Two particle pairing and creation of composite fermions in Fermi-Bose gases.

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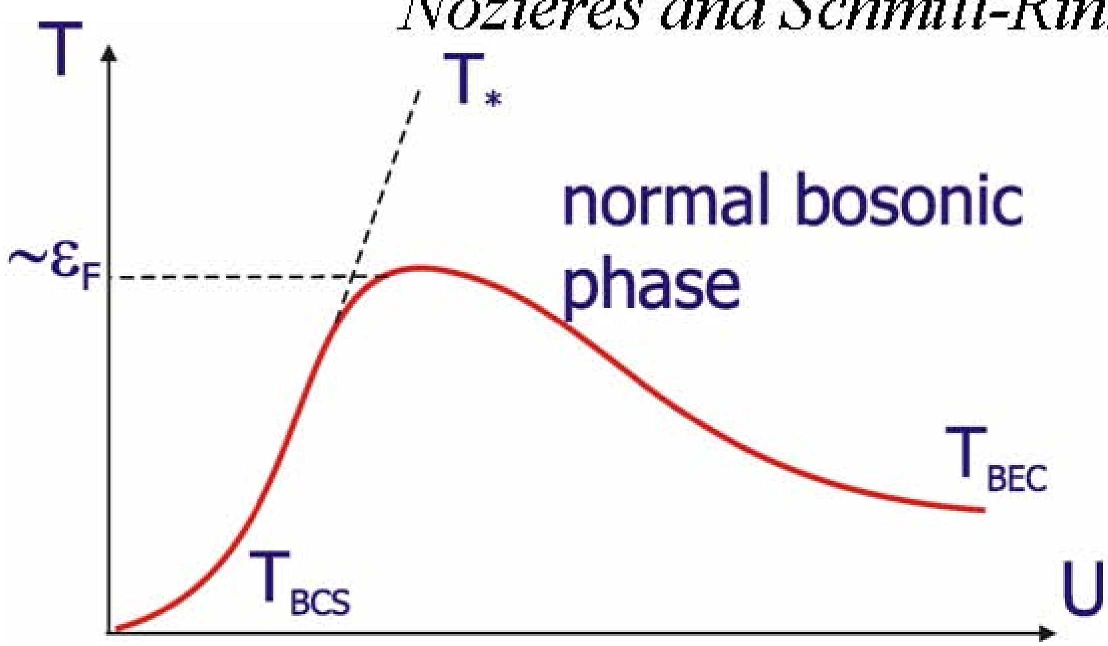
Attractive-U Hubbard model

$$\hat{H} = -t\sum_{<ij>} c_{i\sigma}^{+} c_{j\sigma} - U\sum_{i} n_{i\uparrow} n_{i\downarrow}; \quad n_{i\sigma} = c_{i\sigma}^{+} c_{i\sigma}$$

short range attraction between fermions

3D case

Leggett (1980), Nozieres and Schmitt-Rink (1985)



T_* - a temperature of the creation of bosonic pairs

T_{BEC} - a temperature of the Bose-Einstein condensation

T_{BCS} - a temperature of the BCS transition

Attracting fermions in 3D

a) Weak coupling case $U < U_c \sim W = 12t$

$$T_c = T_{BCS} \approx 0.28\,\varepsilon_F \exp\left[-\frac{\pi}{2\,|a|\,p_F}\right]$$

$$|a|\,p_F << 1$$

Gor'kov, Melik-Barchudarov (1961)

coherent length $\xi \sim \hbar v_F / T_{BCS} >> d$

$$\mu \sim \varepsilon_F > 0$$

- For $|U| << W$:

$$|a| = \frac{m\,|U|\,d^3}{4\pi} \sim d\,\frac{|U|}{W} << d$$

s-wave scattering length

In the absence of a lattice:

$W \rightarrow 1/2mr_0^2$

$d \rightarrow r_0$;

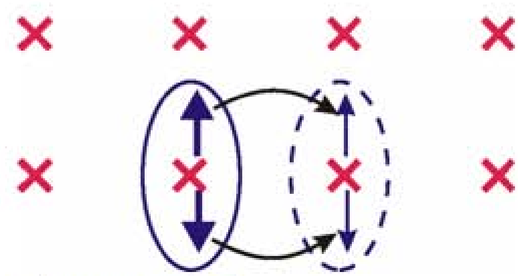
$nd^3 \rightarrow nr_0^3 << 1$

r_0- radius of the potential

b) 3D Strong coupling case $U > U_c$

$$T_{BEC}^{gas} = 3.31\frac{\hbar^2 (n/2)^{2/3}}{4m}$$ in the absence of a lattice (bose gas)

$$T_{BEC}^{lat} \sim \frac{W}{|U|} T_{BEC}^{gas}$$



$$m^* \sim m|U|/W$$

Crossover temperature T_*

Saha formula A+B□AB

E_b – a binding energy of a composite boson

$$n = n_F + 2n_B$$

$$n_F^2 / n_B \sim T^{3/2} \exp\{-|E_b|/T\}$$

$$\text{for} \quad T = T_* : \; n_F = 2n_B = n/2$$

$$T_* \sim \frac{|E_b|}{\frac{3}{2}\ln\left(|E_b|/T_0\right)}$$

$$\left\{ \begin{array}{l} U >> W = 12t : \\ |E_b| = (4/3\pi)U \end{array} \right\}$$

