Three challenges of AMO physics:
Hannover variations on strongly correlated systems

Hannover – Theoretical Quantum Optics

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Theoretical Quantum Optics

at the University of Hannover

Cold atoms and cold gases:

- Weakly interacting Bose and Fermi gases (solitons, vortices, phase fluctuations, atom optics, quantum engineering)
- Dipolar Bose and Fermi gases
- Collective cooling, CW atom laser, quantum master equation
- Strongly correlated systems in AMO physics

Quantum Information:

- Quantification and classification of entanglement
- Quantum cryptography
- Implementations in quantum optics

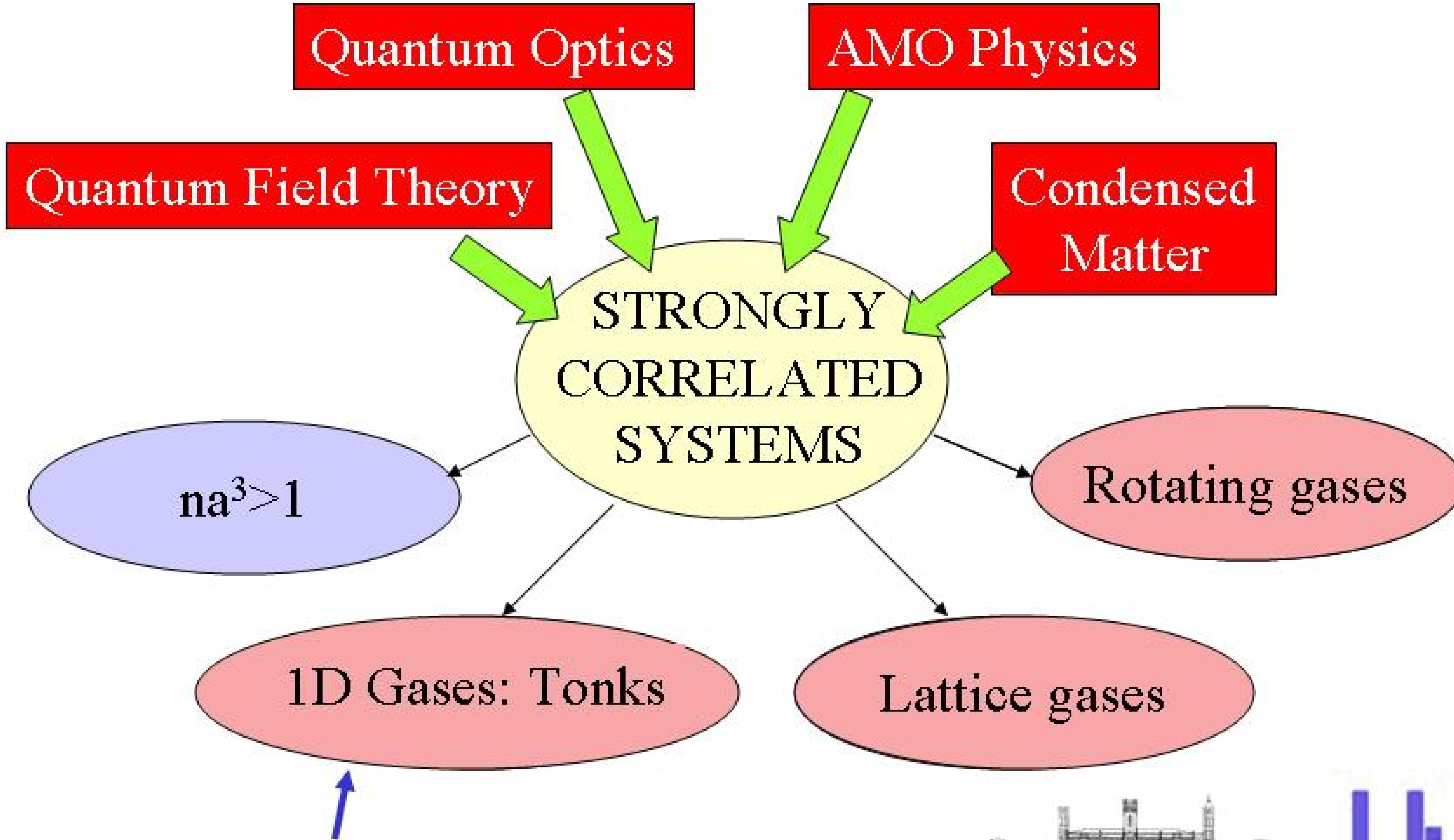
Matter in strong laser fields:

- **High harmonic generation, above threshold ionisation,multielectron ionization**
- **Attosecond physics**

Atomic physics beats condensed matter physics ?

(attributed to Wolfgang Ketterle)

Strongly correlated systems in Hannover



Series of papers by Luis Santos et al.

Sonderforschungsbereich 407
Quantenlimitierte Messprozesse mit Atomen, Molekülen und Photonen

Outline

1. Introduction:

- Bose gas in an optical lattice: From superfluid to Mott insulator

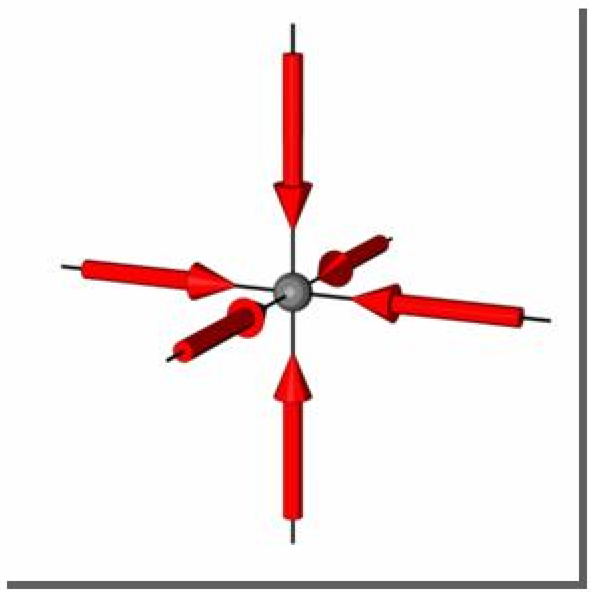
2. Three challenges of the physics of lattice gases:

