr \Psi_{\sigma}^{\dagger}(r)[\frac{p^2}{2m} + V_{trap}^{Fermion}(r)]\Psi_{\sigma}(r) - U\int dr \Psi_{\uparrow}^{\dagger}\Psi_{\downarrow}^{\dagger}\Psi_{\downarrow}\Psi_{\uparrow}$$

$$+\Phi^{\dagger}(r)[\frac{p^2}{2M} + 2\nu + V_{trap}^{Boson}(r)]\Phi(r)$$

$$+g\sum_{\sigma}\int dr[\Phi^{\dagger}(r)\Psi_{\downarrow}(r)\Psi_{\uparrow}(r) + h.c.]$$

Feshbach resonance





Conservation of the total number of atoms

$$N = N_{atom} + 2N_{molecules}$$

This constraint is incorporated into the Hamiltonian as

$$H \Rightarrow H - \mu(N_F + 2N_B)$$

$$\equiv H - \mu N_F - \mu_B N_B$$

chemical potential of molecules

$$\mu_B = 2\mu$$

Fermi and Bose fields lead to two superfluid order parameters:

$$\Delta(r) \equiv U \left\langle \Psi_{\downarrow}(r)\Psi_{\uparrow}(r) \right\rangle$$ **BCS Cooper pair**

$$\phi_m(r) \equiv \left\langle \Phi(r) \right\rangle$$ **BEC molecular condensate**

However, these are NOT independent due to the Feshbach resonance:

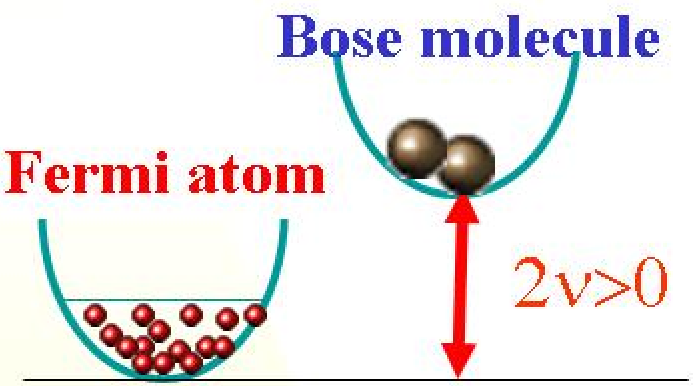
$$\frac{g}{U}\Delta(r) = -[\frac{p^2}{2M} + V_{trap}^{B}(r) + 2\nu - 2\mu]\phi_m(r)$$

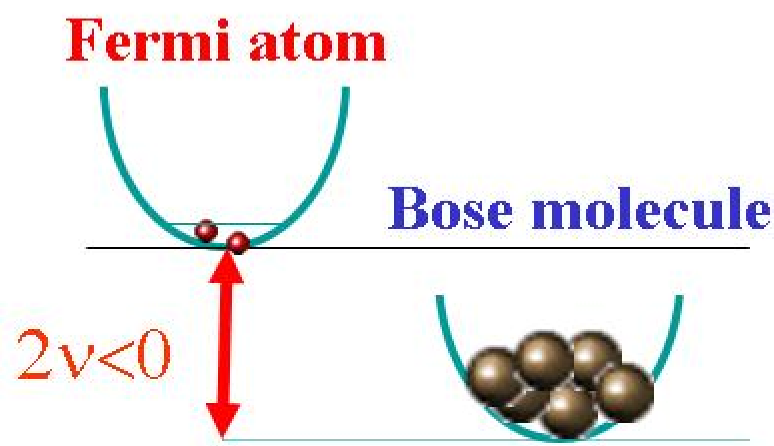
As a result, superfluidity is described by only one composite order parameter given by

$$\boxed{\tilde{\Delta}(r) = \Delta(r) - g\phi_m(r)}$$

BCS regime and BEC regime are determined by which component is dominant.

$$\tilde{\Delta}(r) = \Delta(r) - g\phi_m(r)$$





$$\Delta(r) >> \phi_m(r)$$

$$\tilde{\Delta}(r) \cong \Delta(r)$$

Cooper-pair BCS

$$\Delta(r) << \phi_m(r)$$

$$\tilde{\Delta}(r) \cong -g\phi_m(r)$$

Molecular BEC

BCS-BEC crossover theroy at T=0 (absence of thermal fluctuations)

Mean-field Hamiltonian

$$H_{HFB} = \sum_{\sigma} \int dr \Psi^{\dagger}_{\sigma}(r)[\frac{p^2}{2m} - \mu - \frac{U}{2} n_F(r) + V^{Fermion}_{trap}(r)]\Psi_{\sigma}(r)$$
$$- \int dr[\tilde{\Delta}(r)\Psi_{\downarrow}(r)\Psi_{\uparrow}(r) + h.c.]$$
$$+ \int dr \delta\Phi^{\dagger}(r)[\frac{p^2}{2M} + \nu - 2\mu + V^{Boson}_{trap}(r)]\delta\Phi(r)$$

(The composite order parameter determines Fermi excitation gap.)

