THE "BCS-BEC CROSSOVER" PROBLEM

Always assume:

free space, 3D

"open" channel s-wave

$a_s, n^{-1/3} \gg r_0$

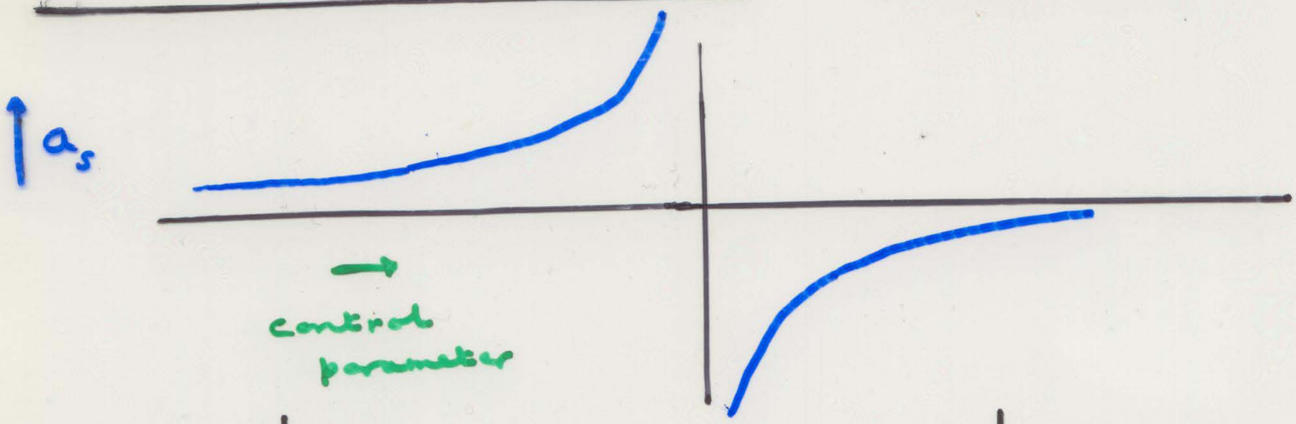
s-wave scattering length

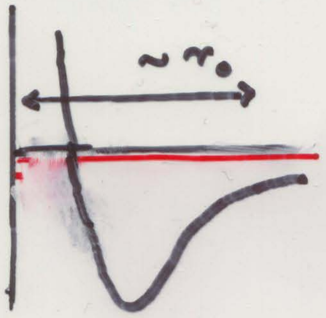
density

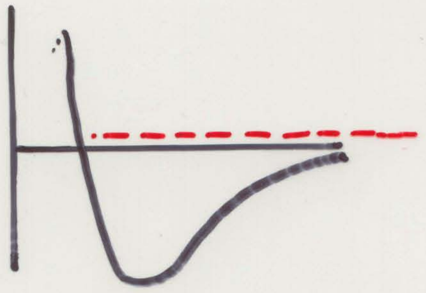
typical range of interatomic potential

only 2 internal states ("$\sigma_z = \pm 1/2$") relevant in open channel.

A. SINGLE-CHANNEL CASE : 2-body problem







only char. length for 2-body problem : $|a_s|$

" " energy : $\hbar^2/m|a_s|^2$

Adiabatic condition : $T \gg ma_s^2/\hbar$

timescale

→ ∞ at onset of b.s.

(SINGLE-CHANNEL CASE, cont.)

MANY-BODY PROBLEM:

only dimensionless parameter is $n^{1/3} a_s$, or equiv.

$$\xi \equiv -1/k_F a_s$$

Fermi w.v. of free gas ($\propto n^{1/3}$)

$\xi \to -\infty$ corr. molecular b.s., radius $\ll n^{-1/3}$

$\xi \to +\infty$ " BCS limit

($\xi = 0$: onset of bound state in 2-body problem)

T = 0

Plausible ansatz for many-body groundstate: (generalized BCS function):

$$\Psi_N \sim \mathcal{A}\,\varphi(r_1 r_2 \sigma_1 \sigma_2)\,\varphi(r_3 r_4 \sigma_3 \sigma_4) \dots \varphi(r_{N-1} r_N \sigma_{N-1} \sigma_N)$$

$$\varphi(r_1 r_2 \sigma_1 \sigma_2) = \frac{1}{\sqrt{2}}(\uparrow_1 \downarrow_2 - \downarrow_1 \uparrow_2)\, f(|\underline{r}_1 - \underline{r}_2|)$$

form of $f(r)$ det. by minimizing energy for fixed N

Qualitatively correct in limits $\xi \to +\infty$ ($\Rightarrow$ BCS wave function) and $\xi \to +\infty$ (Bose condensate of diatomic molecules)

⚠ : does not give GMB correction to prefactor of Δ in BCS limit.

does not give Bogoliubov-level corrections in BEC limit.

