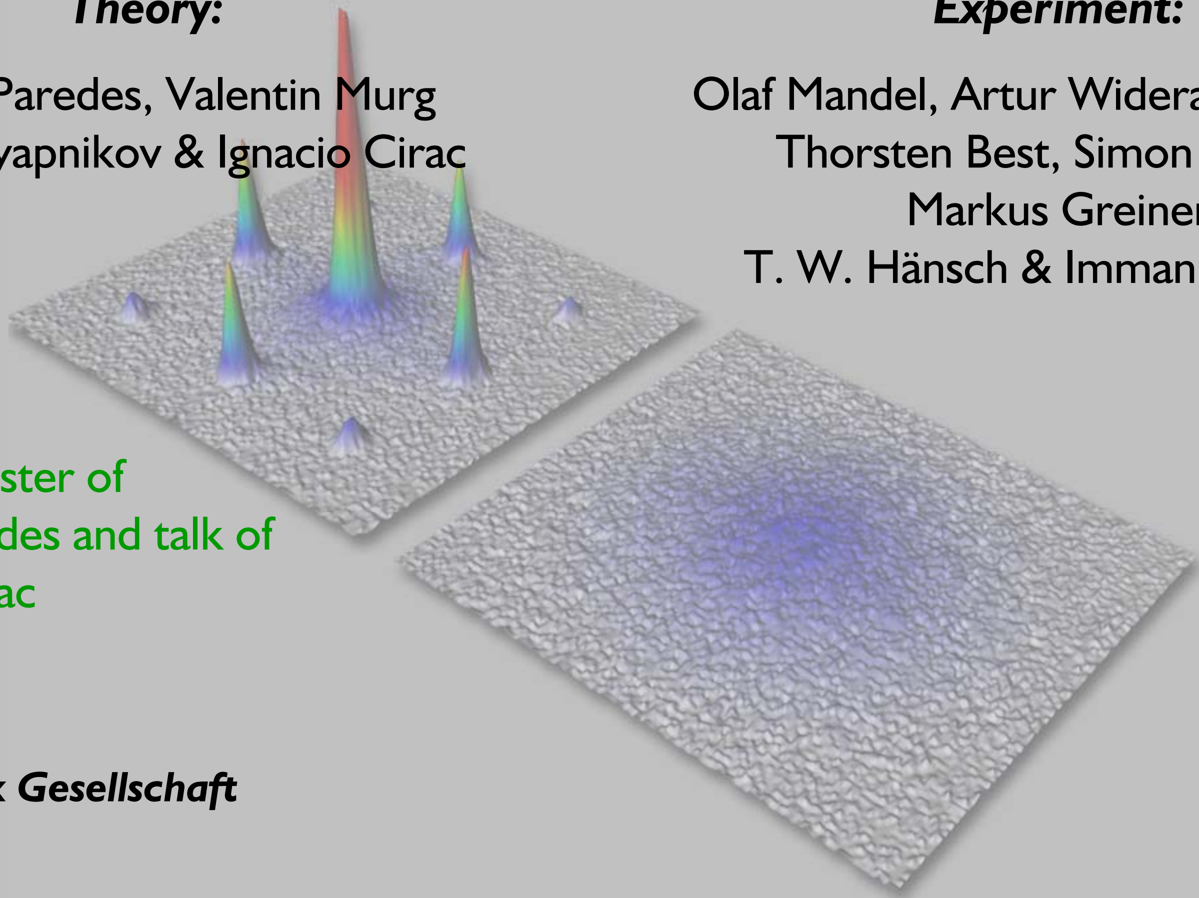
Fermionizing Bosons in Optical Lattices

Theory:

Belén Paredes, Valentin Murg
Gora Shlyapnikov & Ignacio Cirac

Experiment:

Olaf Mandel, Artur Widera, Tim Rom,
Thorsten Best, Simon Fölling,
Markus Greiner
T. W. Hänsch & Immanuel Bloch



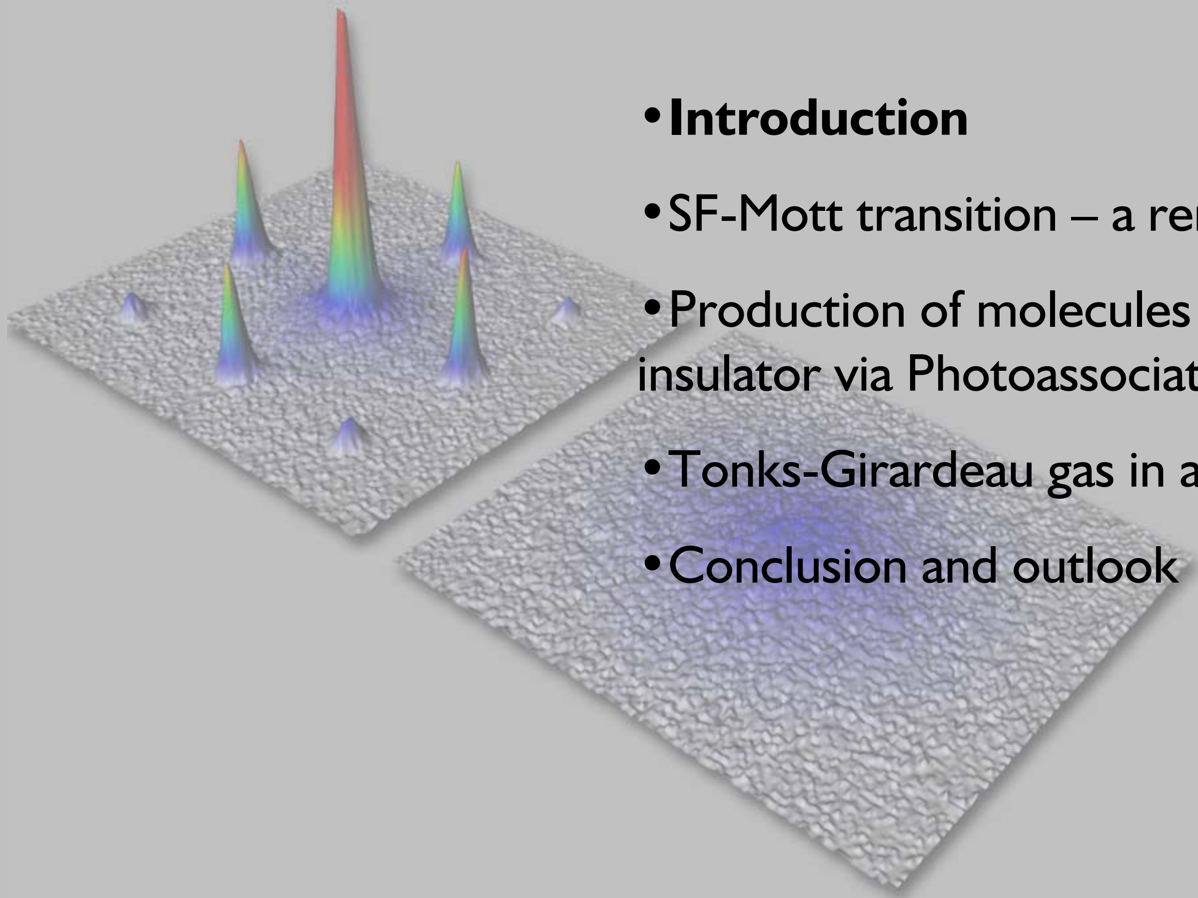
see also poster of
Belén Paredes and talk of
Ignacio Cirac

funding:
€ DFG, EU,
Max-Planck Gesellschaft
$ AFOSR

Ludwig-Maximilians-Universität & Max-Planck-Institut für Quantenoptik, Munich

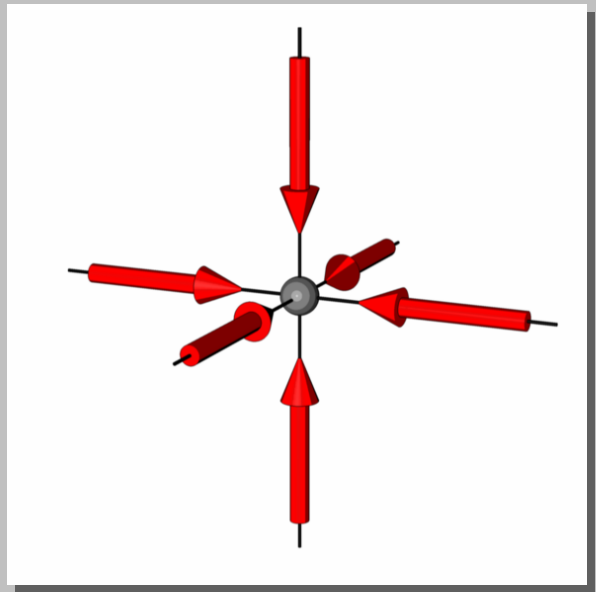
Johannes Gutenberg-Universität, Mainz, Germany