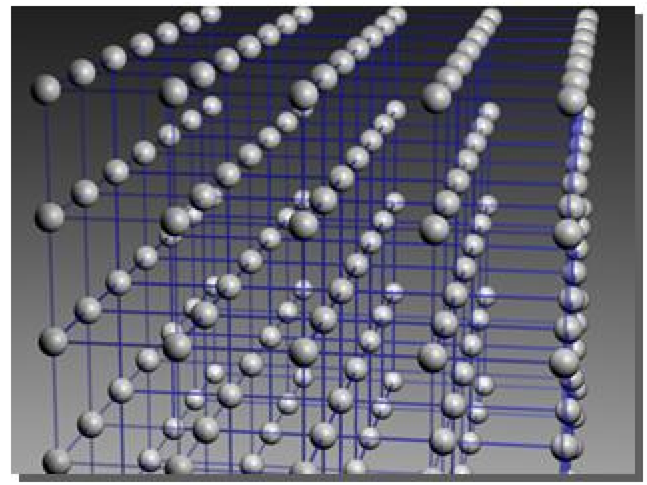
- Fermi-Bose mixtures in optical lattices: Atomic physics goes beyond condensed matter physics!!!
- Fermi-Bose mixtures in random lattices: From Fermi glass to fermionic spin glass and quantum percolation
- From triangular to Kagomé lattice: generating a spin liquid with resonating valence bonds states "in vivo"

3. Quantum fractional Hall effect in rotating dipolar gases:

- Survival of energy gap in the thermodynamical limit

3D Lattice Potential (by courtesy of M. Greiner, O. Mandel, I. Bloch, and T. Hänsch)





- ***Resulting potential consists of a simple cubic lattice***
- ***BEC coherently populates more than 100,000 lattice sites***

V_0 ***up to 22 E_{recoil}***

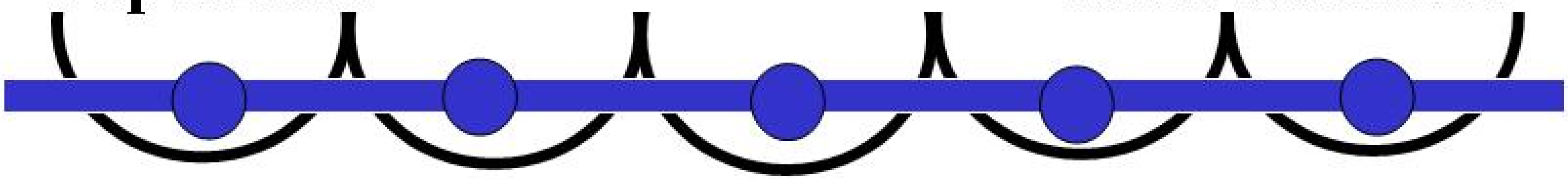
ω_r ***up to $2\pi \times 30$ kHz***

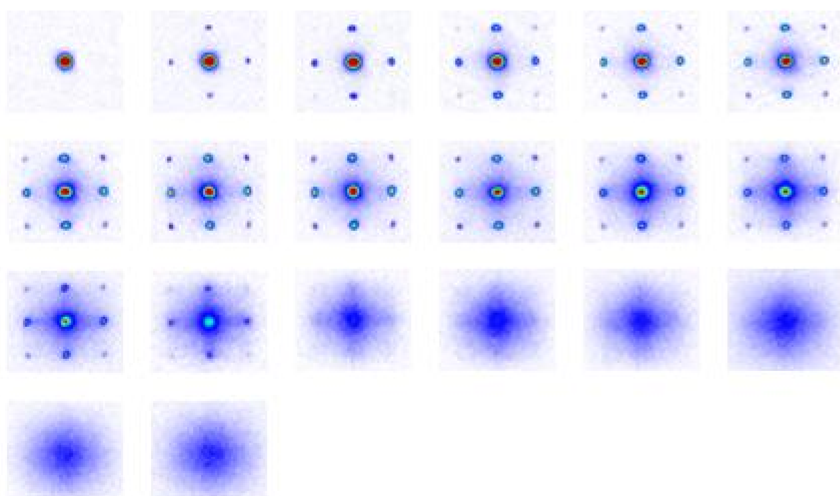
n ⊕ ***1-5 atoms on average per site***

Bose gas in an optical lattice

Idea: D. Jaksch, C. Bruder, J.I. Cirac, C.W. Gardiner and P. Zoller

Superfluid **Mott insulator**





Bose-Hubbard model

$$H=\frac{1}{2}\sum_{i} U n_i(n_i-1)-\frac{1}{2}\sum_{<ij>} t b_i^{+} b_j + h.c. +$$

By courtesy of M. Greiner, I. Bloch, O. Mandel, and T. Hänsch

Challenge Nr. 1

Bose-Fermi mixtures in optical lattices

M. Lewenstein, L. Santos, M. Baranov, and H. Fehrmann

cond-mat/0306180, PRL **92**, 050401 (2004)

H. Fehrmann, M. Baranov, B. Damski, M. Lewenstein, and L. Santos

cond-mat/0307635, submitted to Opt. Comm. (2004)

H.Fehrmann, M. Baranov, M. Lewenstein, and L. Santos

Optics Express **12**, 55 (2004)

Why is it challenging?

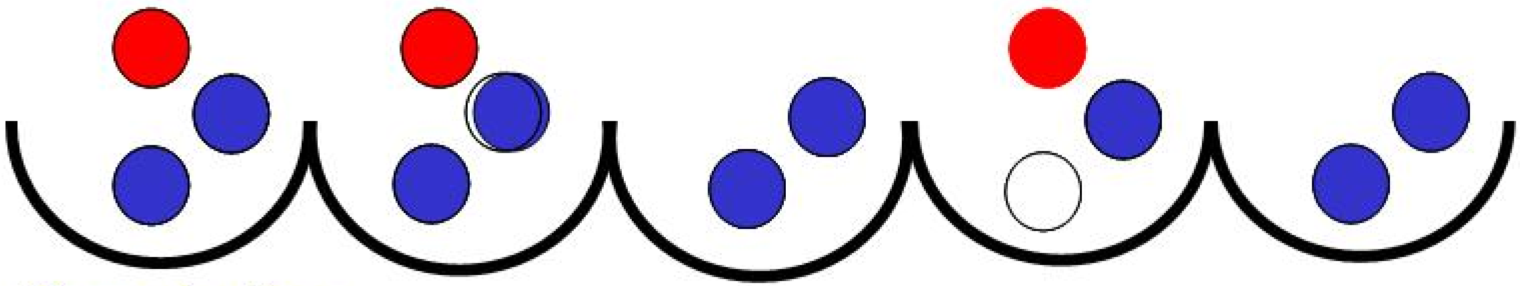
Fermi-Bose mixtures in optical lattices:

- Fermions and bosons on equal footing in a lattice: Atomic physics goes beyond condensed matter physics!!!
- Novel quantum phases and novel kinds of pairing: Fermion-boson pairing!!!
- Novel possibilites of control of the system