(SINGLE-CHANNEL CASE, contd.)

Standard BCS treatment of ansatz for CSWF:

$$\Psi_N \to \Psi_{BCS} \equiv \prod_k (u_k + v_k a^+_{k\uparrow} a^+_{-k\downarrow})|vac\rangle$$

$(v_k/u_k = \overbrace{c_k)}$ Fourier component of f(r).

Minimize $\langle H\rangle - \mu\langle N\rangle$.

Result is gap eqn. for $F_k \equiv u_k v_k$:

$$F_k = \frac{1}{2E_k}\sum_{k'} V_{kk'} F_{k'} \qquad E_k \equiv \sqrt{(\epsilon_k - \mu)^2 + |\Delta_k|^2}$$

$\hbar^2k^2/2m$

compare Schr. eqn.:

$$c_k = \frac{1}{2(\epsilon_k - E_0/2)}\sum_{k'} V_{kk'} c_{k'}$$

Eliminate V in terms of zero-energy t-matrix V_{eff}: $\equiv \frac{4\pi\hbar^2 a_s}{m}$

$$-\frac{1}{V_{eff}} = \frac{1}{2}\sum_k (E_k^{-1} - \epsilon_k^{-1})$$

ultraviolet & infrared convergent in 3D

+ condⁿ. of no. consⁿ.:

$$N = \sum_k (1 - \epsilon_k/E_k)$$

Dimensionally, $\mu = \epsilon_F \tilde{\mu}(\xi)$, $\Delta = \epsilon_F \tilde{\Delta}(\xi)$

gap eqn. in dimensionless form:

$$\xi = \text{const.}\int d\epsilon\, \epsilon^{1/2}\left\{[(\epsilon-\tilde{\mu})^2 + |\tilde{\Delta}|^2]^{-1/2} - \epsilon^{-1}\right\} \quad \text{etc.}$$

No singularity at $\xi = 0$ (onset of 2p b.s.)

$$E_{min} = \left\{\begin{array}{l} |\Delta|,\ \mu > 0 \\ (|\mu|^2 + |\Delta|^2)^{1/2},\ \mu < 0 \end{array}\right\} \Rightarrow$$ weak singularity at $\mu = 0$?

(SINGLE-CHANNEL CASE, cont.)

FINITE T

Qualitatively, expect 2 types of excitation:

(a) fermionic, with en. gap $\begin{cases} |\Delta|, & \text{if } \mu > 0 \\ (|\mu|^2 + |\Delta|^2)^{1/2}, & \text{if } \mu < 0. \end{cases}$

speed of sound

(b) bosonic, with spectrum $E_k \sim \hbar c_s k$

In BCS limit, contributions to sp. ht. are of order

$$\frac{C_F}{N} \sim k_B (T/T_F) \exp - \Delta/kT$$

$$\frac{C_B}{N} \sim k_B (T/T_D)^3 \qquad (T_D \sim \hbar c_s n^{1/3}/k_B \sim T_F)$$

⇒ fermionic contr.^{n} dominant except for $T \ll T_c$

(regime almost attainable in ^{3}He-B!)

In BEC limit,

$$C_F/N \sim \exp - E_0/2kT$$

$$C_B/N \sim k_B (T/T_D)^3$$

⇒ bosonic excit.^{ns} dominate for $T \ll E_0/k_B$.

The \$64K qn: what is max. value of T_c/T_F?

(and for what value of ξ is it attained?)

Failing that, what is max. value of condensate fraction N_0/N as $f(T/T_F, \xi)$?

Hohenberg's lemma gives very weak upper bound

$(N_0/N \sim (T_F/T)^{3/2})$. Can one do better?

POSSIBLE DEFINITIONS OF "DEGREE OF PAIRING"

	VALUE IN LIMITS:			
DEF^N	BEC, $T=0$	BEC, $T>T_c$	BCS $T=0$	BCS, $T>T_c$
1. Normaliz^n of pair w.f. $N^{-1}\sum_k (\lvert\Delta_k\rvert^2/E_k^2)\times\tanh^2..$	1	0	$\sim\Delta/\epsilon_F$	0
2. Square of order parameter $\lvert\langle\psi_\uparrow(0)\psi_\downarrow(0)\rangle\rvert^2$	~ 1	0	$\sim(\Delta/\epsilon_F)^2$ $(\times \ln^2..)$	0
3. Superfluid fraction $N^{-1}\sum_k(\hbar k)^2(\partial n/\partial E_k) - 1$	1	0	1	0
4. Degree of occupation of "ground pair" state $u_k\lvert 00\rangle + v_k\lvert 11\rangle$	1	0	1	≈ 1