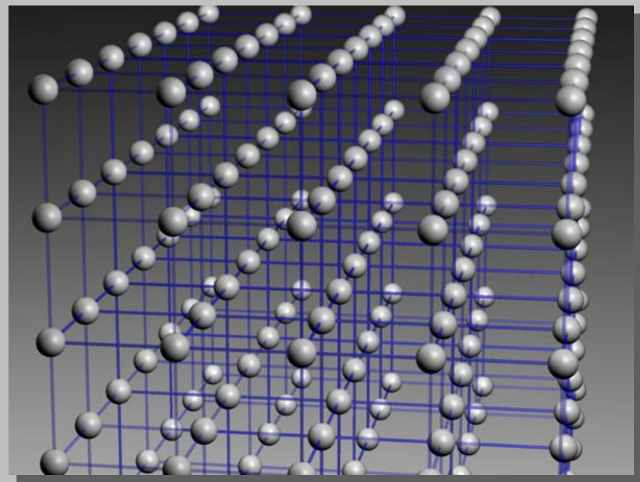
Outline



- **Introduction**
- SF-Mott transition – a reminder
- Production of molecules from a Mott insulator via Photoassociation
- Tonks-Girardeau gas in an optical lattice
- Conclusion and outlook

3D Lattice Potential





- ***Resulting potential consists of a simple cubic lattice***
- ***BEC coherently populates more than 100,000 lattice sites***

V_0 ***up to 40 E_{recoil}***

ω_r ***up to $2\pi \times 45$ kHz***

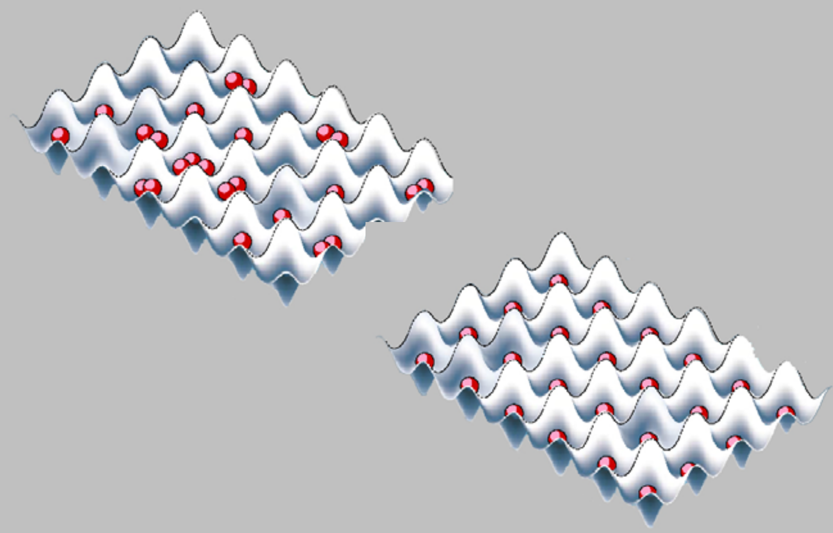
$n \approx$ ***1-5 atoms on average per site***

The SF-Mott Insulator Transition

$$H = -J\sum_{\langle i,j\rangle} \hat{a}_i^\dagger \hat{a}_j + \frac{1}{2}U\sum_i \hat{n}_i(\hat{n}_i - 1)$$

Characteristic parameter for the behaviour of the system

U/J



M. Greiner et al., Nature, 415, 39 (2002)

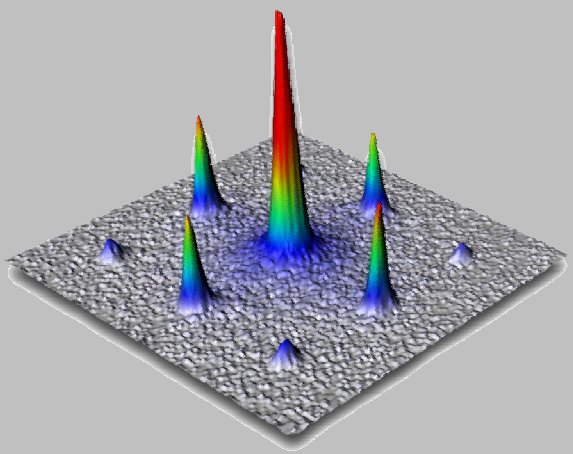
D. Jaksch et al. *PRL*, M. Fisher et al. *PRB*, R. Roth & K. Burnett, K. Braun-Munzinger, B. Svistunov et al., M. Lewenstein, L. Santos et al. M. Kasevich, Yale, W.D. Phillips, NIST, T. Esslinger ETHZ

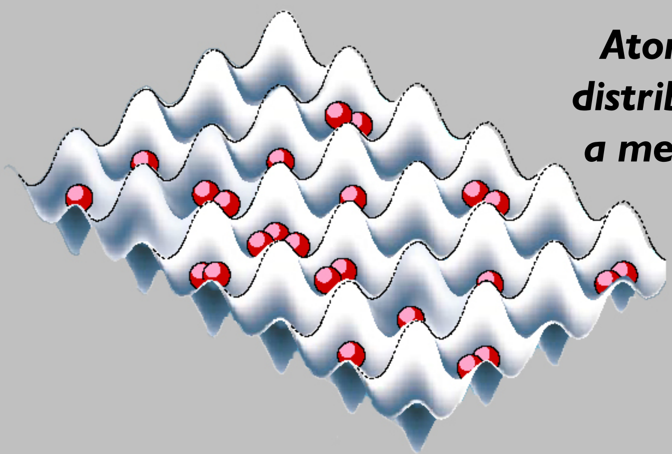
Superfluid Limit

$$H = -J\sum_{i,j}\hat{a}_i^{\dagger}\hat{a}_j + \frac{1}{2}U\sum_i \hat{n}_i(\hat{n}_i - 1)$$

Atoms are delocalized over the entire lattice !
Macroscopic wave function describes this state very well.

$$\left|\Psi_{SF}\right\rangle \propto \left(\sum_{i=1}^{M}\hat{a}_i^{\dagger}\right)^N \left|0\right\rangle$$





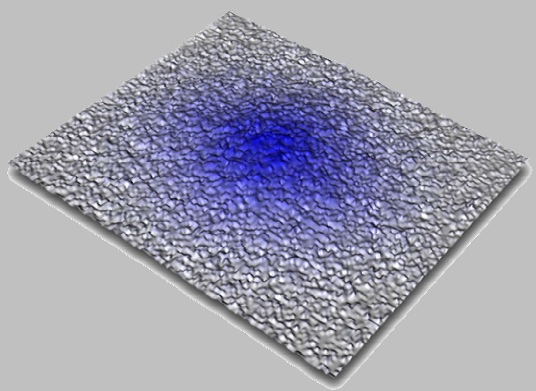
Atom number distribution after a measurement

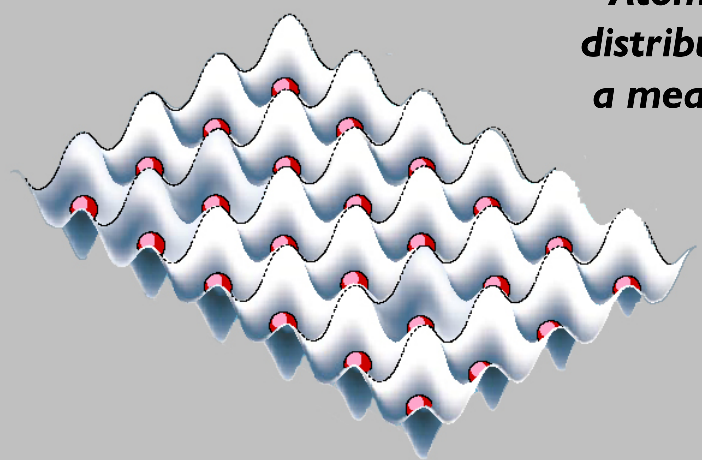
"Atomic Limit" of a Mott-Insulator

$$H = -J\sum_{i,j}\hat{a}_i^\dagger \hat{a}_j + \frac{1}{2}U\sum_i \hat{n}_i(\hat{n}_i - 1)$$

Strong repulsion between atoms leads to a kind of „fermionization"