total number of atoms

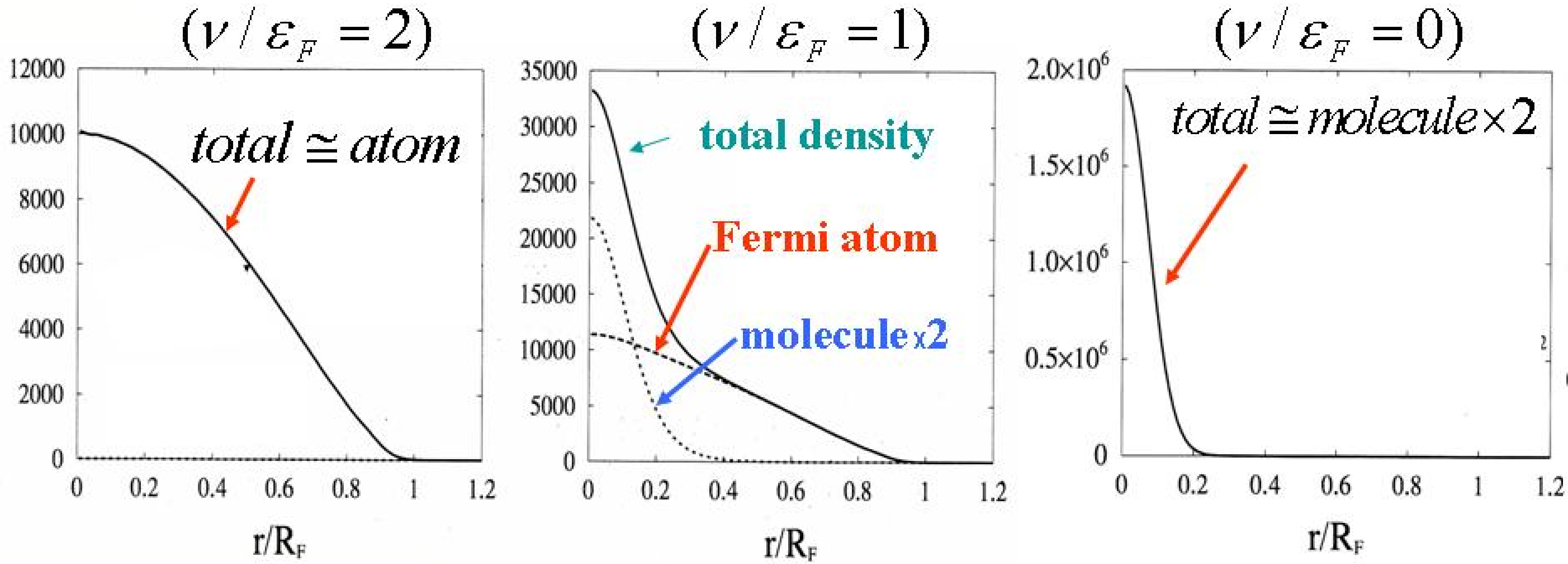
$$N(\mu) = N_{atoms} + 2N_{molecules}$$

Chemical potential

$(\tilde{\Delta}, n_F, \mu)$ are determined self-consistently.

Density profile

BCS → **crossover** → **BEC**



(R_F: Thomas Fermi radius)

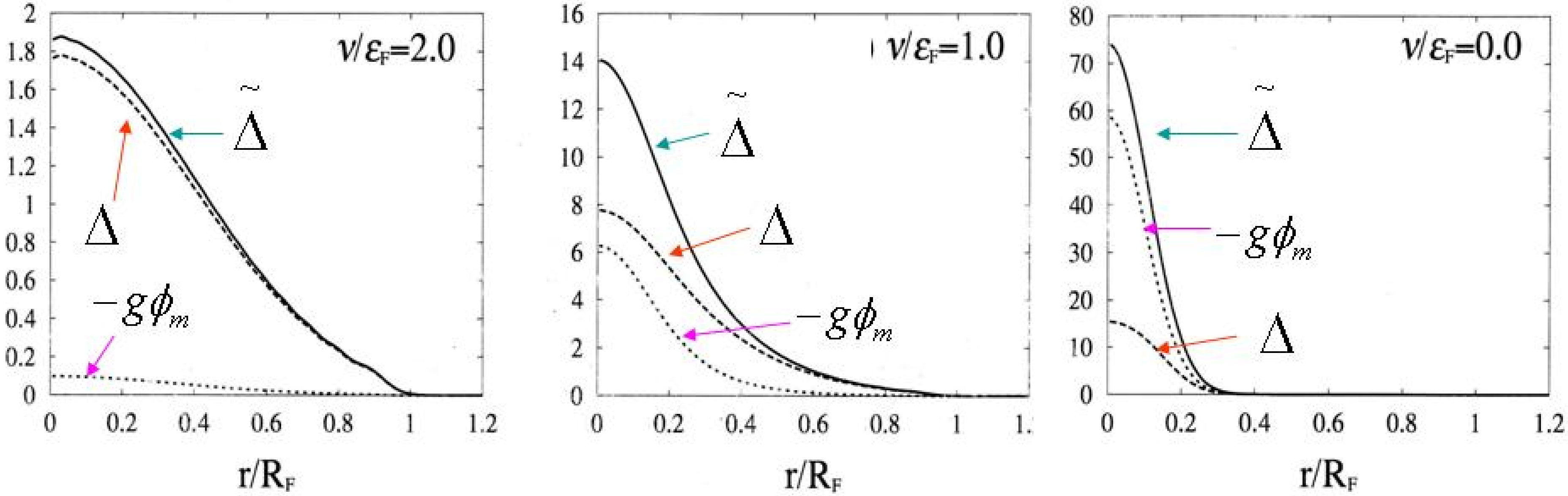
Fermi atoms form Cooper-pair bosons, which does not affect the density profile in the BCS regime.

Density profile shrinks in the BEC regime due to the absence of Pauli Principle between molecules associated with the F.R.