Mayor players in the (theory) game:

- A. Albus, J. Eisert (Potsdam), F. Illuminati (Salerno), H.P. Büchler (Innsbruck), G. Blatter (ETH), A.B. Kuklov, B.V. Svistunov (Amherst/Kurchatov) M.Yu. Kagan, D.V. Efremov, A.V. Klaptsov (Kapitza), M.-A. Cazalilla (Donostia), A.F. Ho (Birmingham)

Quantum phases of the Bose-Fermi Hubbard model



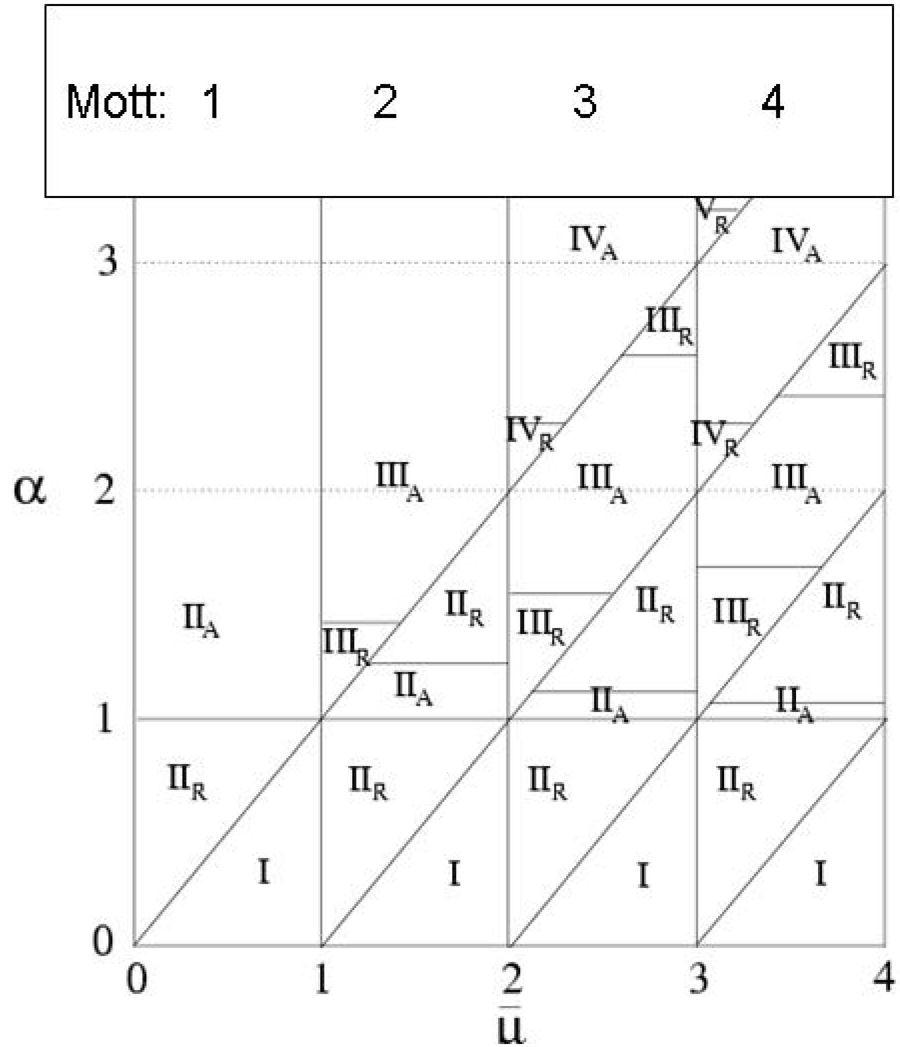
Description:

i) Bose-Fermi Hubbard model
ii) Phase I – Mott (n) plus Fermi gas of fermions with NN interactions
iii) Phase II – Interacting composite fermions (fermion + bosonic hole)
iv) Phase III – Interacting composite fermions (fermion + 2 bosonic holes)

Quantum phases of the Bose-Fermi Hubbard model

Description:

i) Effective spinless Fermi Hubbard model
ii) Phase I – Mott (n) plus Fermi gas of fermions with NN interactions
iii) Phase II – Interacting composite fermions (fermion + bosonic hole)
iv) Phase III – Interacting composite fermions (fermion + 2 bosonic holes)

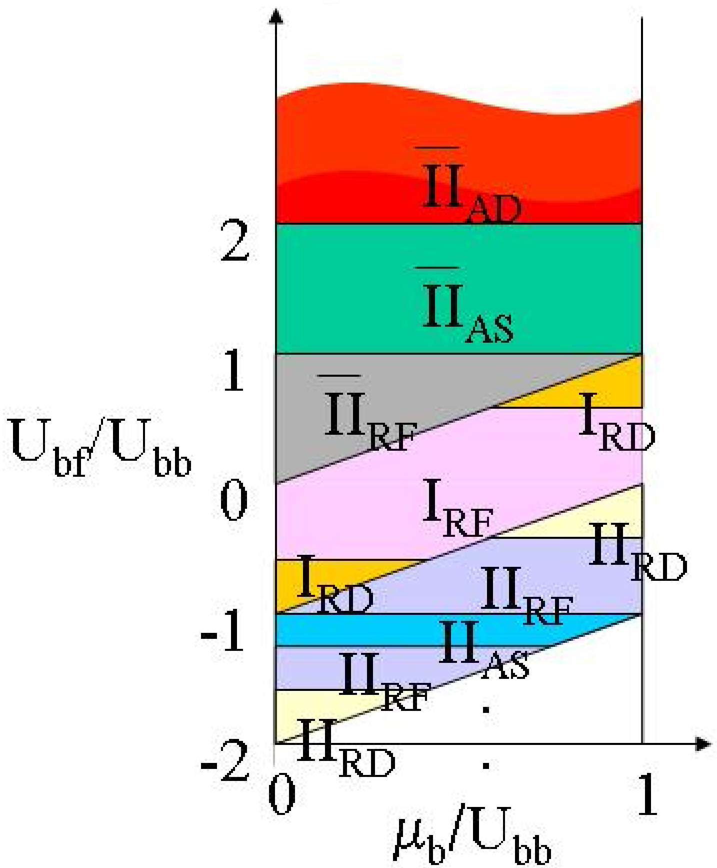


α = ratio of fermion-boson to boson-boson interaction energy,
μ = chemical potential in units of boson-boson interaction energy