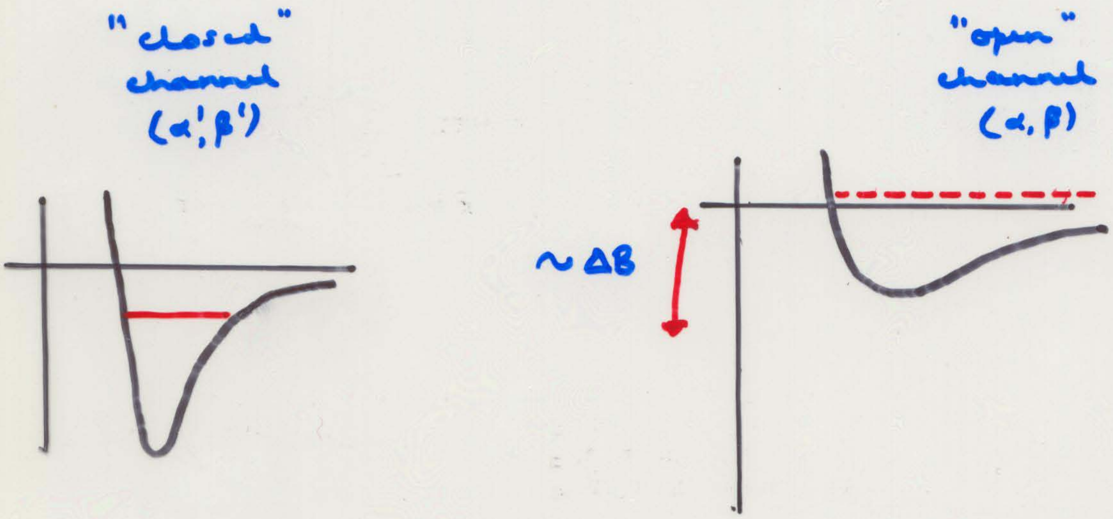
WHICH OF THESE QUANTITIES REMAINS INVARIANT UNDER ADIABATIC CHANGE OF THE CONTROL PARAMETER?

(assume thermal occupation factors unchanged)

1. no
2. no
3. yes, but ...
4. yes

SO: WHAT DOES THE JILA EXPT. ACTUALLY MEASURE?

CASE OF A FESHBACK RESONANCE



for $\Delta B \approx 0$, bound state should be linear superposition of two pairs of spin states (α', β') and (α, β)

Qn: are both indices different in the 2 cases?

If yes, then description in terms of "atom-pair-molecule" hybridizn presumably justified.

If no, ... ?*

In either case, formally can try ansatz

$$\Psi_N \sim \mathcal{A}\,\varphi(r_1 r_2 \sigma_1 \sigma_2)\,\varphi(r_3 r_4 \sigma_3 \sigma_4) \dots \varphi(r_{N-1} r_N \sigma_{N-1} \sigma_N)$$

but dependence on σ_1, σ_2 ($\neq \pm \frac{1}{2}$!) no longer trivial

Adiabatic condition prob.

$$T \gg \hbar/\Delta \leftarrow f(\Delta B), \text{ exp'ly small for } \Delta B \to \infty$$

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* cf. difficulty in Bose case for $a_s \to +\infty$.

CAN WE PROBE INTERNAL STRUCTURE OF C. PAIRS?

(i.e. the dependence of $\langle\psi_\uparrow(\underset{\sim}{r})\psi_\downarrow(\underset{\sim}{r}')\rangle$ on relative coordinate $|\underline{r}-\underline{r}'|$?)

What depends on this?

classic ($l=0$) superconductors: electromagnetic screening (λ vs λ_L)., vortex core structure.

superfluid ^{3}He: dipole energy etc.

Fermi alkali gases:

ESR analog of ^{3}He NMR?

optical absorption?

FINAL NOTE: THE KINETICS OF BEC

At $T=0$, only obvious timescale is $\hbar/\Delta(0)$.

But near T_c, must

(a) adjust gap to new value

(b) adjust quasiparticle distribution ($f(\Delta_{old}) \rightarrow f(\Delta_{new})$)

(a) presumably takes $\sim \hbar/\Delta(T)$

(b) " " $\sim \tau$ ← q.p. collision time.

Except <u>very</u> close to T_c, $\tau \gg \hbar/\Delta(T)$.

⇒ qp relax^n is probably bottleneck (cf. sup. ^{3}He)