Composite order parameter

BCS → crossover → BEC

$\tilde{\Delta} \cong \Delta$ $\tilde{\Delta} = \Delta - g\phi_m$ $\tilde{\Delta} \cong -g\phi_m$

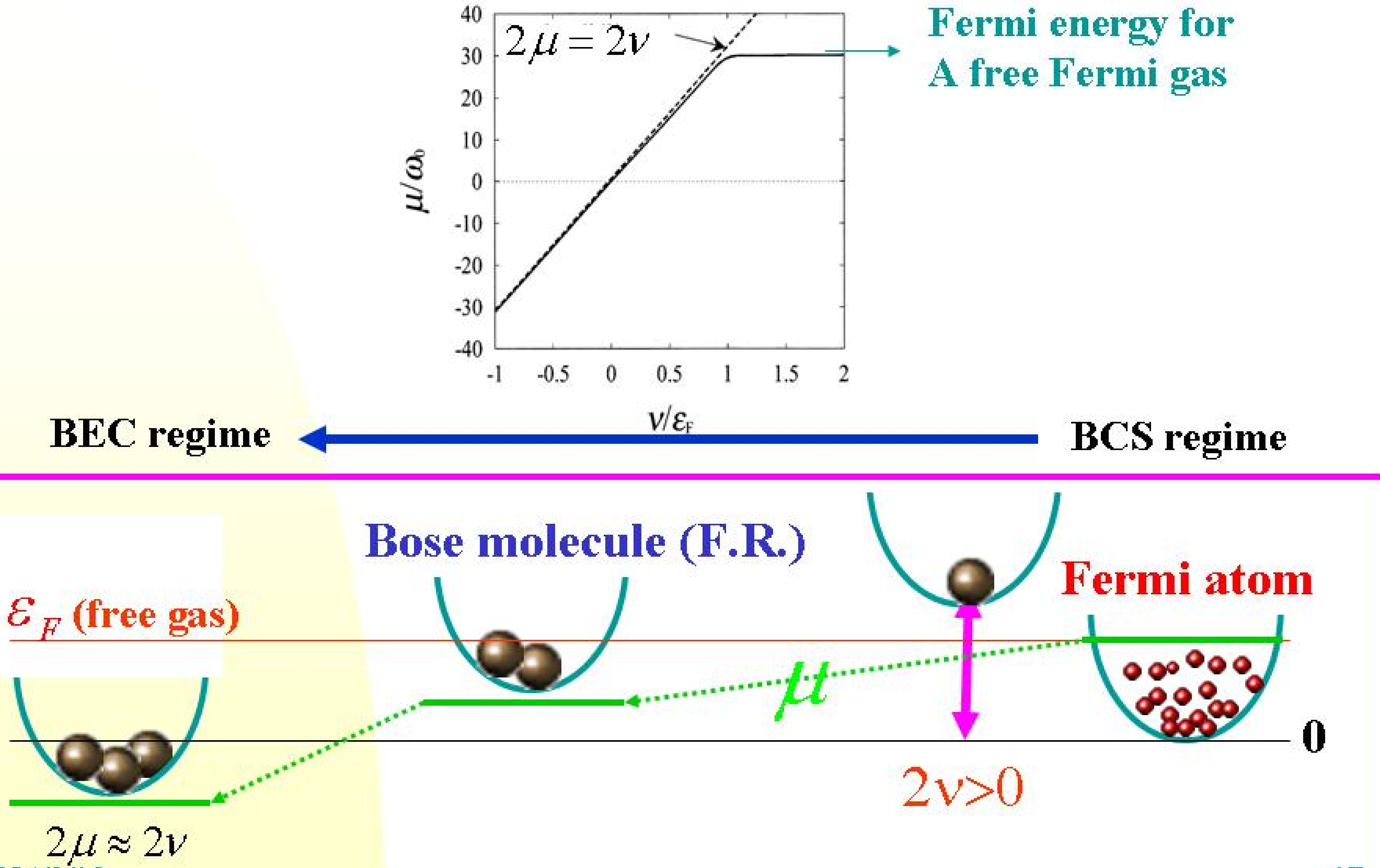


Cooper-pair condensate → molecular condensate

$$\Delta = U\left\langle \Psi_{\downarrow}\Psi_{\uparrow} \right\rangle$$

$$\phi_m = \left\langle \Phi \right\rangle$$

Chemical potential

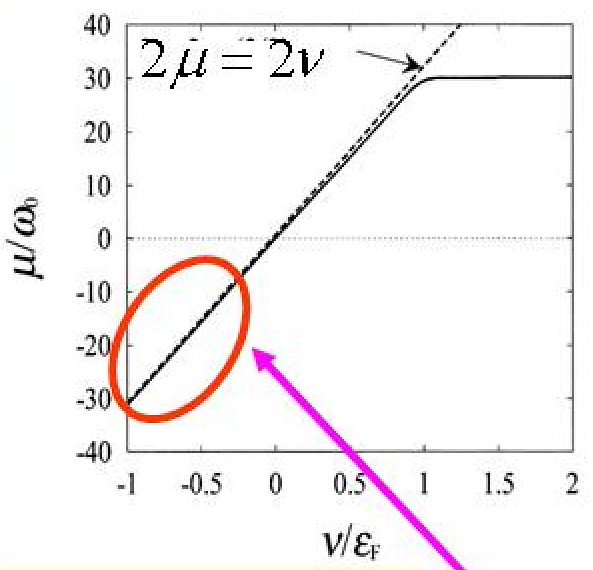


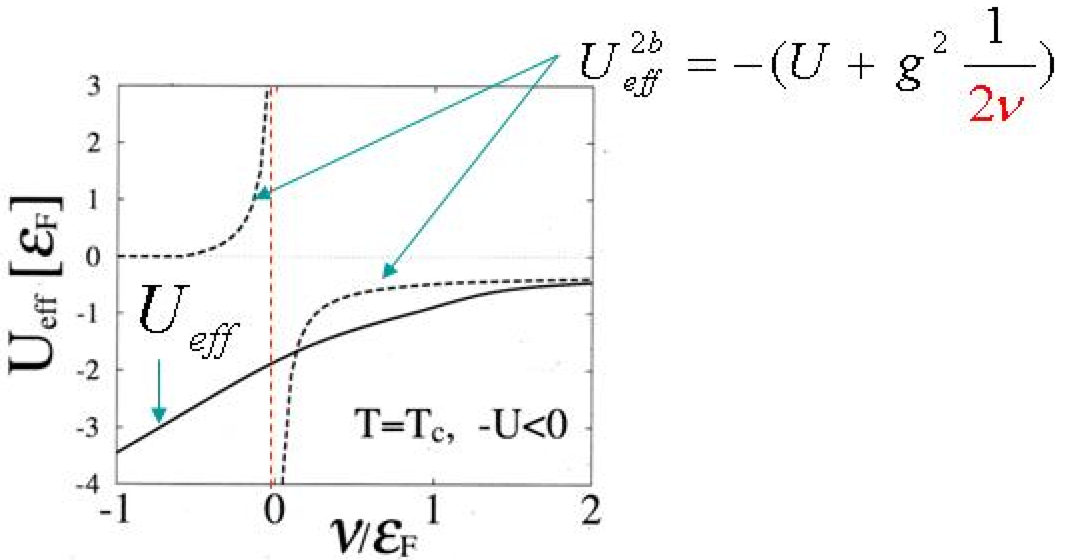