Repulsion mimics Pauli principle, but connection still vague





Atom number distribution after a measurement

Short Resume of Work with the Mott State

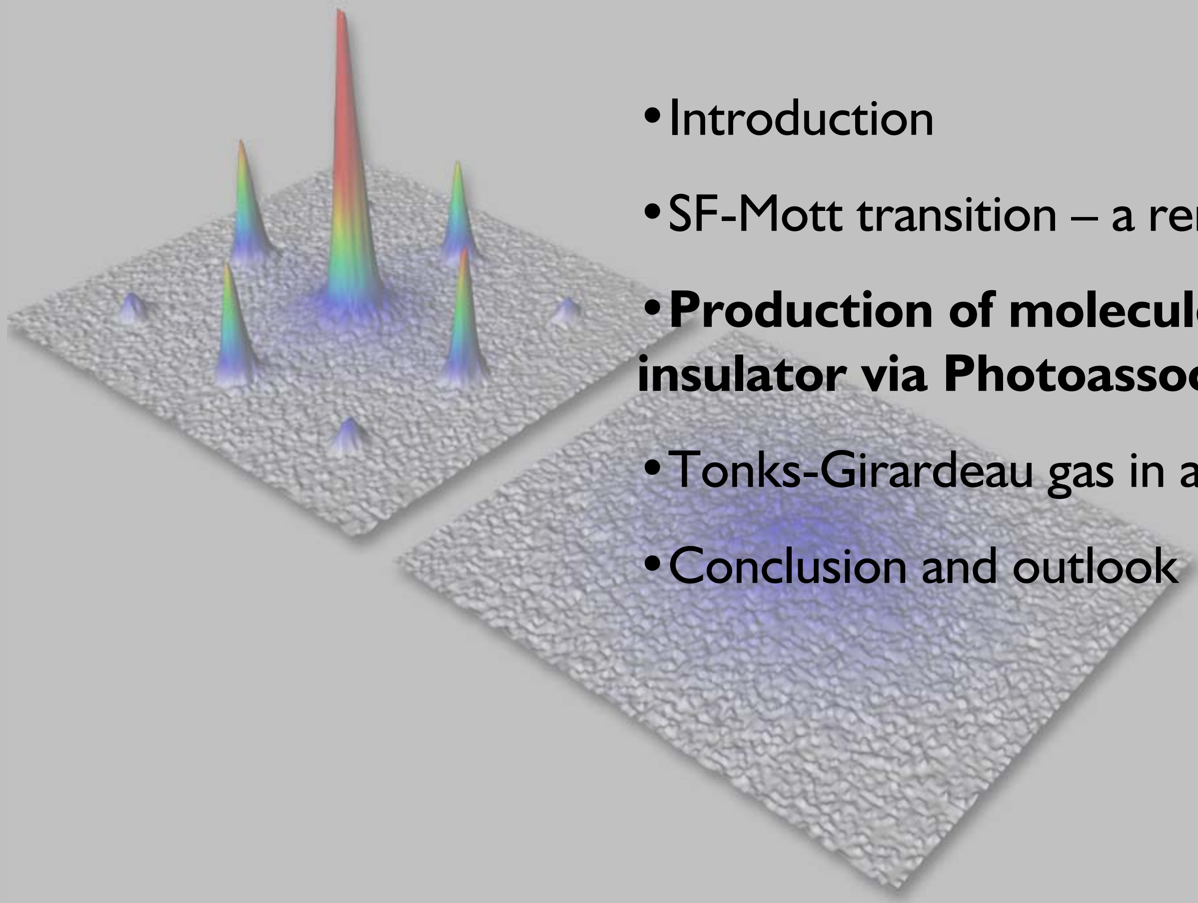
Quantum Information applications – The Mott state as a quantum register

- ***Spin dependent potentials for individual atoms***
  O. Mandel et al., PRL **91**, 010407 (2003)
- ***Collaps and revival of the matter wave field of a BEC***
  M. Greiner et al., Nature, **419**, p. 51, 2002.
- ***Controlled collisions – massively parallel quantum gate array – controllable Ising interaction***
  O. Mandel et al. Nature, **425**, p. 937, (2003)

Atomic/Molecular Physics

- ***Entanglement Interferometry – Precision measurement of atomic scattering properties***
  A. Widera et al., PRL (in press)
- ***Full quantum control over molecule production via Photoassociation in optical lattices***
  (in preparation)

Outline



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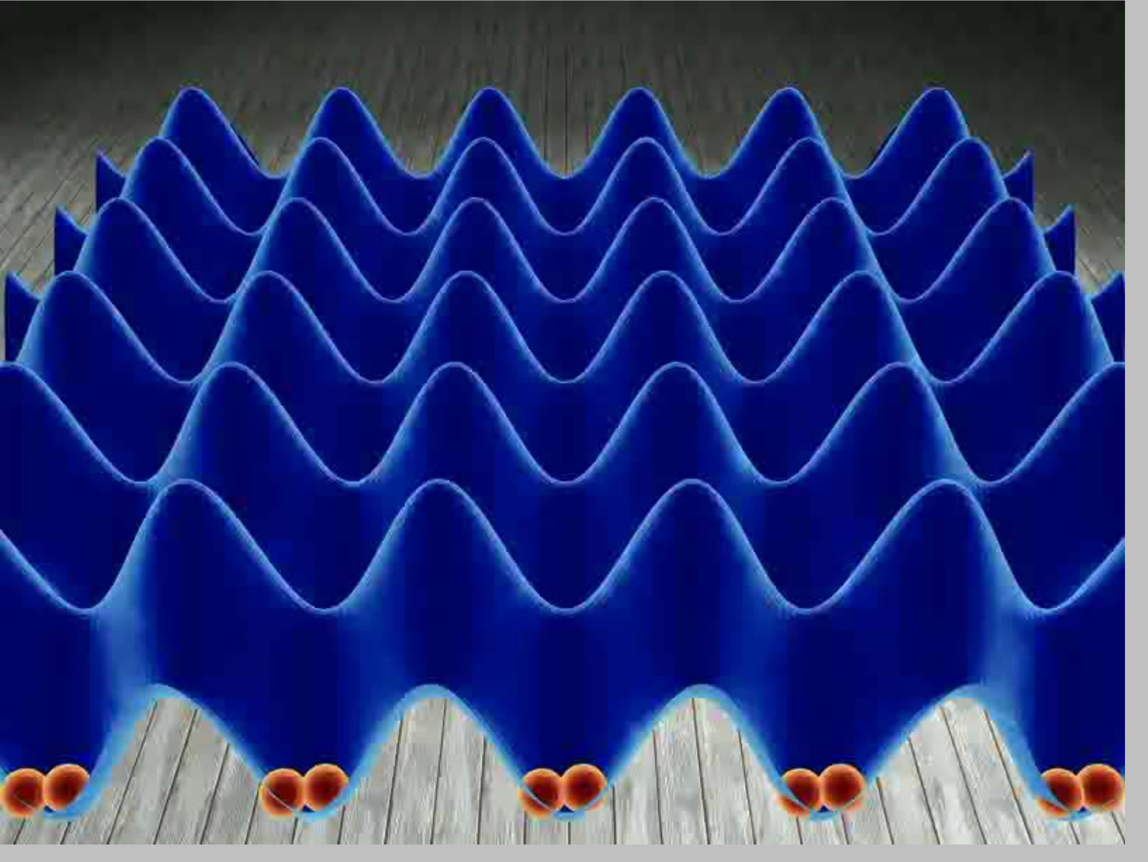
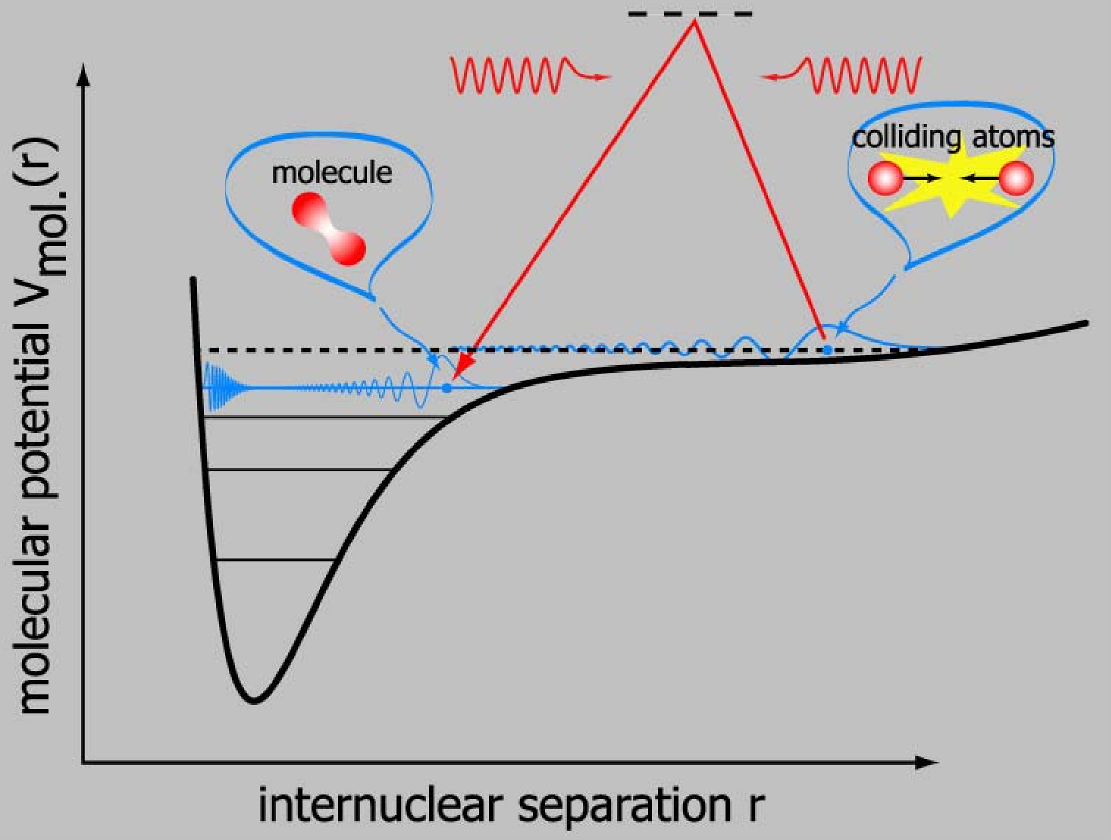
Microarray of Identical Two-Particle Systems



Great environment to form molecules !