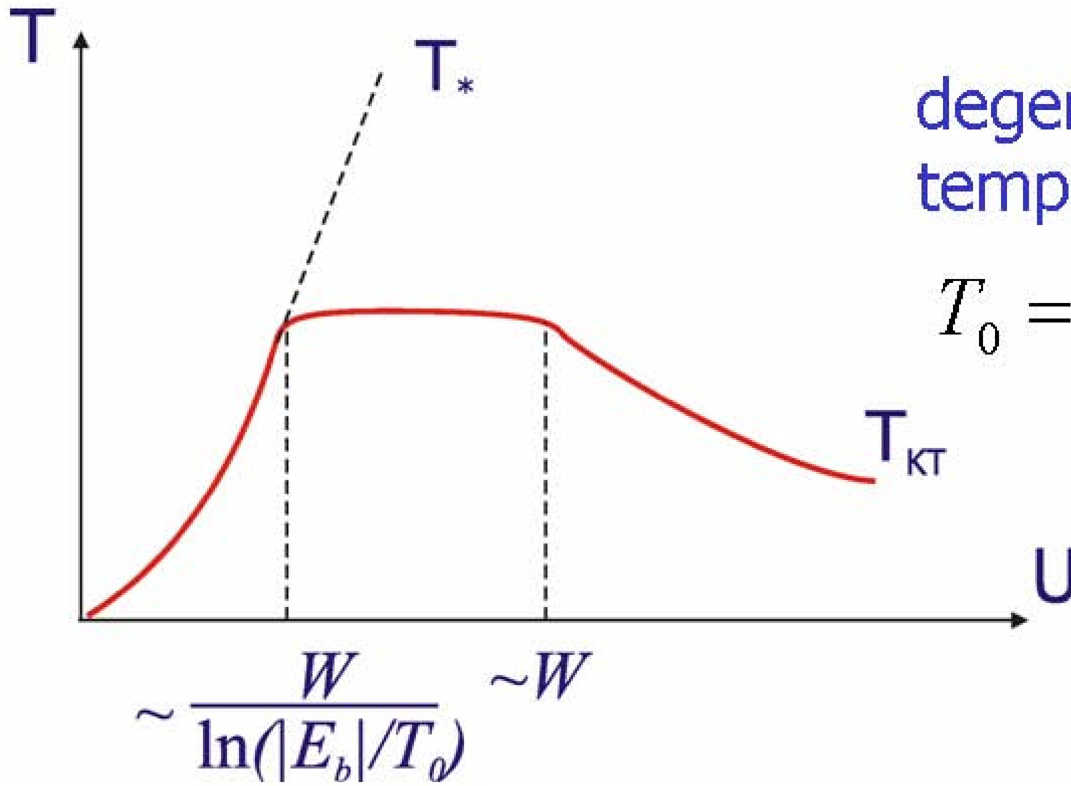
$$T_{BEC} < T < T_* :$$

$$\mu \approx -|E_b|/2 - (3/2)T\ln\left(|E_b|/T_0\right)$$

$$\text{at } T = T_{BEC} : \; \mu_B = 2\mu + |\mathrm{E_b}| = 0$$

Attracting fermions in 2D

(more relevant for HTSC)



degeneracy temperature

$$T_0 = \frac{2\pi n}{m}$$

$$T_* = \frac{|E_B|}{\ln(|E_B|/T_0)}$$ - crossover temperature

a) Weak coupling case $|U| < W/\ln(|E_b|/T_0)$

$$T_c = T_{BCS} = \sqrt{2T_0|E_b|} \quad \textit{Miyake (1983)}$$

$$|E_b| \approx W \exp\{-2W/|U|\} << T_0$$

Note that $\left(T_{KT} - T_{BCS}\right)/T_{BCS} << 1$

Mooji (1983)

b) 2D Intermediate coupling case

$$W / \ln(|E_b| / T_0) < |U| < W$$

$$T_{KT} = \frac{T_0}{2 \ln \ln(|E_b| / T_0)}$$

Fisher, Hohenberg(1988)

$$|E_b| = \frac{1 / md^2}{\exp(4\pi / m |U|) - 1} < W$$

Kagan, Beck(1998)