Effective pairing interaction

$$U_{eff} = -(U + g^2 \frac{1}{2\nu - 2\mu})$$

This interaction appears in the gap equation.

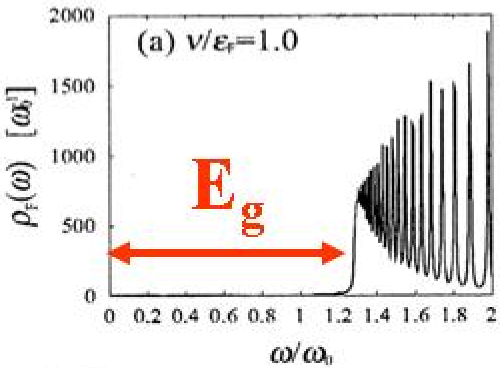
Timmermans et al. (2001), Holland et al. (2001), O&G (2002)

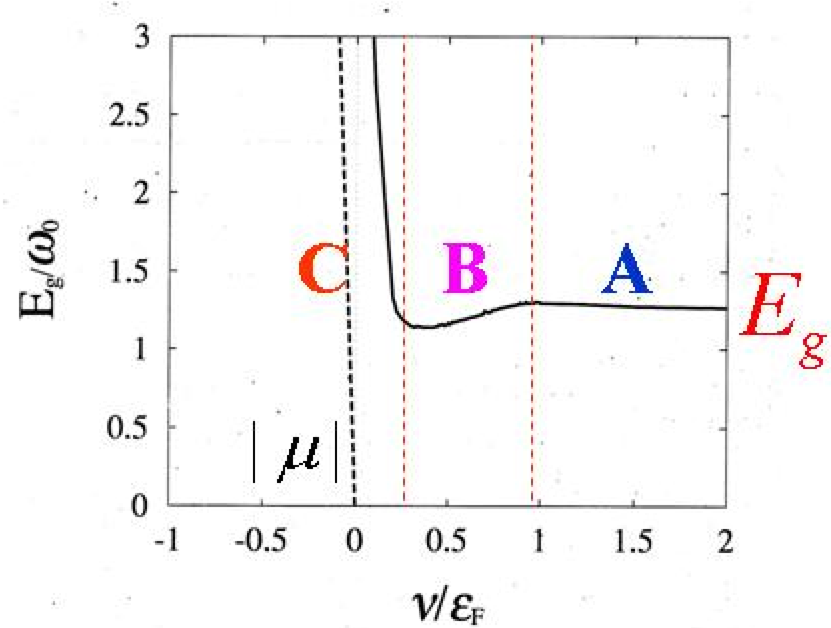




U_{eff} becomes strong in the BEC regime because $2\mu \sim 2\nu$.

Single-particle excitations (DOS)

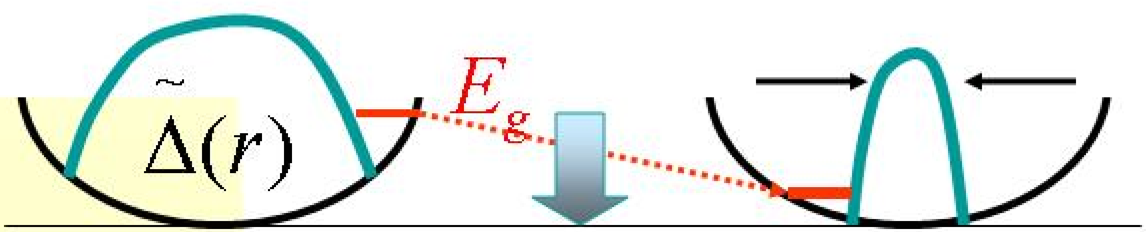




(Sharp peaks come from discrete levels in a hamonic potential.)