Ultracold molecules via Feshbach resonances:
C. Wieman, D. Jin, Ch. Salomon, R. Grimm, G. Rempe, W. Ketterle

Molecule Formation via Two-Photon Photoassociation
- Controlling the Internal Degrees of Freedom -



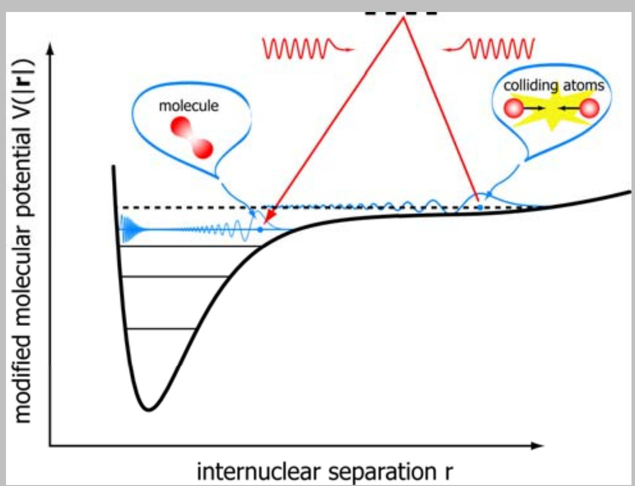
A selection of groups performing PA experiments:
D. Heinzen, P. Pillet, J. Weiner, M. Leduc & C. Cohen-Tannoudji, W.D. Phillips, R. Hulet

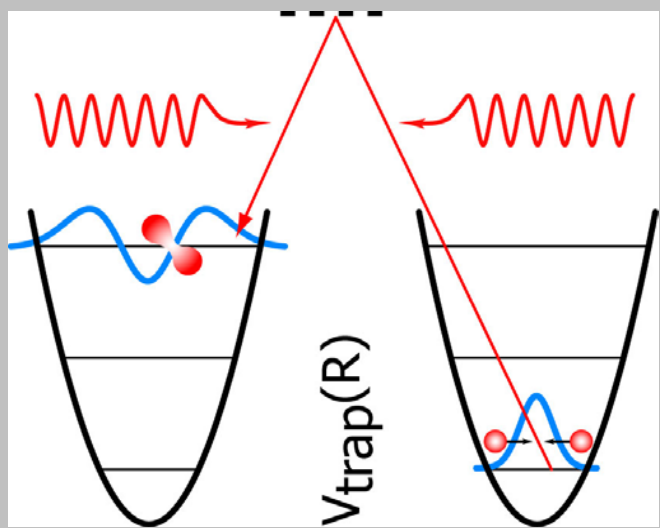
Ultracold molecules can also be created through Feshbach resonances:
D. Jin, Ch. Salomon, R. Grimm, G. Rempe

Resolving the External Quantum States

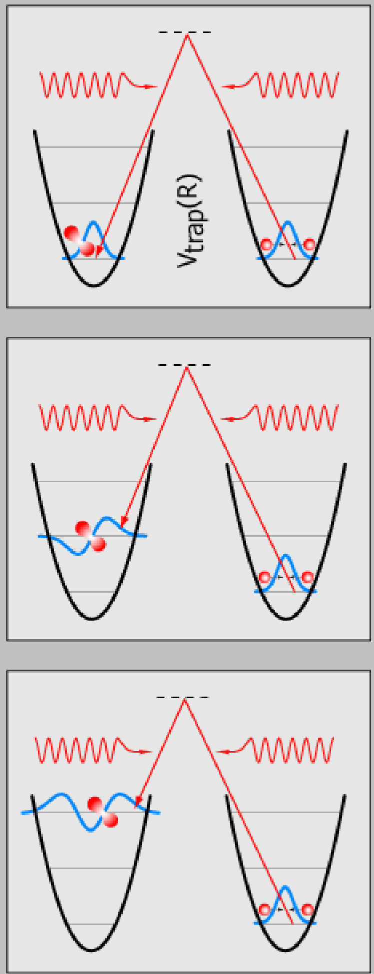
The Raman spectrum additionally resolves the quantized vibrational levels of the trapped molecules.

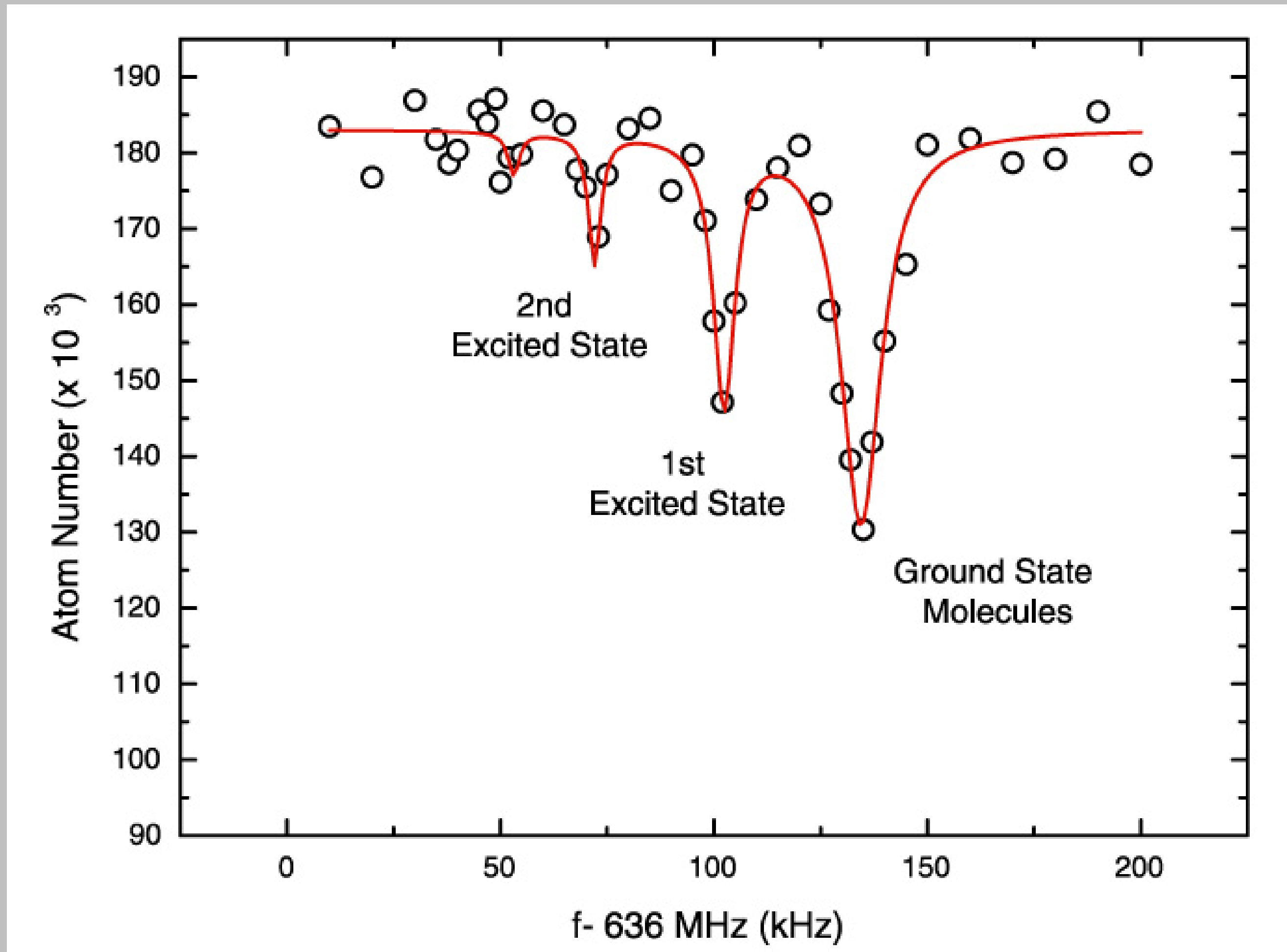
relative motion	center-of-mass motion
$H_{rel.} = -\frac{\hbar^2}{2\mu}\nabla_{\vec{r}}^2 + \frac{1}{2}\mu\omega^2\vec{r}^2 + V(\|\vec{r}\|)$	$H_{c.m.} = -\frac{\hbar^2}{2M}\nabla_{\vec{R}}^2 + \frac{1}{2}M\omega^2\vec{R}^2$