$$T_{KT} < T < T_* : \ \mu \approx -|E_b| / 2 - T \ln(|E_b| / T_0)$$

For $T = T_{KT} : \ \mu_B = 2\mu + |E_b| = f_0 T_0$

where $\mu_B = T_0 \exp(-T_0 / T)$; $\quad f_0 = \dfrac{1}{\ln(|E_b| / T_0)}$

c) 2D Strong coupling case $|U| >> W$

$$|E_b| \sim |U|$$

$$T_{KT}^{lat} \sim \frac{W}{|U|} T_0 \sim T_{BEC}^{lat} \text{ in 3D}$$

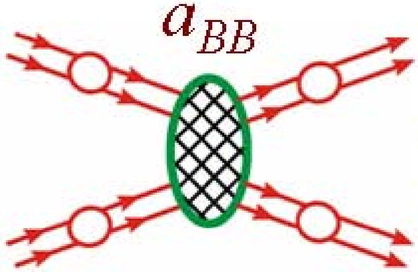
Composite boson interaction

3D case

resonance situation:

$$U < U_c; \quad |a| >> r_0; \quad \frac{1}{ma^2} = |E_b|$$

a_{BB} - scattering length of two composite bosons:

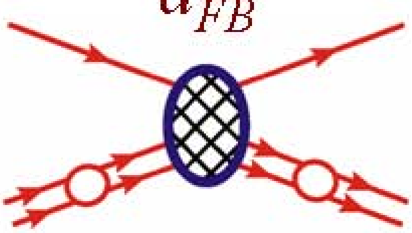


$a_{BB} = 2|a|$ *Hausmann (1993)*

$a_{BB} = 0.75|a|$ *Pieri, Strinati (2000)*

$a_{BB} = 0.6|a|$ *Shlyapnikov et al.(2003)*

a_{FB} - scattering length of composite boson and fermion:



$a_{FB} = 1.2|a|$ *Petrov (2003)*

2D case

$$f_{BB} = \frac{4\pi}{m}$$

Drechsler, Zwerger (1992); exchange approximation

Repulsive-U Hubbard model

$$\hat{H} = -t \sum_{<i\,j>} c_{i\sigma}^{+} c_{j\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow}; \quad n_{i\sigma} = c_{i\sigma}^{+} c_{i\sigma}$$

3D case

For $nd^3 < 1$: triplet p-wave pairing due to Kohn-Luttinger effect

$$T_{c1}^{p} \sim \varepsilon_F \exp\left(-13/\lambda^2\right)$$

$$\lambda = 2ap_F/\pi; \quad a \approx d \text{ - for the lattice,} \quad U >> W$$

In the absence of a lattice and far from the resonance $a \sim r_0$ (radius of the potential)

2D case

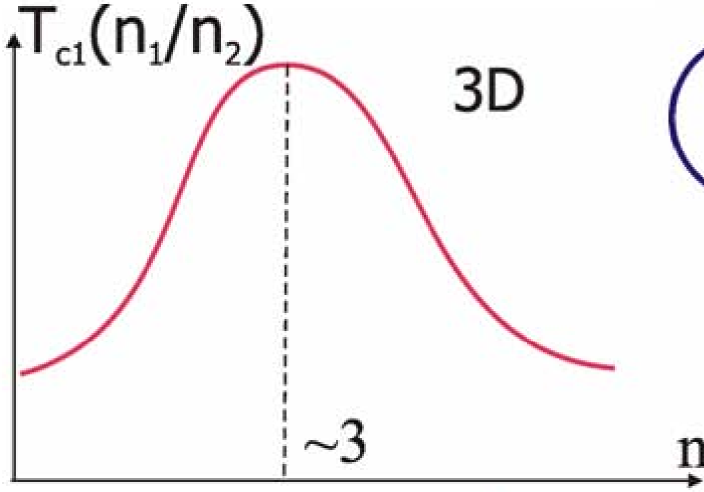
$$T_{C1} = \varepsilon_F \exp\left(-1/6.1 f_0^3\right),$$

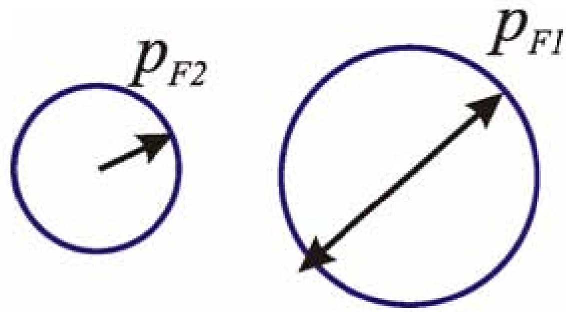
$$\text{where} \quad f_0 = 1/2\ln\left(p_F r_0\right)$$