A: $\tilde{\Delta} \Uparrow \longrightarrow E_g \Uparrow$

B:

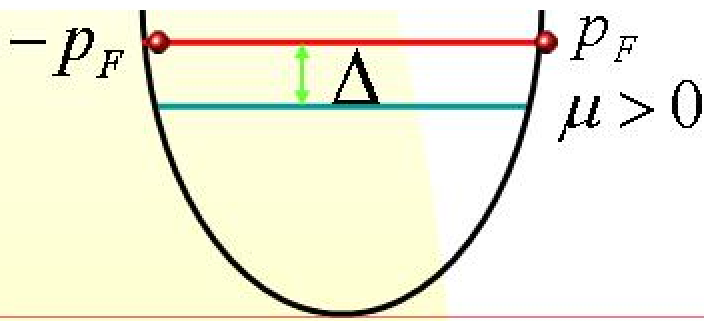


C: Increase of E_g in the BEC regime (ν<0)

$$\boxed{E_p = \sqrt{(\varepsilon_p - \mu)^2 + \tilde{\Delta}^2}}$$

BCS regime: $\mu > 0$

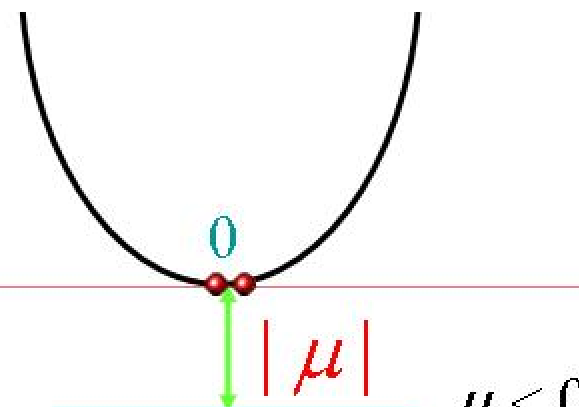
$$E_g = \tilde{\Delta}$$



BEC regime: $\mu < 0$

$$E_g = \sqrt{\tilde{\Delta}^2 + \mu^2} \sim |\mu|$$

$$\sim |\nu|$$



(Leggett (1980))

Collective modes

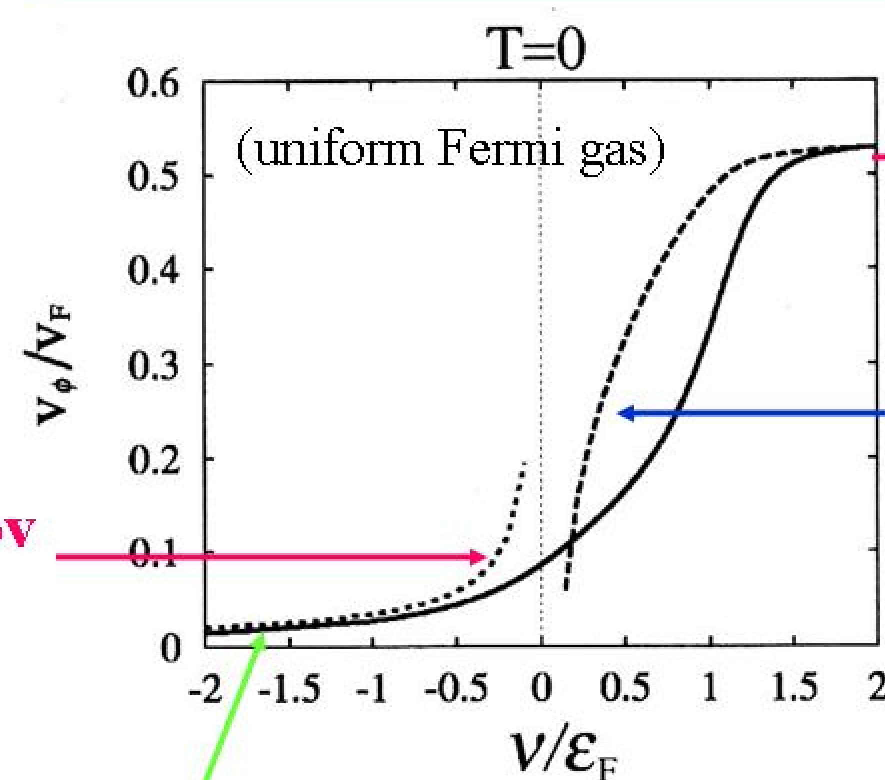
We calculate the density correlation function coupled with superfluid fluctuations in RPA, and obtain the poles due to collective modes.

$$\Pi_{ij} = \langle \rho_i \rho_j \rangle = \begin{pmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} \\ \Pi_{21} & \Pi_{22} & \Pi_{23} \\ \Pi_{31} & \Pi_{32} & \Pi_{33} \end{pmatrix} = \sum_{L,M} Y_{LM}(\Omega) \Pi^L_{ij}(r,r') Y_{LM}(\Omega)$$

- $\rho_i(r) = \Psi^\dagger(r)\tau_i\Psi(r)$ **(1:ampliton, 2:phason, 3,density)**
- $$\Pi^L(r,r') = \Pi^L_0(r,r') - \frac{U}{2}\int dr'' r''^2 \Pi^L(r,r'')\Pi^L_0(r'',r') + \frac{g^2}{2}\int dr'' r''^2 \int dr''' r'''^2\, \Pi^L(r,r'') D_L(r'',r''') \Pi^L_0(r'',r''')$$