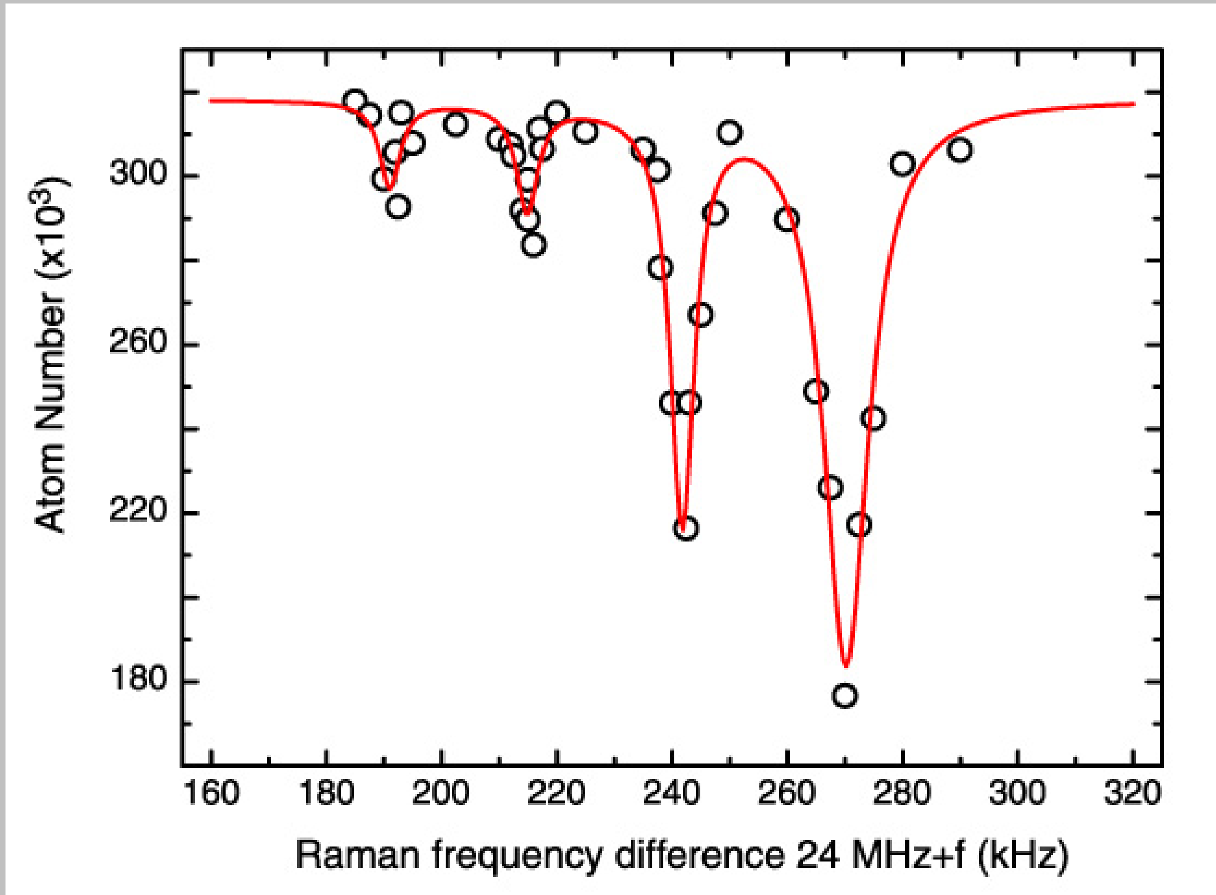
Full Control over all Internal and External Quantum Degrees of a Chemical Reaction Should be Possible





Final Quantum State is Now Completely Controllable !

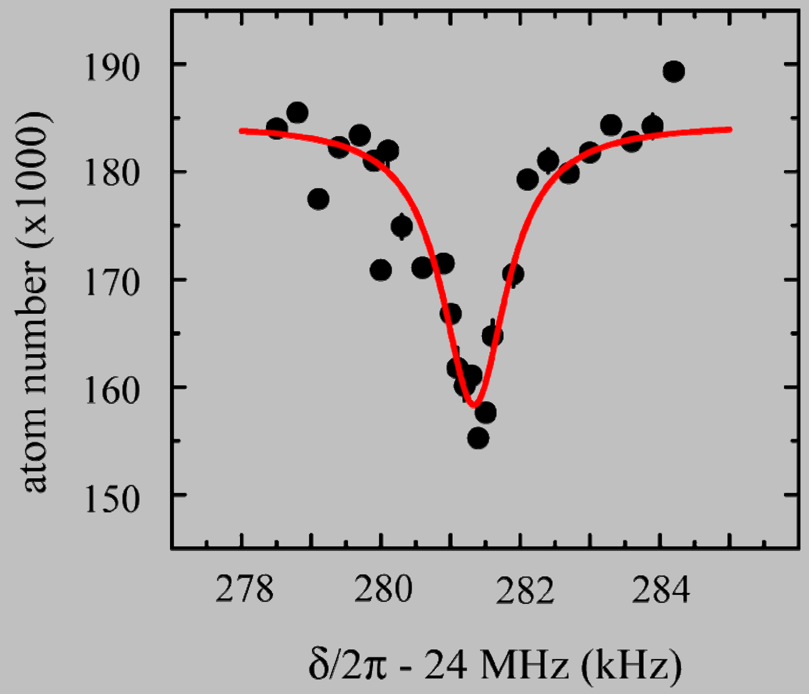
Photoassociation to the last bound vibrational state



No Mean Field Broadening !

High Resolution Spectroscopy at High Densities: ***Peak density 2*10^{15} cm^{-3}***

Compare e.g. Dan Heinzen experiments : ***Peak density 2*10^{14} cm^{-3}***

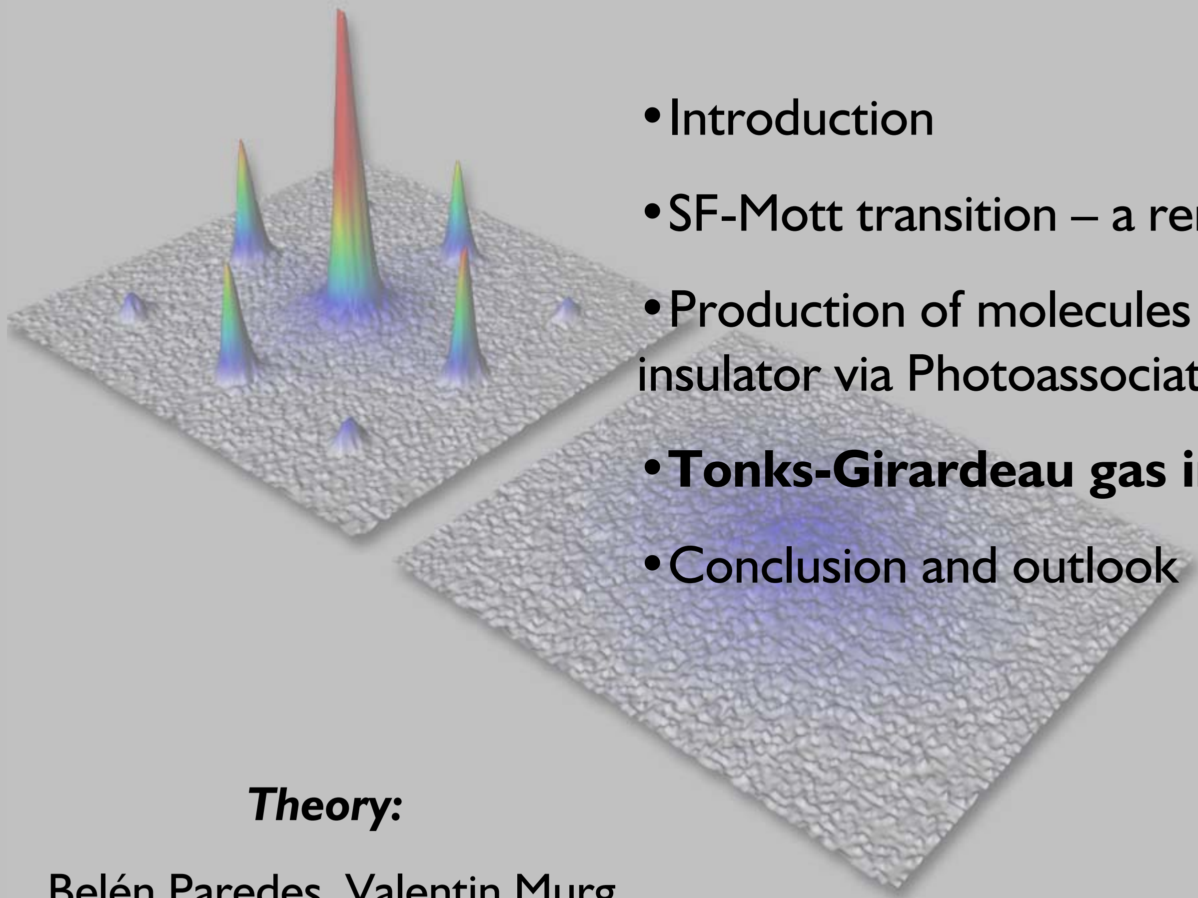


Linewidth 1.1 kHz

Outlook
-Photoassociation-

- High resolution spectroscopy of deeply bound states
- Formation of ground state molecules
- STIRAP processes to coherently form molecules
- Rabi-Oscillations between Atoms and Molecules
- Create Mott Insulator of Molecules
- Optical Feshbach Resonances
- PA as a Sensitive Probe for Two-Body Correlation Functions
- Formation of heteronuclear molecules
- Combination of Feschbach resonances & PA

Outline



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The Tonks-Girardeau Gas
– A Fermionized 1D Quantum Gas -