Lattice gases: Bose-Fermi mixtures

- Low tunneling $J << U_{bf}, U_{bb}$
- Effective Fermi-Hubbard Hamiltonian

$$H_{eff} = \sum_{<ij>} \left(-J_{eff} \left[C_i^+ C_j + h.c. \right] + U_{eff} N_i N_j \right)$$



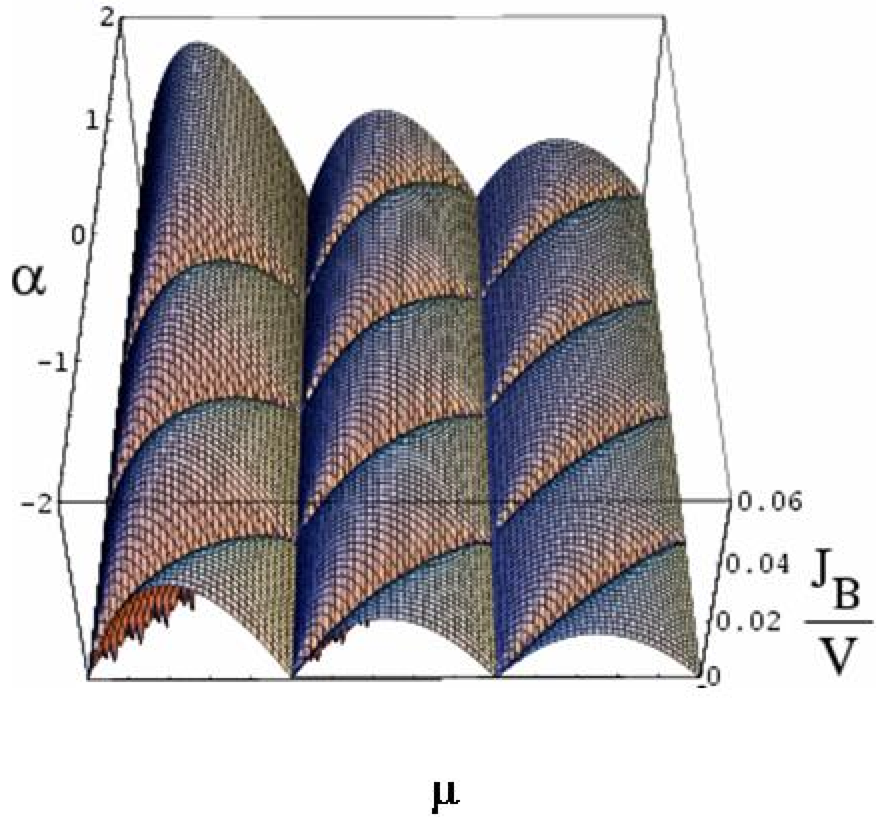
<u>Different quantum phases</u>

Attractive: Superfluid
fermionic Domains

Repulsive: Fermi liquid
Density modulations

Quantum phases of the Bose-Fermi Hubbard model – boundaries of the strongly correlated phases

Generalization of the „handbook" result of Fisher *et al.* from 1989



Challenge Nr. 2

Bose-Fermi mixtures in random optical lattices: From Fermi glass to fermionic spin glass and quantum percolation

A. Sanpera, A. Kantian, L. Sanchez-Palencia, J. Zakrzewski, and M. Lewenstein

cond-mat/0402375, and submitted to Phys. Rev. Lett. (2004)

Why is it challenging?

Fermionic spin glasses in optical lattices:

- Spin glasses (SG) are spin systems with random (disoredered) interactions: equally probable to be ferro- or antiferromagnetic. The spin behaviour is dominated by frustration!!! The nature of ordering in SG poses one the most outstanding open questions of classical (*sic!*) and quantum statistical mechanics.
- In Parisi´s picture SG has very many possible ground state configurations (SG phase consists of very many pure thermodynamic phases). This picture is presumably correct for SG with infinite range interactions.
- In the „droplet´s" picture SG has one, or few similar ground states.

How do the short range SG behave?

How to make a quantum SG with atomic lattice gas?

Use spinless fermions or bosons with strong repulsive interactions:

- There can be $N_i = 1$ or $= 0$ atoms at a site!! We can define Ising spins $s_i = 2 N_i - 1$. What we need are RANDOM NEXT NEIBOUR INTERACTIONS,

$$H_{QSG} = \Sigma K_{nm} s_n s_m + \text{quantum tunneling terms} + \ldots$$

Generate random K_{nm} using random on-site potential and tunneling induced interactions between composite fermions, bosons etc.

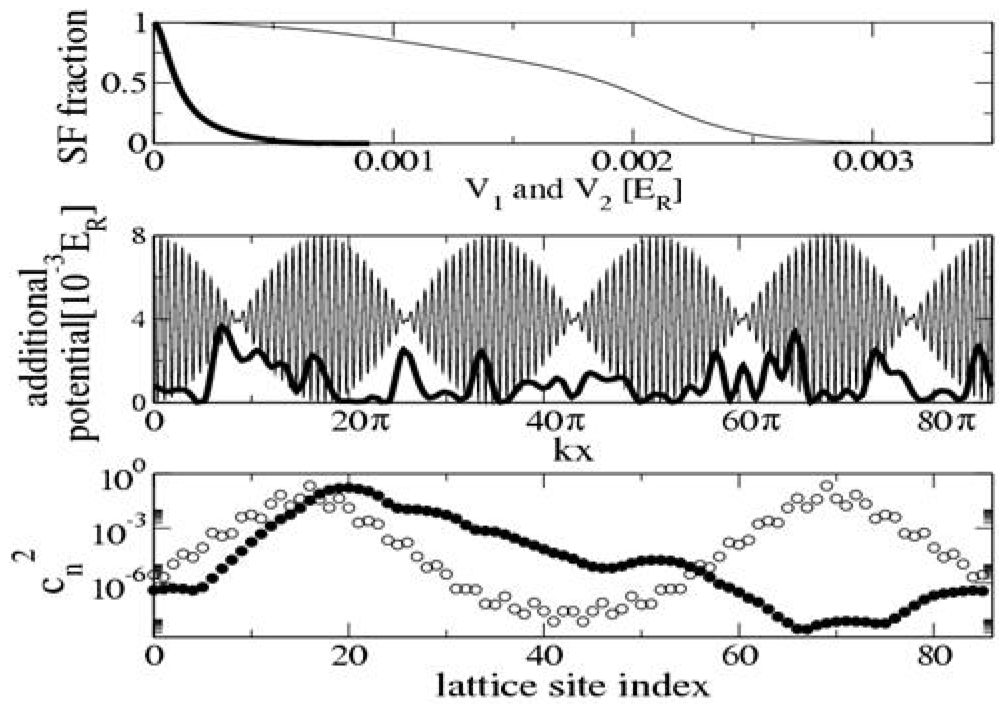
- We have applied this idea to Bose-Fermi mixtures in optical lattices with random on-site potential with appropriate deterministic inhomogeneous part (employing superlattices).
- But, before discussing the results for BF mixture let us look at a simple lattice Bose gas in a random on-site potential.