Feshbach molecule Green function

Velocity of Goldstone phonon

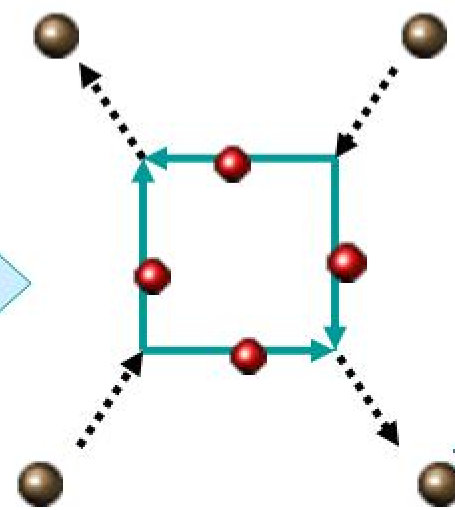


$$v_\phi = \frac{1}{\sqrt{3}} v_F$$

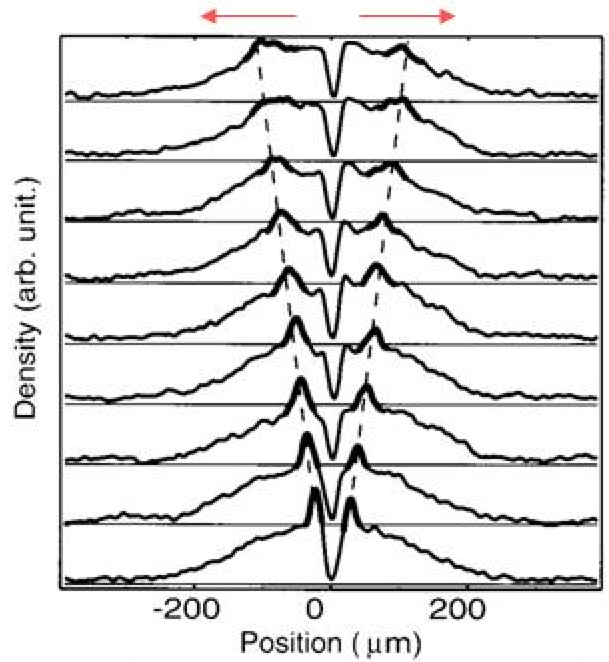
Anderson-Bogoliubov mode

Bogoliubov phonon

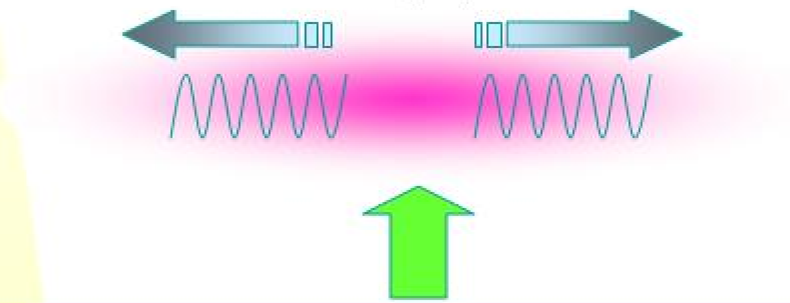
Effective repulsive interaction between molecules is induced by Fermi atoms



Bogoliubov phonon in BEC (Na)