Requirements (1): ***1D bosonic quantum gas, tightly confined in two dimensions and only weakly confined along the axial direction***

$$\mu \ll \hbar\omega_{\perp}$$

$$T < T_d \approx N\hbar\omega_{ax}$$

In Experiments here: aspect ratio typically100-200

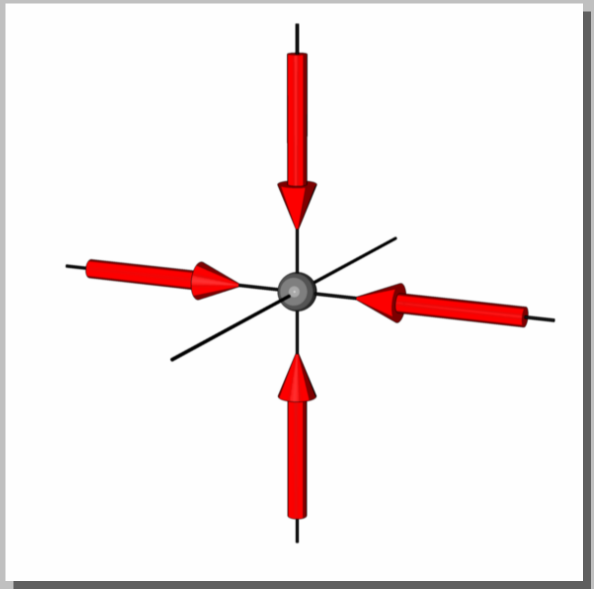
Experiments with 1D condensates:

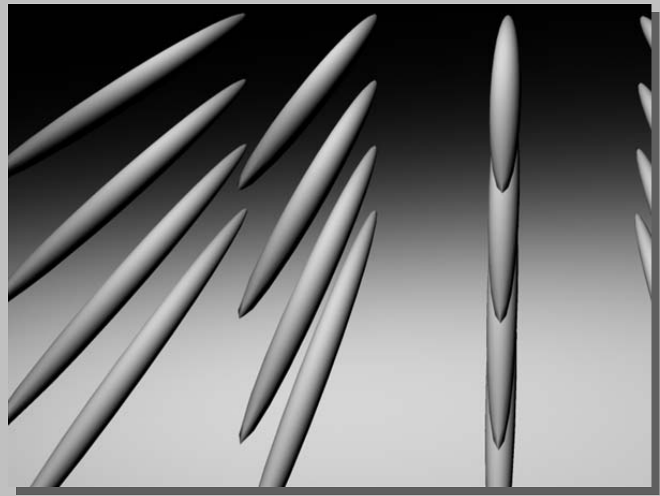
A. Goerlitz et al., PRL (2001), F. Schreck et al. PRL (2001), M. Greiner et al. PRL (2001)

more recently:

H. Moritz et al., PRL (2003), B. Laburthe Tolra et al., cond-mat (2003)

2D Lattice Potential





- *Resulting potential consists of an array of tightly confing potential tubes*
- *BEC is split into up to 10,000 1D quantum gases (radial motion confined to zero point oscillations)*

V_0 *up to 30* E_{recoil}

ω_r *up to* $2\pi \times 35$ *kHz*

n ≈ *up to 20 atoms per tube*

Tonks-Girardeau Gas

Requirements (2): ***Strong repulsive interactions between atoms***

$$\boxed{\gamma \approx I / K \gg 1}$$

Homogeneous case

Interaction energy $I \sim ng$

Kinetic energy $K \sim \dfrac{\hbar^2 n^2}{m}$ $\Longrightarrow$ $\boxed{\gamma = \dfrac{mg}{\hbar^2 n}}$

see e.g. D.S. Petrov et al. PRL (2000) and V. Dunjko et al. PRL (2001)

Tonks-Girardeau Gas - Fermionization

In 1D for strongly interacting bosons, the many-body wave function can be mapped on to the one of non-interacting fermions.

(M.D. Girardeau, J. Math. Phys. 1960) ***This lies at the heart of a TG gas!***

$$\Psi_B\left(x_1,\ldots,x_N\right)=\left|\Psi_F\left(x_1,\ldots,x_N\right)\right|$$

For example:

$$\Psi_B\left(x_1,\ldots,x_N\right)=\left|\det\left[\varphi_i\left(x_j\right)\right]\right| \qquad i,j=1\ldots N$$

- ***Slater determinant ensures that two particles cannot be placed at the same position in space!***
- ***Absolute value ensures symmetrization***

Bosons behave like Fermions – Not Quite

Density distribution:

$$\left|\Psi_B(x)\right|^2 = \left|\Psi_F(x)\right|^2$$

identical to the one of free fermions!
(absolute value of det does not matter)

Correlation function:

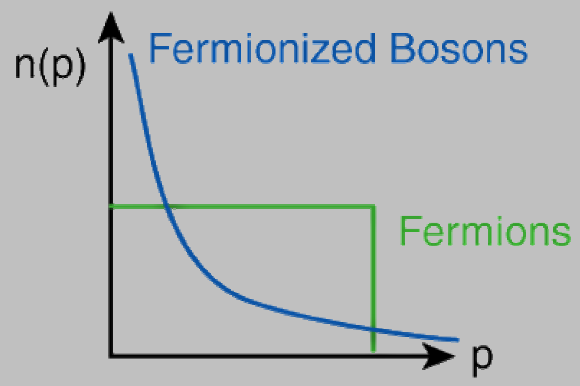
$$g^{(1)}(x) = \left\langle \Psi_B^\dagger(0)\Psi_B(x)\right\rangle \neq \left\langle \Psi_F^\dagger(0)\Psi_F(x)\right\rangle$$

different to the one of free fermions!
(absolute value of det matters)

Momentum Distribution:

$$n(p) \propto \int e^{-ipx} g^{(1)}(x)\, dx$$

different to the one of free fermions!
(FT of correlation function)



Momentum distribution is characteristic for a Tonks-Girardeau gas!

Status of Experiments

So far, experiments in 2D optical lattices have achieved $\gamma \approx 0.5$-1,

Still 1D mean-field regime (see H. Moritz et al. PRL (2003)), allthough correlations begin to be modified
(see B. Laburthe Tolra et al. cond-mat/0312003)

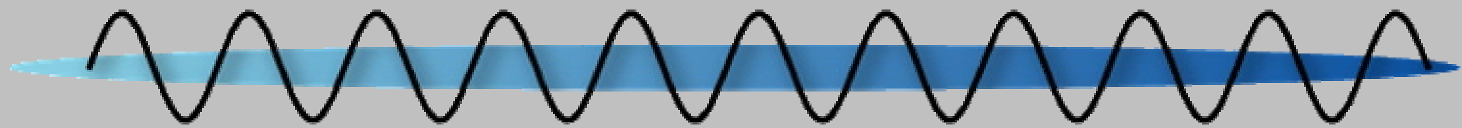
$$\gamma = \frac{mg}{\hbar^2 n}$$

<u>Ways to increase γ:</u>

1. ***Increase Interaction strength*** $g = 2\, a\, \hbar\omega_\perp$
2. ***Decrease density*** n
3. ***Increase mass*** m

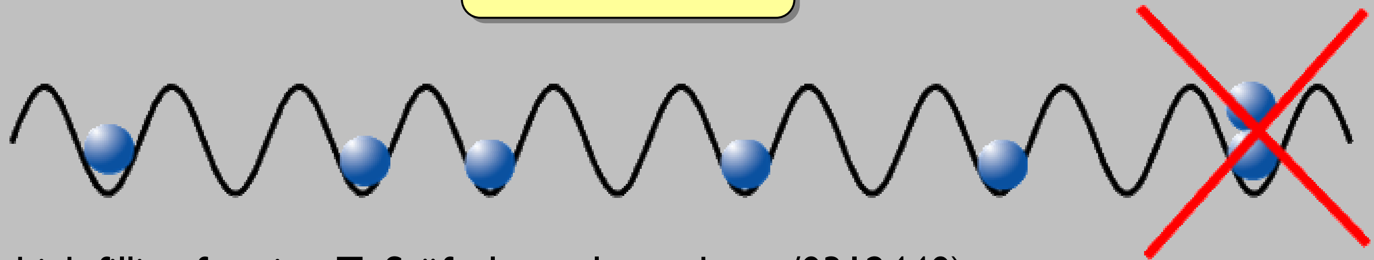
Increasing the Mass

Addition of lattice along the axial direction leads to an increase in the effective mass m*!



However, in order to apply Fermionization, we need to work in a regime, where:

$$\nu \leq 1$$

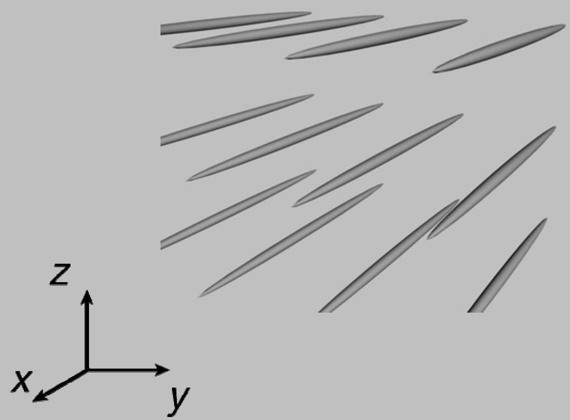


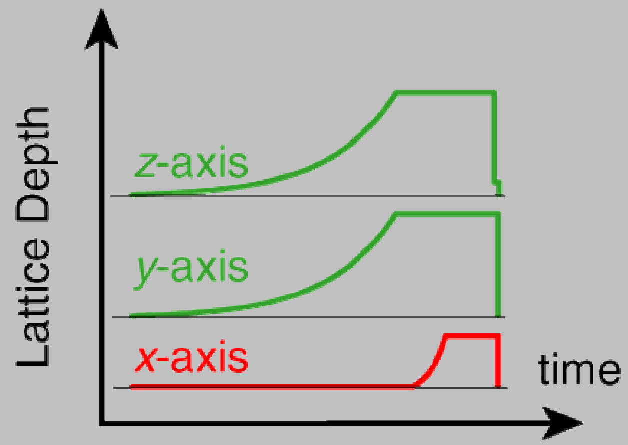
(exp. with high filling fraction T. Stöferle et al. cond-mat/0312440)