Both in 3D and in 2D T_{c1} can be strongly enhanced in a strong magnetic field or in the case of two or more sorts of fermions (two or more components of nuclear spin I)

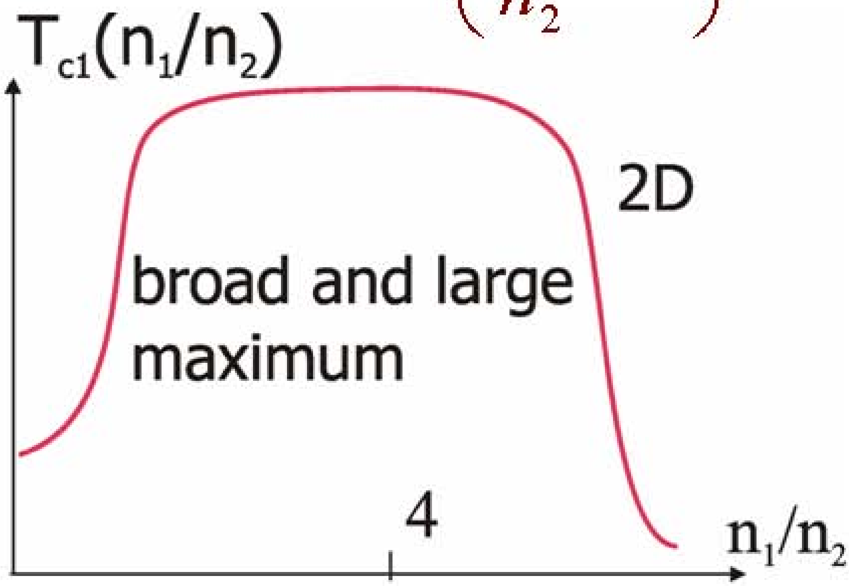
Two sorts of fermions

Two particles of sort 1 form Cooper pairs, while particles of sort 2 prepare effective interaction





$$\text{In 3D } T_{c1}^{\max} = T_{c1}\left(\frac{n_1}{n_2} \approx 3\right) = \varepsilon_{F1} \exp\left(-\frac{7}{\lambda^2}\right)$$

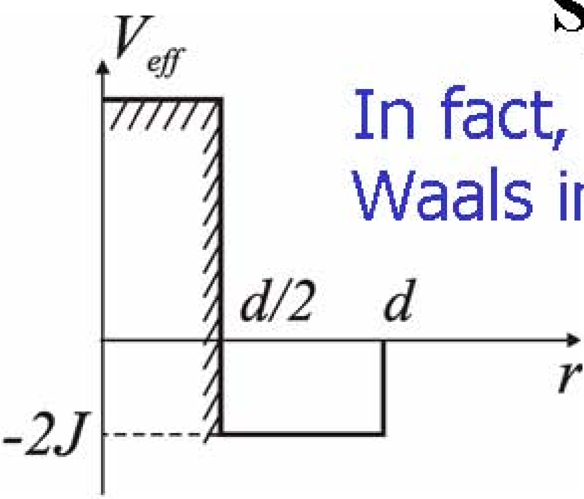


$$\text{In 2D } T_{c}^{\max} = T_{c1}\left(\frac{n_1}{n_2} = 4\right) = \varepsilon_{F1} \exp\left(\frac{-1}{6.1 f_0^3 + 2 f_0^2}\right)$$