Anderson and Bose glasses in (“quasi”) random optical lattices

B. Damski, J. Zakrzewski, L.Santos, P. Zoller,
and M. Lewenstein,

PRL **91**, 080403 (2003)

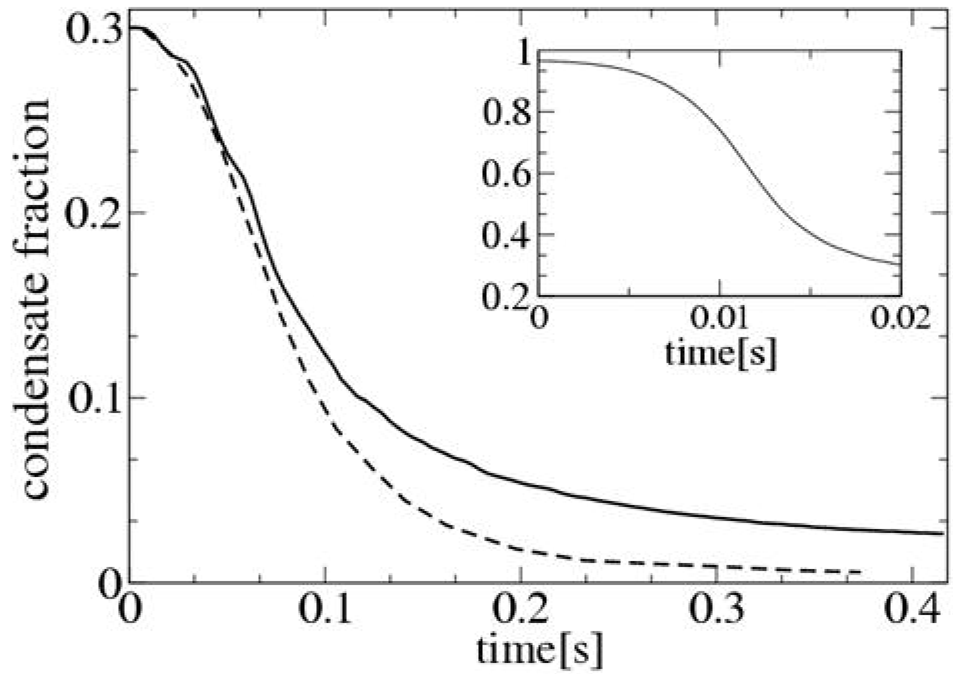
Creating Anderson glass in a (“quasi”) random lattice



Description:

i) Hubbard model with random on-site energies
ii) negligible on-site interactions
iii) „boost“ method to calculate the SF fraction
iv) localization of the condensate wave functions

Bose glass in a (“quasi”) random lattice



Description:

i) time-dependent Hubbard model with random on-site energies
ii) growth of the disorder
iii) „boost“ method to calculate the SF fraction
iv) rapid decrease of the SF and the condensate fraction

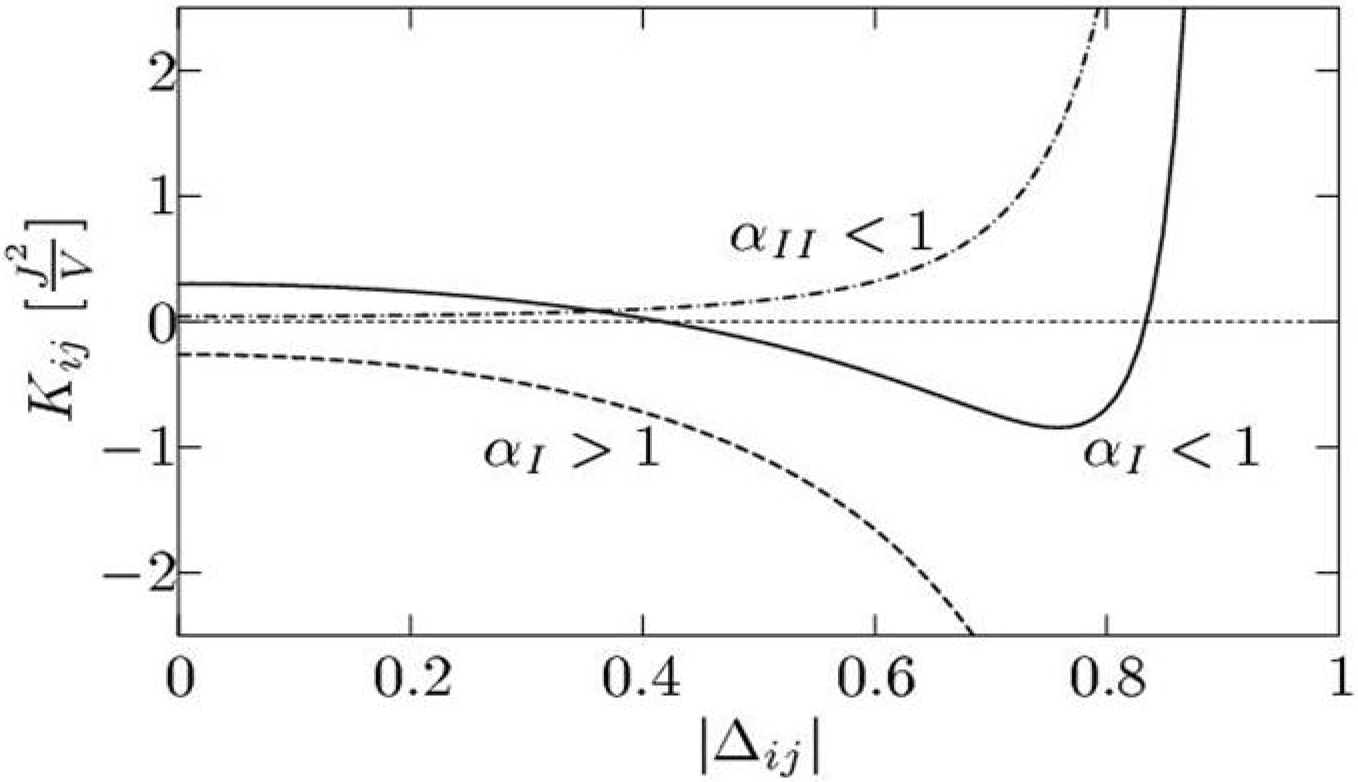
Challenge Nr. 2

Bose-Fermi mixtures in random optical lattices: From Fermi glass to fermionic spin glass and quantum percolation