Tonks-Parameter in a lattice:

$$\gamma = U / J$$

Experimental Sequence to Prepare the 1D Quantum Gases





(1) ***Create array of 1D quantum gases***

(2) ***Add lattice along axial direction***

Experimental parameters:

V_0 (2D) approx 27 E_r

V_{ax} =0-19 E_r

Lattice Wavelengths 825 nm

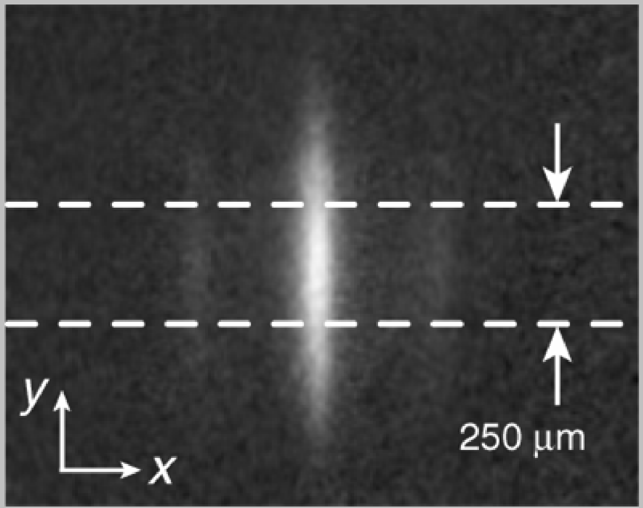
Atom number < 4×10^4

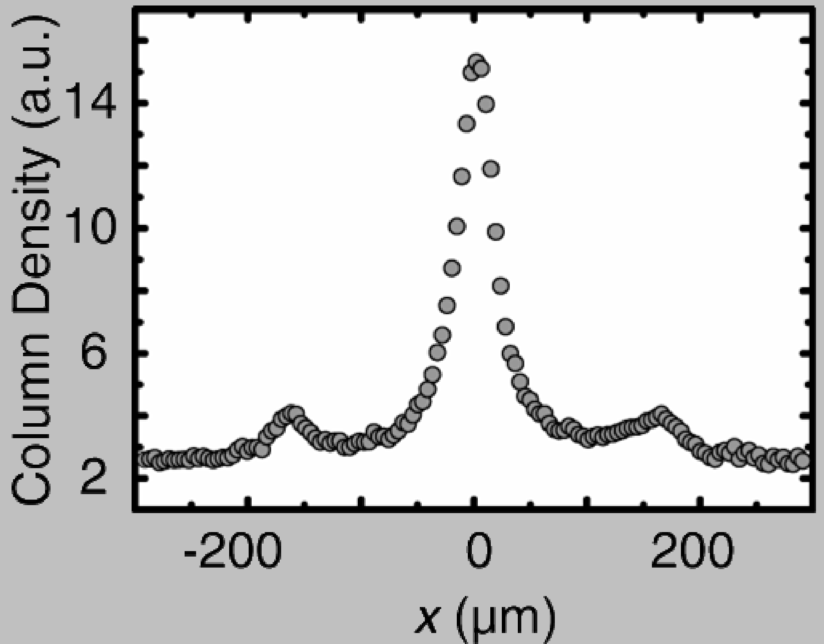
Harmonic confinements:

$$\omega_{ax} = 2\pi\times 60\,\text{Hz}$$
$$\omega_{\perp} = 2\pi\times 35\,\text{kHz}$$

Typical Absorption Images After Time Of Flight

Observe fast expansion in radial direction





Due to the low atom number we average horizontal profiles within the white dashed lines.

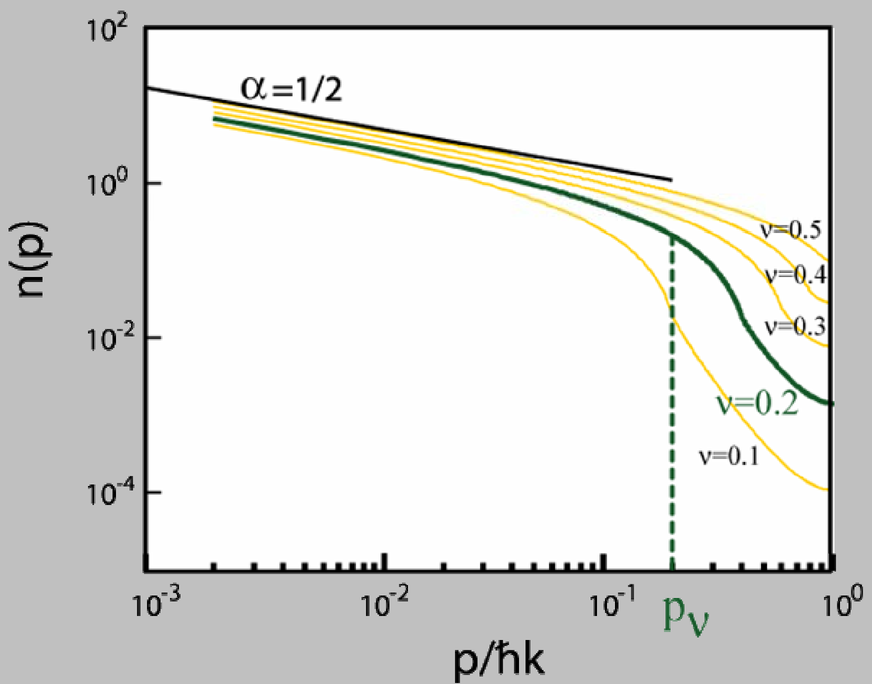
Challenge:
Fully explain momentum distributions!

Momentum Distribution of a (Lattice) 1D Gas

Important momentum scale (1/average interparticle spacing):

$$p_\nu = \hbar \times \frac{2\pi\nu}{\lambda}$$

ν : filling factor



(a) For $p \ll p_\nu$ the slope tends to 1/2

$$n(p) \propto \frac{1}{\sqrt{p}}$$

(b) For $p \gg p_\nu$ the momentum distribution is affected by short range correlations, which tend to **increase** the slope

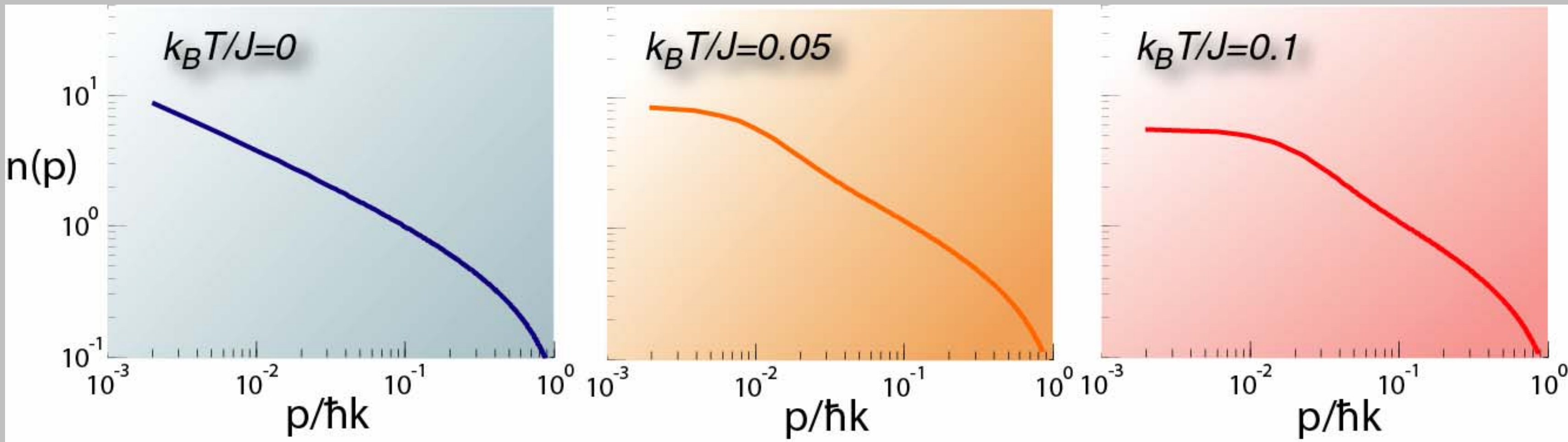
cp. M. Olshanii, PRL **91** (2003)

Finite Temperature Effects in a (lattice) 1D gas

Important momentum scale (1/thermal phase coherence length):

$$p_T = \hbar \times \frac{\pi}{L_\phi}$$

$$L_\phi \approx \lambda J / k_B T \times \sin(\pi\nu)$$



$\nu = 1/2$, 1000 sites

Finite temperature affects low momenta (long range coherence) and broadens peaks

Summary of Important Momentum Scales

$$p_\nu / \hbar = \frac{2\pi\nu}{\lambda} \approx n \approx k_F$$

Short range – long range correlations (change slope)

$$p_T / \hbar = \frac{\pi}{L_\phi}$$

Thermal effects (broaden momentum peaks)

$$p_L / \hbar = \frac{\pi}{L}$$

Finite size effects

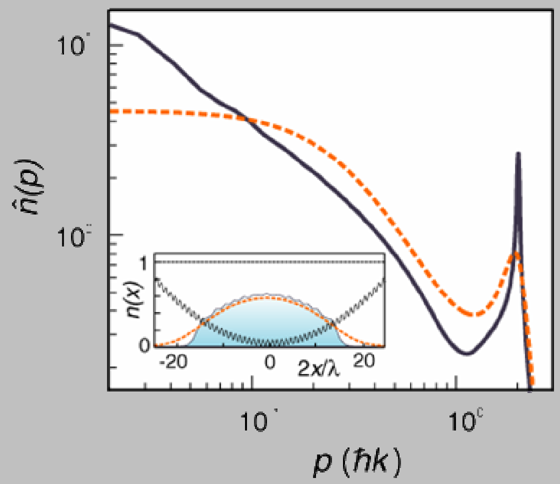
For our experimental parameters, we find:

$$p_L < p_T \sim p_\nu$$

Finite size effects are dominated by finite temperature effects!