t-J model

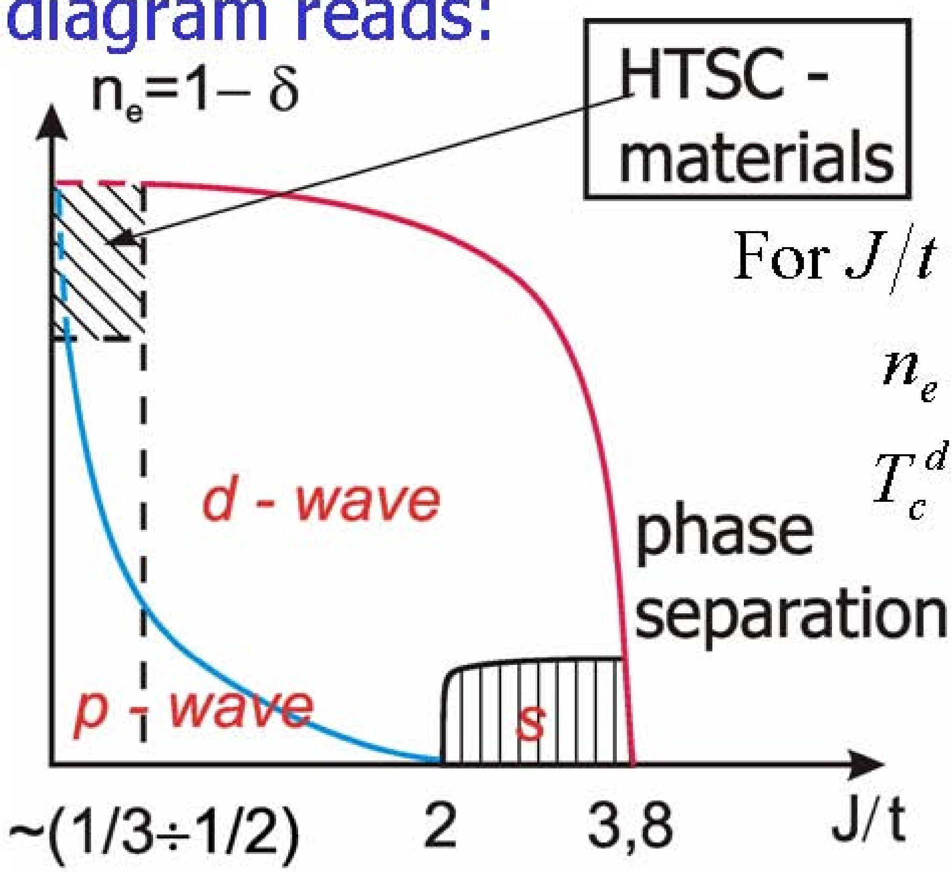
$$\hat{H} = -t\sum_{\langle i\,j\rangle} c^{+}_{i\sigma}c_{j\sigma} + U\sum_{i} n_{i\uparrow}n_{i\downarrow} - J\sum_{\langle i,j\rangle}\mathbf{S}_i\mathbf{S}_j$$

$$\mathbf{S}_i = c^{+}_{i\mu}\sigma_{\mu\nu}c_{i\nu}$$



In fact, it is a model with Van der Waals interaction on the lattice

For $U >> \{J,t\}$ superconducting phase-diagram reads:



For $J/t \sim (1/3 \div 1/2)$;

$n_e \sim 0.85$;

$T_c^d \sim 10$ K

Applications of the theory

In pure ^{3}He 20 % enhancement of $T_{c1}^{\uparrow\uparrow}$ (A1-phase) was experimentally observed by Frossati.

In ^{3}He-^{4}He mixtures the scattering length a changes sign at ^{3}He concentration x=1-2%.
For x<1 % a<0 – attraction (s-wave pairing *Bashkin, Meyerovich*). $T_{C0}=10^{-4}$-10^{-5} K for H=0 and $x \sim 1$ %.
For x>2 % a>0 – repulsion (p-wave pairing *Kagan, Chubukov*). $T_{C1}=10^{-5}$ K for H=15 T and $x_{max} \sim 9.5$ %.

For fermionic ^{6}Li $a = -2.3\times10^3$ Å – attraction.
$T_{C0}=10^{-6}$ K – predicted by Stoof.
s-wave pairing is suppressed when $(n_1 - n_2)/(n_1 + n_2) > T_{C0}/\varepsilon_F$, where n_1 and n_2 are densities of two components of nuclear spin I (*M.Yu.Kagan, Yu.Kagan, M.Baranov*)
p-wave pairing arises:

$$T_{c1} \sim \varepsilon_{F1} \exp\left(-7/\lambda^2\right) \sim 10^{-7}\ \mathrm{K}$$

Bosonic model with Van der Waals interaction

$$\hat{H} = -t\sum_{<ij>} b_i^{+} b_j + \frac{U}{2}\sum_i n_i^2 - \frac{V}{2}\sum_{<ij>} n_i n_j$$

There is a threshold for s-wave bound state even in 2D

$V > V^s_{crit} = 4t$ *Kagan, Efremov (2002)*

$$\left|E^s_b\right| = 4W \exp\left\{-\frac{\pi V}{(V - V^s_{crit})}\right\}$$

$V > V^d_{crit} \approx 14t >> V^s_{crit}$ $\quad T_0 = \frac{2\pi n}{m}$

Two boson pairing and their Bose condensation

$\langle bb \rangle \neq 0$ $\quad \langle b \rangle = 0$

$$T_* = \frac{\left|E_b\right|}{\ln(\left|E_b\right|/T_0)}$$

Creation of the boson pairs (crossover temperature)

$$T_B = \frac{T_0}{2\ln\ln(\left|E_b\right|/T_0)}$$

Condensation of boson pairs *(Fisher, Hohenberg 1988)*

Phase separation in 2D

Total phase separation on two clusters:

$n \to 1$ and $n \to 0$

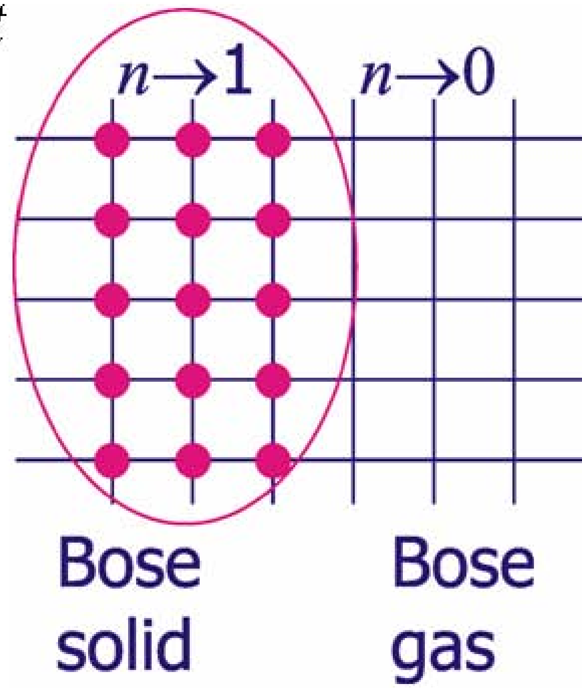
Energy per particle in the cluster with $n \to 1$

$$E_{PS} = -2\eta V, \quad \eta \sim 1$$

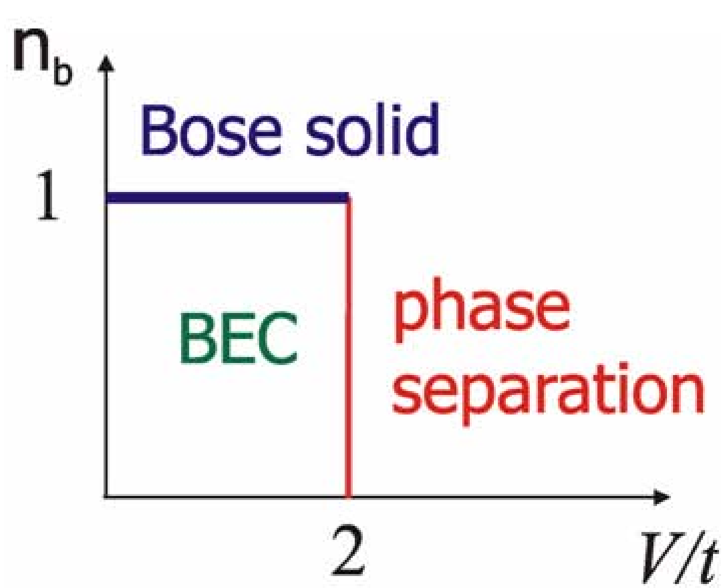
Energy due to delocalization in the cluster with $n \to 0$: $E_1 \approx -W/2 = -4t$

Condition of total phase separation

$$E_{PS} < E_1 \Leftrightarrow V > V_{PS} = 2t$$



Phase separation takes place earlier than s-wave two-boson pairing



Two sorts of bosons in 2D

$$\hat{H} = -t_1 \sum_{<ij>} b_{1i}^{+} b_{1j} - t_2 \sum_{<ij>} b_{2i}^{+} b_{2j} +$$

$$+\frac{1}{2} U_1 \sum_i n_{1i}^2 + \frac{1}{2} U_2 \sum_i n_{2i}^2 - U_3 \sum_i n_{1i} n_{2i}$$

U_1, U_2 strong repulsive interactions to prevent collapse

U_3 intermediate attraction between bosons of different sorts

Energy of a bound state $m_{12} = \frac{m_1 m_2}{m_1 + m_2}$

$$|E_b| = \frac{1}{2 m_{12} d^2} \frac{1}{\exp\left(2\pi / m_{12} |U_{12}|\right) - 1}$$

For $|E_b| > \left\{ T_{01} = \frac{2\pi n_1}{m_1}; T_{02} = \frac{2\pi n_2}{m_2} \right\}$

Non-diagonal two-boson pairing of the type $\langle b_1 b_2 \rangle \neq 0 \; \left(\langle b_1 \rangle = 0, \langle b_2 \rangle = 0 \right)$ takes place

For $n_1 = n_2 = n$

$$T_{012} = \frac{2\pi n}{m_{12}}$$

$$T_c = \frac{T_{012}}{2 \ln \ln\left(|E_b| / T_{012} \right)}$$

Fermi-Bose mixture in 2D (I)

$$\hat{H} = -t_F \sum_{<ij>} f_{i\sigma}^{+} f_{j\sigma} - t_B \sum_{<ij>} b_i^{+} b_j +$$

$$+ U_{FF} \sum_i n_{i\uparrow}^F n_{i\downarrow}^F + \frac{1}{2} U_{BB} \sum_i \left(n_i^B\right)^2 - U_{FB} \sum_i n_{i\sigma}^F n_i^B$$