A. Sanpera, A. Kantian, L. Sanchez-Palencia, J. Zakrzewski, and M. Lewenstein

cond-mat/0402375, and submitted to Phys. Rev. Lett. (2004)

Effective n.n. coulings in FB mixture in a random optical lattice



Here $\Delta_{ij} = \mu_i - \mu_j$

Physics of Fermi-Bose mixtures in random optical lattices

Regime of small disorder (weak randomness of on-site potential)

- With weak repulsive interactions we deal essentially with a Fermi glass (i.e. an analog of Fermi liquid, but with Anderson localized quasi-particle states)
- With attractive interactions we deal with the interplay of superfluidity and disorder
- Both situations might occur simultaneously with quantum site percolation (some sites might be „blocked“)

Regime of strong disorder

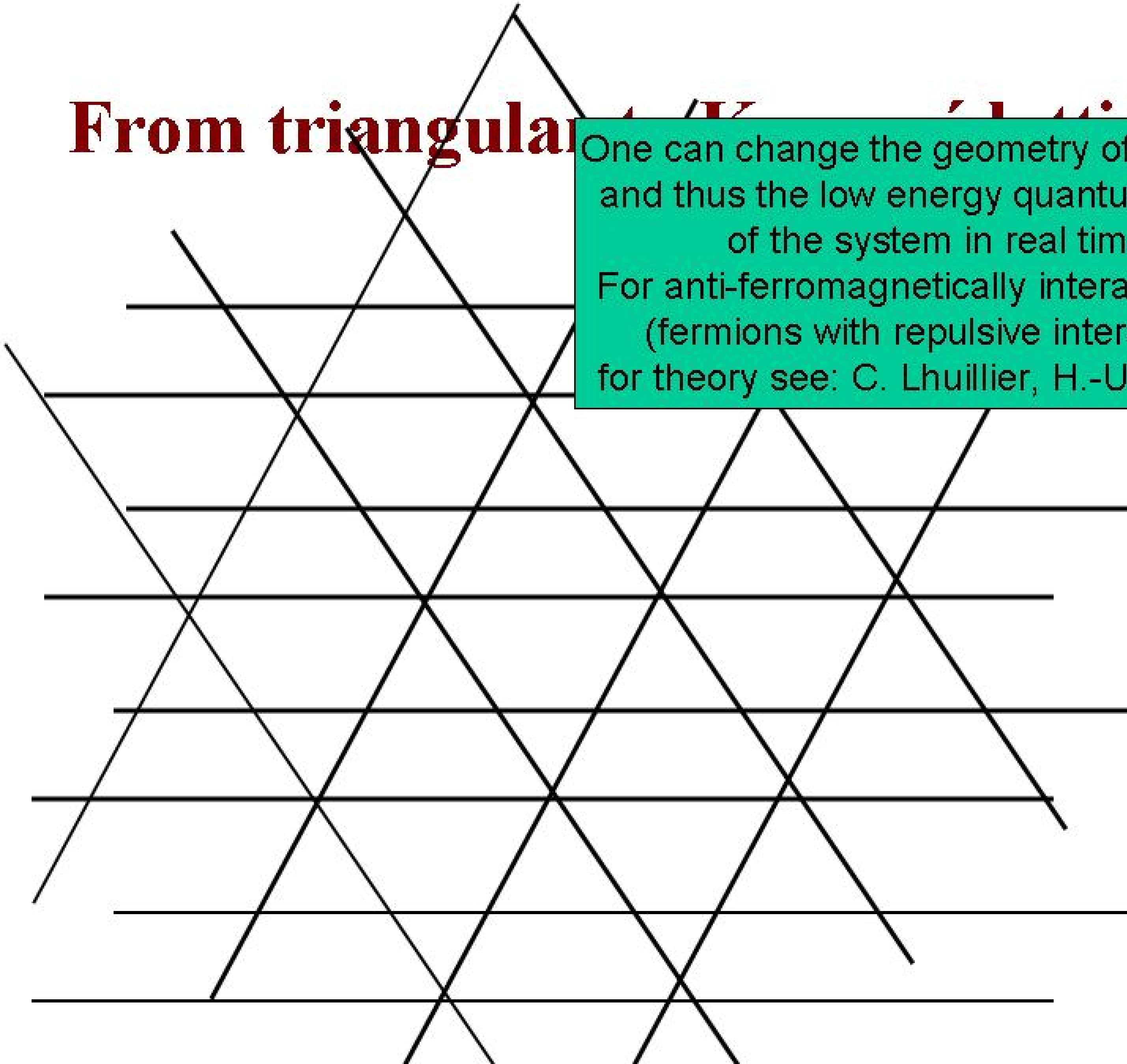
- Using the superlattices method we may make local potential to fluctuate on n.n. sites strongly, being zero on the mean. This leads to quantum fermionic spin glass
- There is a possibility of novel metallic phases at the interplay between disorder, hopping and n.n. interactions

Challenge Nr. 3

From triangular to Kagomé lattice: generating an „exotic“ spin liquid “in vivo“

L. Santos, M.A. Baranov, J.I. Cirac, H.-U. Everts,
H. Fehrmann and M. Lewenstein,
cond-mat/0401502, and submitted to Phys. Rev. Lett. (2004)

From triangular



One can change the geometry of the lattice, and thus the low energy quantum physics of the system in real time!!!
For anti-ferromagnetically interacting spins (fermions with repulsive interactions)
for theory see: C. Lhuillier, H.-U. Everts...

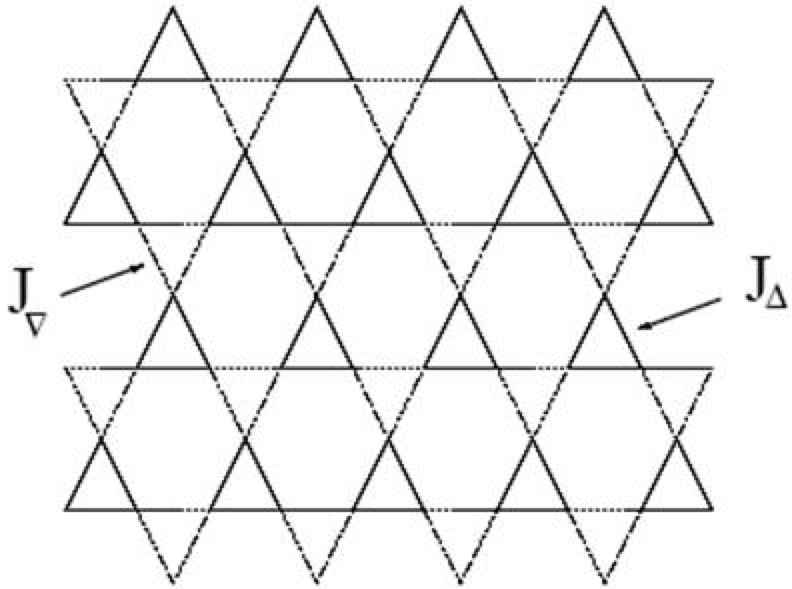
Why is it challenging?