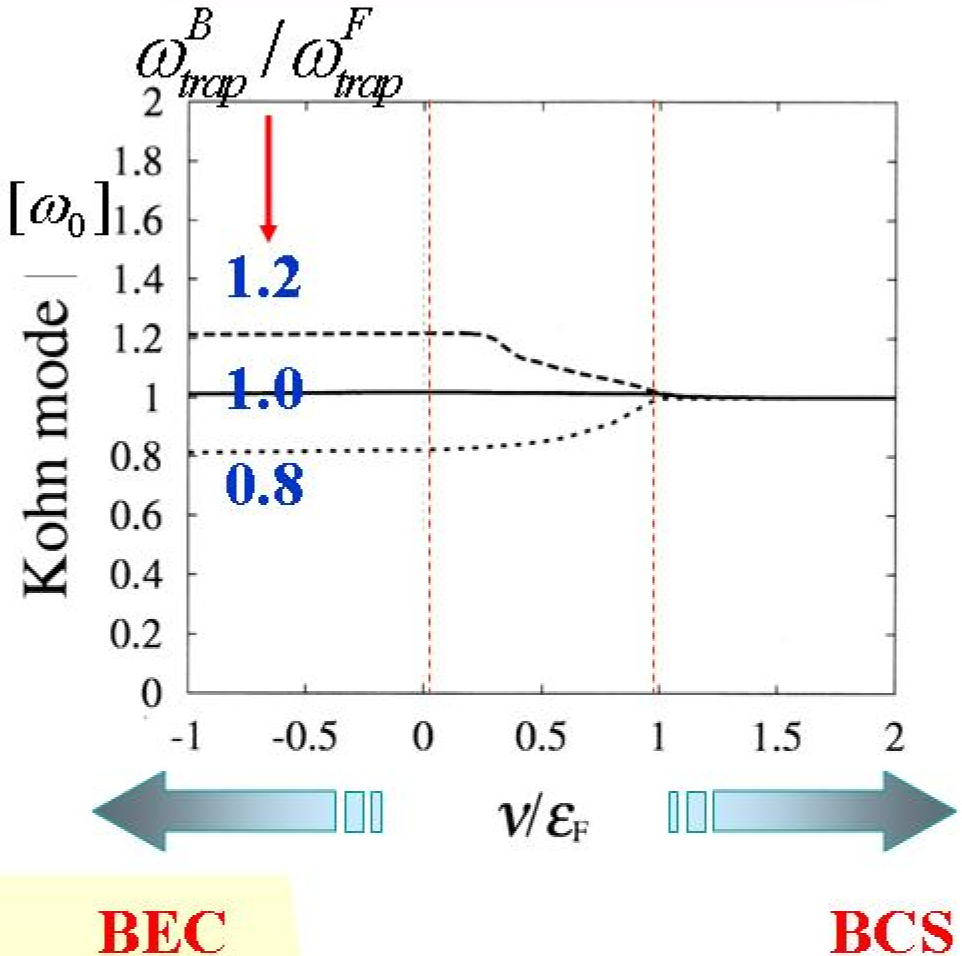


Andrew et al. (1997)



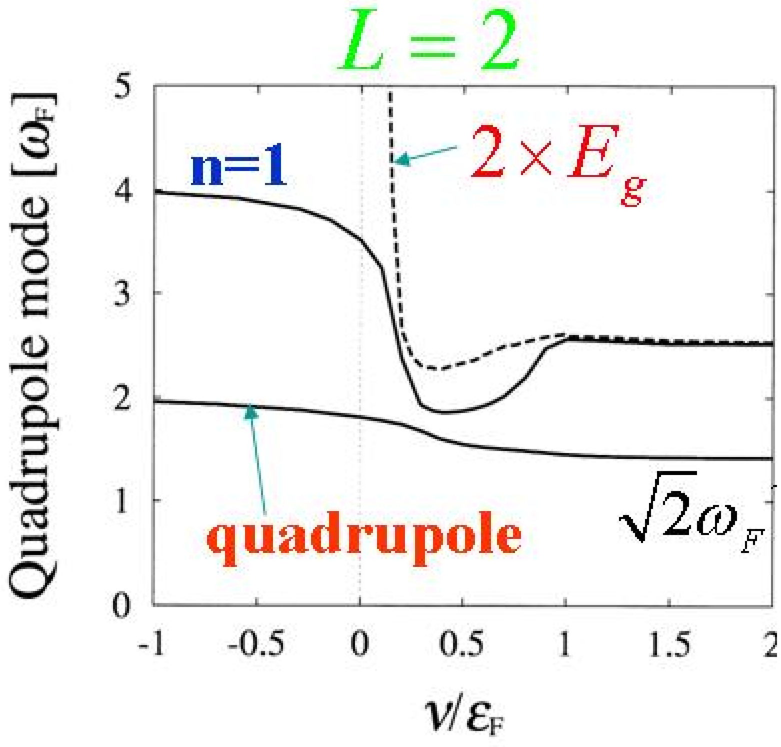
A similar experiment would be also useful for Fermi gases!

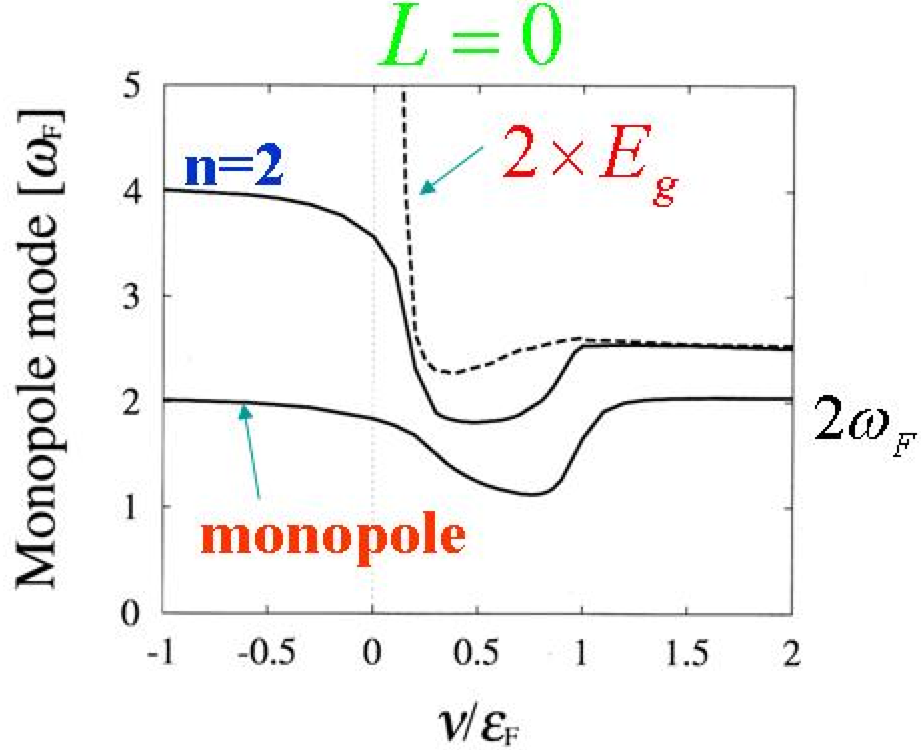
Kohn mode (L=1)



	trap frequency
molecule	ω^B_{trap}
atom	ω^F_{trap}

ν-dependent Kohn mode may be useful as an evidence of the molecular condensate in the BEC regime.