U_{FF}, U_{BB} strong repulsion between particles of the same sort

U_{FB} attraction between bosons and fermions

Energy of a bound state $\qquad m_{FB} = \frac{m_B m_F}{m_B + m_F}$

$$|E_b| = \frac{1}{2 m_{FB} d^2} \frac{1}{\exp\left(2\pi / m_{FB} |U_{FB}|\right) - 1} < W_{FB}$$

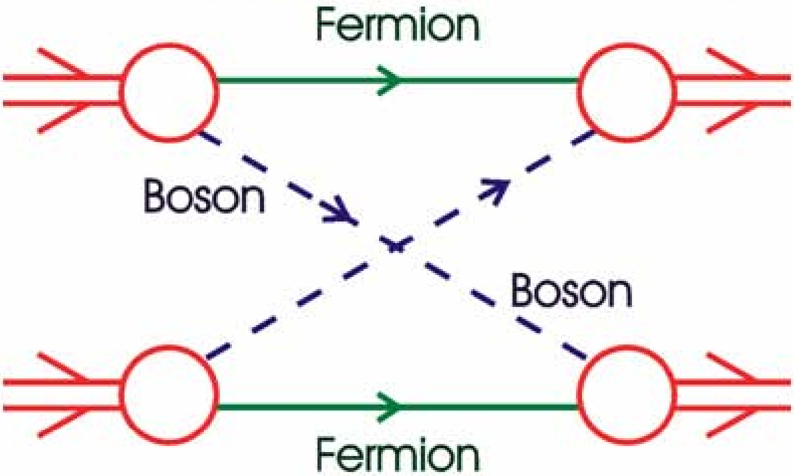
$$\text{For} \quad |E_b| > \left\{ T_{0F} = \frac{2\pi n_F}{m_F}; T_{0B} = \frac{2\pi n_B}{m_B} \right\}$$

$$n_B = n_F = n$$

$$T_{0FB} = \frac{2\pi n}{m_{FB}} \qquad T_* = \frac{|E_b|}{\ln\left(|E_b| / 2T_{0FB}\right)}$$

Fermi-Bose mixture in 2D (II)

Interaction between composite fermions



$$m_B = m_F = m$$

$$|E_b| = 1/ma^2$$

Interaction is attractive

$$U_4^{(0)}(q) \approx -\frac{4\pi}{m}\frac{1}{\left(1+q^2a^2/4\right)^2}$$

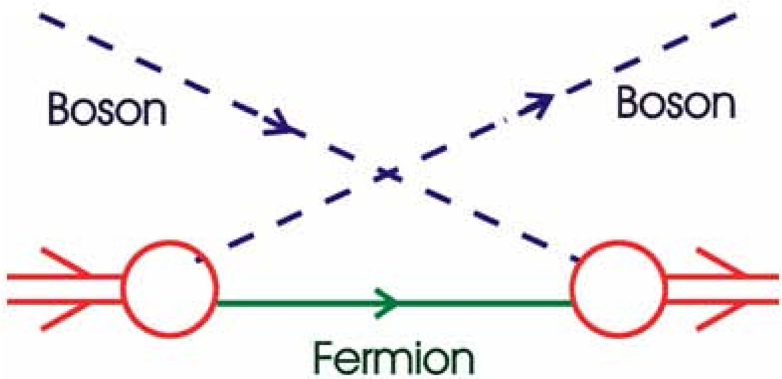
Hence quartets are created in the system

$$|E_4| \approx 3|E_b|$$

$$T_{**}^{(4)} = \frac{|E_4|}{\ln|E_4|/2T_0}$$

Fermi-Bose mixture in 2D (III)