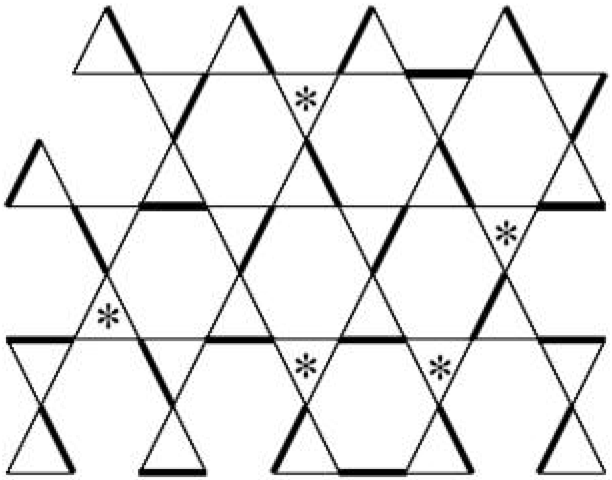
Frustrated quantum antiferromagnets:

- Frustrated antiferromagnets lie at heart of modern condensed matter theory, and play essential role in theory of high T_c superconductivity. In 1973 P.W. Anderson proposed that low T states of such systems are resonating-valence bond (RVB) states, for which the lattice is covered with dimers, i.e. singlet states of pairs of spins. Colossal contributions from P.W. Anderson, F.D.M. Haldane and others...
- Physics of Heisenberg antiferromagnet on the Kagomé lattice has been shown to be governed by RVB states, that form continuous bands of singlet and triplet excitations, separated by a small gap (if any).
- The system forms a rather exotic spin liquid at low T

Trimerized Kagomé lattice and relevant RVB states

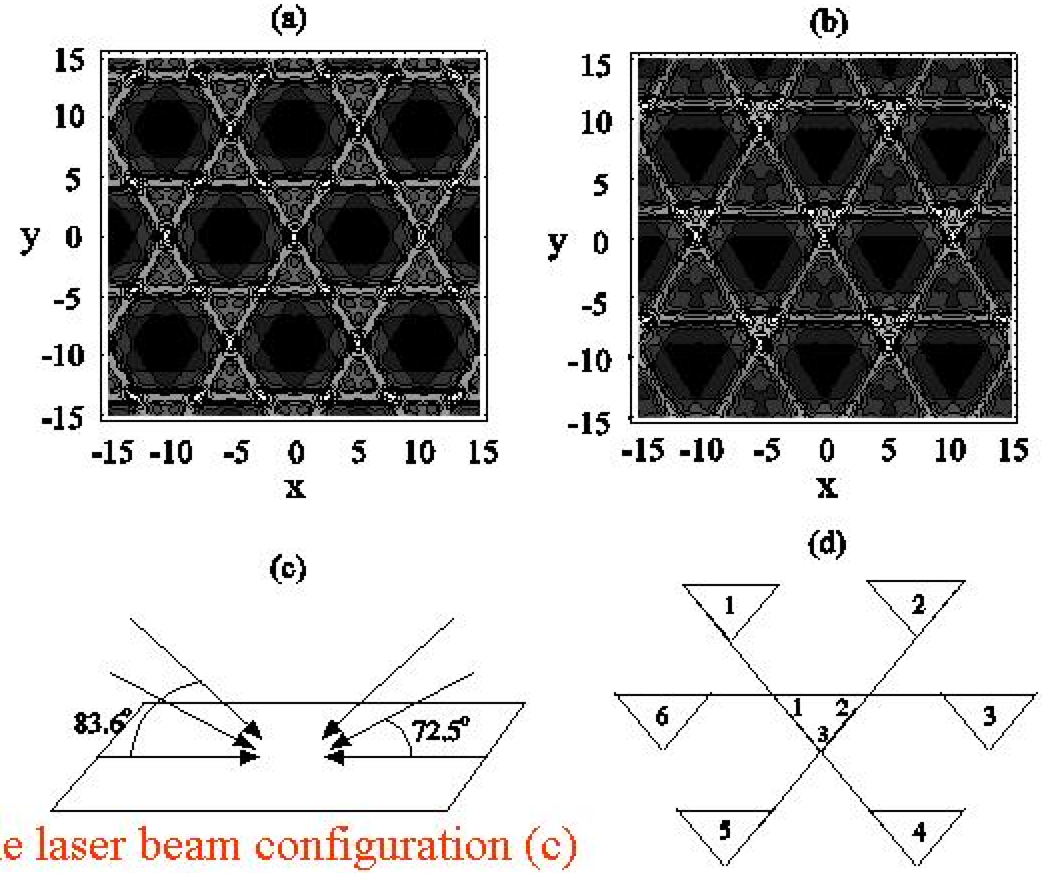
In trimerized Kagomé lattice we deal with a triangular lattice of trimers connected via weaker bonds.





Typical dimer covering of the trimerized Kagomé lattice

How to generate the Kagomé and *trimerized* Kagomé lattice?



One may use the triple laser beam configuration (c) in three directions differing by $2\pi/3$ in the xy-plane, and phase shift one of the triplets!

Physics of atomic gases in trimerized Kagomé lattice

- An atomic Bose gas in TKL exhibits a novel Mott insulator states with fractional filling 1/3 or 2/3. On the trimers the bosons (bosonic holes) a W-state.

- A spinless interacting Fermi gas (i.e. for instance gas of composite Fermi-Bose fermions) behaves as spinless interacting Fermi gas on the underlying triangular lattice at 1/3 filling. At 2/3 filling it behaves as quantum magnet on the triangular lattice with anisotropic bonds.

- A Fermi-Fermi mixture with half filling for both components represents a frustrated Heisenberg antiferromagnet with resonating-valence-bond ground state and quantum spin liquid behavior dominated by continuous spectrum of singlet anf triplet excitations.

How to prepare a covering of the lattice by singlet dimers?