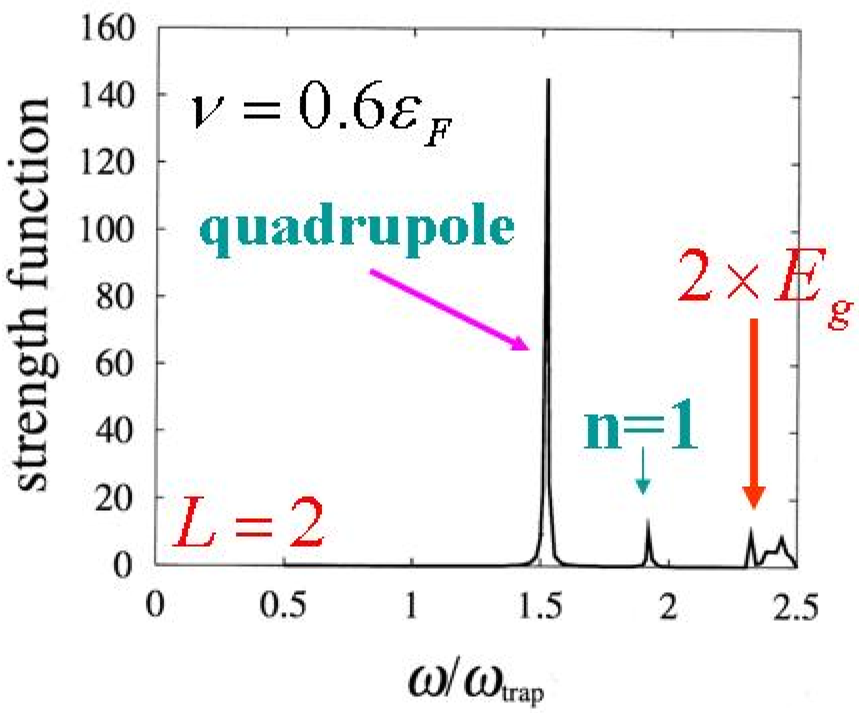


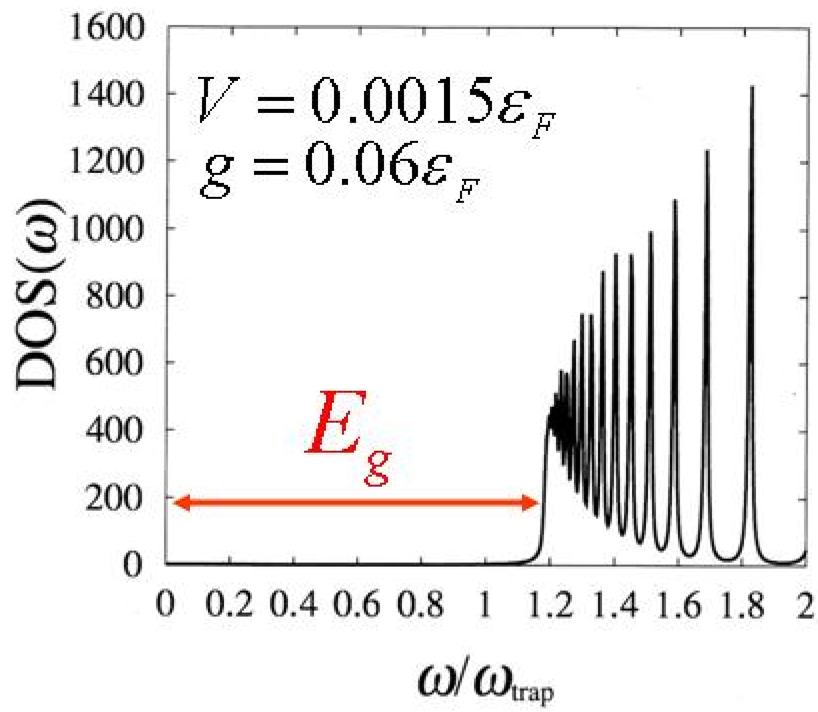


Collective mode energies are suppressed to exist below the two-particle continuum (<$2E_g$). This phenomenon may be useful to distinguish between superfluid phase and hydrodynamic normal phase without a gap.

When one includes interaction between molecules, mode frequencies would agree with hydrodynamic results in the BEC regime. $(BEC : \omega_{quad} = \sqrt{2}\omega_0, \omega_{mono} = \sqrt{5}\omega_0)$

Spectrum of the density correlation function





Spectrum of density correlation function gives
(1) collective mode frequency,
(2) single-particle excitation gap (x2).

Summary

We have discussed single-particle and collective excitations in the BCS-BEC crossover in a trapped Fermi gas with a Feshbach resonance.

The crossover is smooth, where the character of condensed bosons continuously changes from Cooper-pairs to molecules associated with a Feshbach resonance.

Single-particle excitation gap takes a minimum in the Crossover regime, which affects collective mode frequencies.

Summary 2

The Kohn's theorem is broken if the trap frequency of molecules can be different from the trap frequency of atoms (using their different hyperfine states(?)). This phenomenon may be useful as an evidence that the character of the bosons really changes from Cooper-pair to molecule in the crossover.

In the crossover regime, quadrupole and monopole modes are suppressed to exist below the threshold of two-particle continuum, which is twice as large as the single particle excitation gap. We emphasize that this suppression is absent in the hydrodynamic normal phase with no energy gap.

Inclusion of interaction between molecules, and extension to finite temperatures are our future problems.

END