Interaction between composite fermions and elementary boson



$$m_B = m_F = m$$

$$|E_b| = 1/ma^2$$

Interaction is also attractive

$$U_3^{(0)}(q) = -\frac{8\pi}{m}\frac{1}{(1+q^2a^2)}$$

For $\quad m_B = m_F = m \quad |E_3| \approx 1.65|E_b|$

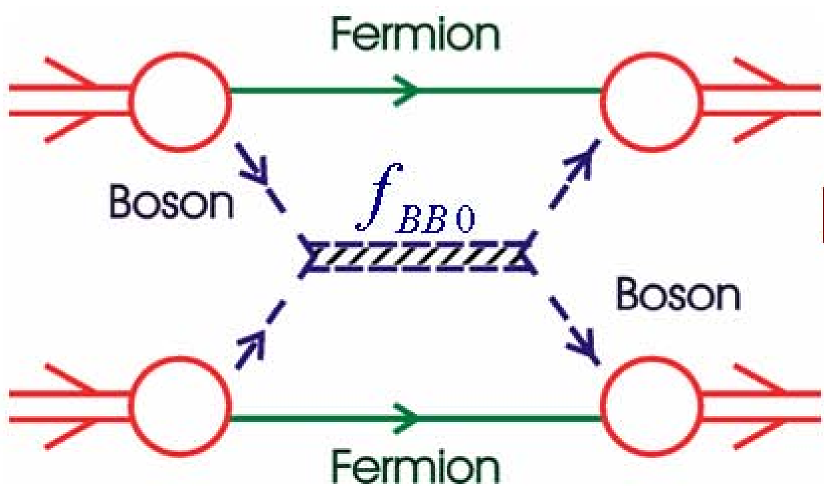
$$T_{**}^{(3)} = \frac{|E_3|}{\ln|E_3|/2T_0}$$

$T_{**}^{(4)} >> T_{**}^{(3)}$ -quartets are dominant in the system

Interaction between elementary fermion and composite fermion is repulsive.

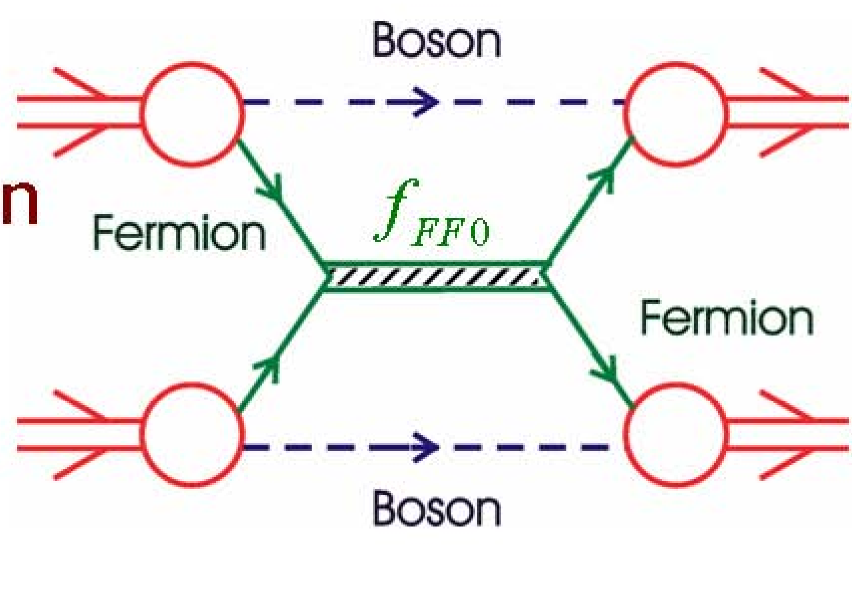
Fermi-Bose mixture in 2D (IV)

Corrections to the interaction between composite fermions



boson-boson interaction

fermion-fermion interaction



Bose-condensation of quartets

For $T < T_{**}^{(4)}$ number of quartets prevails over the number of pairs and trios.

At $T_c = T_0 / 8 \ln\ln(4/na^2)$ the quartets are bose-condensed.

It is important to note that in Feshbach resonance scheme we are usually in the regime $T \sim T_0$, where quartets prevail independently of the ratio between $T_{**}^{(3)}, T_{**}^{(4)}$.

The quartets are in a spin-singlet sate. Spin-triplet quartets are suppressed or reduced by Pauli pribciple.

Triplet p-wave quartets can possibly arise in a strong coupling case.

Conclusions

We consider all the possible averages of the type <ff>, <bb>, and <bf> which arise in Fermi-Bose gases and in Fermi-Bose mixtures.

We discuss the possibility of quartets formation <bf, bf> in Fermi-Bose mixture.

Our investigations enrich superfluid phase diagram in a magnetic trap and are important in connection with recent experiments, in which weakly bound pairs $^{6}Li_2$ and $^{40}K_2$, consisting of two fermions, were observed (*Jin, Regal et al.*).

They are also interesting in connection with recent experiments on Fermi-gas collapse in a Fermi-Bose mixture of ^{87}Rb and ^{40}K atoms with attractive interaction between fermions and bosons (*Madugno et al.*).

Fermi-Bose mixture in 3D

For the experiments on Fermi-Bose mixture more actual is the 3D case.

There is a threshold
for quartet formation

$$U_4(q) = -\frac{\pi}{m_{BF}} \frac{a_{eff}}{\left(1 + q^2/2(m_B + m_F)\,|E_b|\right)}$$

A bound state exists only if

$$a_{eff} > \pi a \left(\frac{m_B + m_F}{m_{BF}} \right)^{3/2}$$

For $\quad m_B = m_F = m$

$$a_{eff} > \frac{\pi a}{4}$$