- Go the 4th floor in Innsbruck. Take a molecular BEC, possibly at the temperature less than 10 nK. Start slowly increasing the scattering length.
- The molecules will start to grow and become less bounded. When their size will be comparable to the lattice constant, start slowly growing the lattice. The molecules should dissociate smoothly, and the resulting two fermions will sit in n.n. sites in the lattice
- The fermion pairs will be in the siglet state of the pseudo-spin, i.e. their spin wave function will be antisymmetric with respect to the exchange of all (both nuclear and electronic) spin variables.
- In this way a desired covering of the lattice with singlet dimers will be achieved!!!

Suplementary challenge Nr. 4

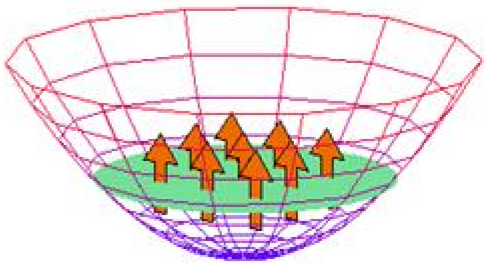
Quantum fractional Hall effect in rotating dipolar gases: survival of the energy gap

M.A. Baranov, K. Osterloh, and M. Lewenstein,

in preparation (2004)

FQHE states in rotating dipolar Fermi gases

- quasi 2D, rapidly rotating ⇨ strong correlations dominate
- rotation close to trap frequency leads to Quantum Hall regime



$$\mathcal{H} = \sum_{j=1}^{N}\left(-\frac{\hbar^2}{2m}\nabla_j{}^2 + \frac{1}{2}\omega_0^2\vec{r}_j^{\,2} - 2\,\vec{\omega}\cdot\vec{L}_j\right) - \sum_{j<k}^{N}\frac{2d^2}{|\vec{r}_j - \vec{r}_k|^3}$$

- dipole-dipole interaction ⇨ gap in the excitation spectrum
- energy gap derived from two-particle distribution function:

$$\Delta\varepsilon = \int \mathrm{d}^2 z_1 \mathrm{d}^2 z_2 \,\frac{2d^2}{|\vec{r}_j - \vec{r}_k|^3}\left(g_{\mathrm{qh}}^{(2)}(z_1,\, z_2) - g_0^{(2)}(z_1,\, z_2)\right)$$

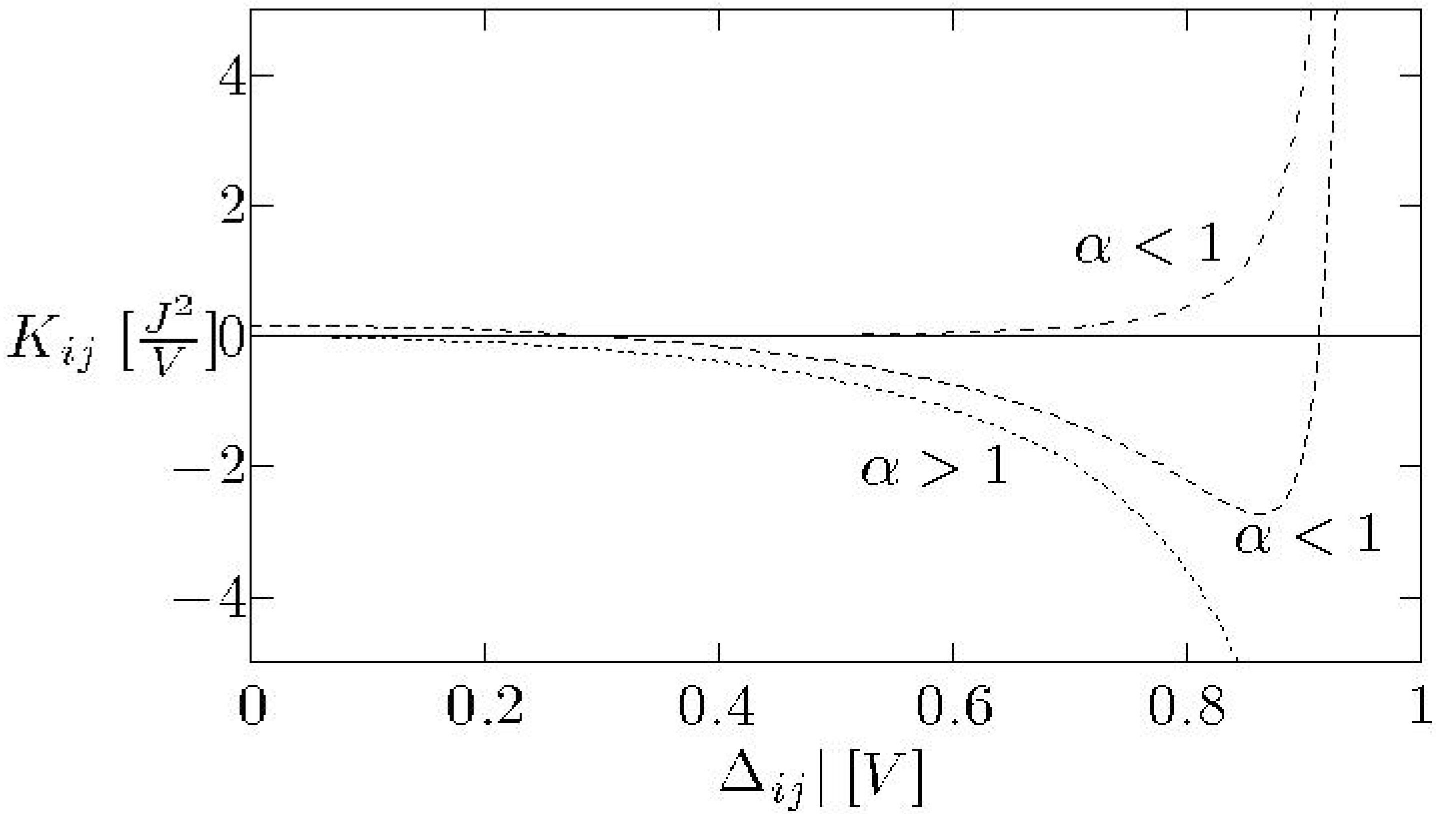
⇨ for Laughlin's state at filling 1/3 ⇨ $\Delta\varepsilon$ ~ 100nK

CONCLUSIONS (The Tragedy of Hamlet, by Shakespeare):

- *There are more thing in heaven and earth, Horatio, than are dreamt of in your philosophy.*

Wow!!!

Effective n.n. coulings in FB mixture in a random optical lattice



Here $\Delta_{ij} = \mu_i - \mu_j$