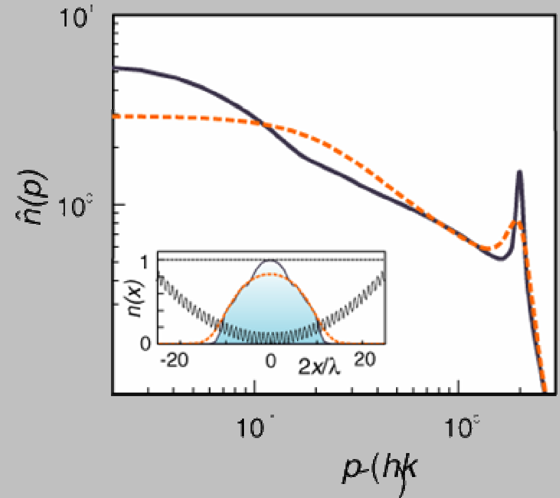
Momentum & Density Distribution for a Fermionized 1D (lattice) gas

$V_{ax}=5\ E_r$



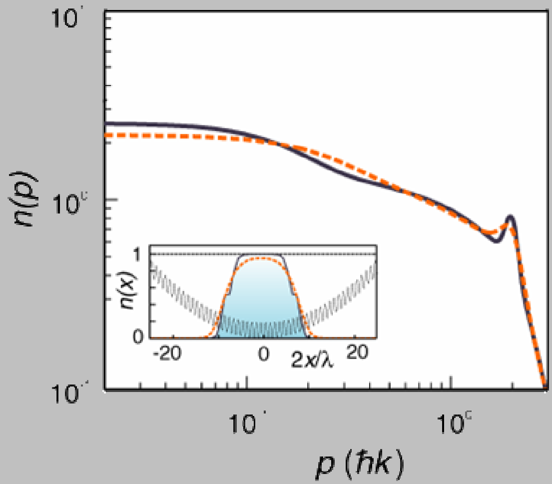
$\alpha=0.8$, T=0

$V_{ax}=9.5\ E_r$



$\alpha=0.5$, T=0

$V_{ax}=12\ E_r$



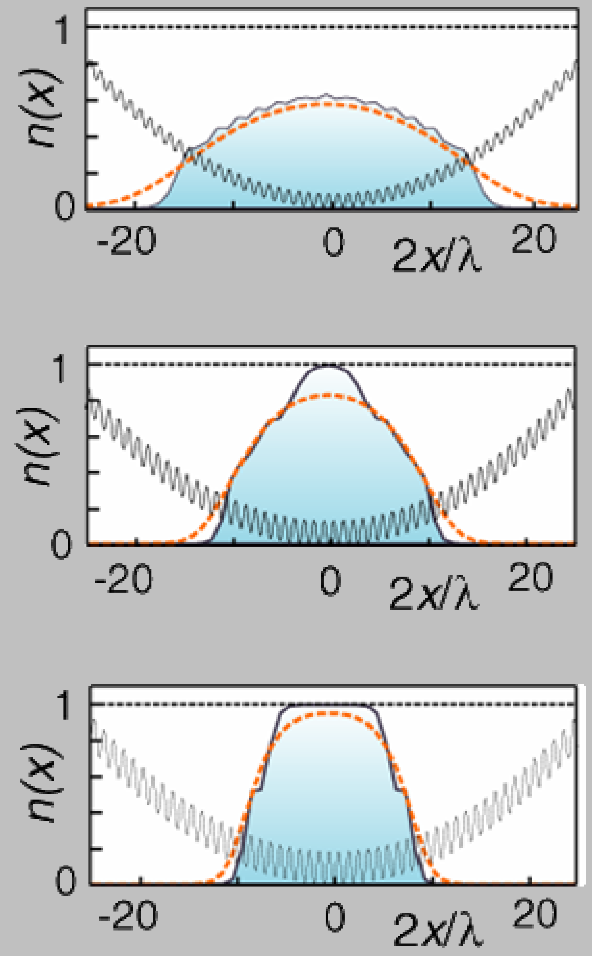
$\alpha=0.2$, T=0

Red curve
$k_BT/J=0.75$

Single tube result: N=15

Increase of Lattice Depth Changes Filling Factor in the Inhomogeneous System

Increasing Lattice Depth
(not harmonic confinement!)

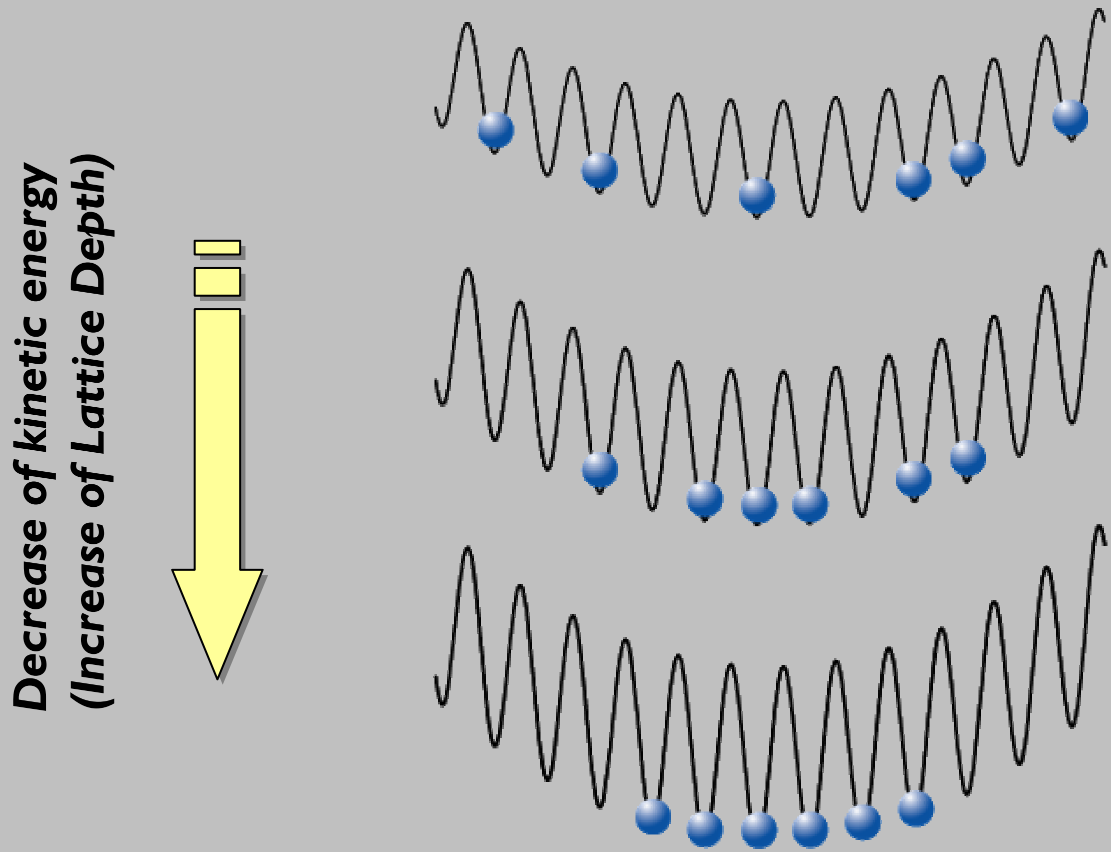


Even for $U \to \infty$ the system spreads out due the kinetic energy J !

⇨ If J decreases (deeper lattice), the system shrinks until a Mott state with n=1 is formed in the center!

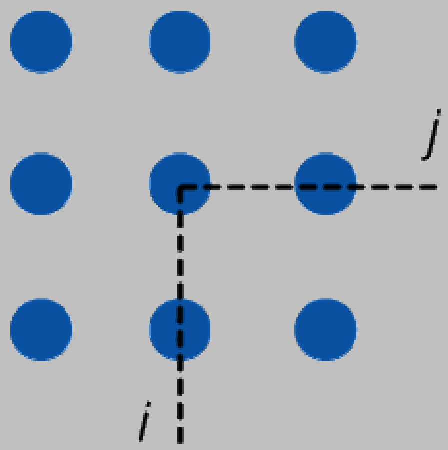
Fermionization describes all filling factor regimes up to n≤1, provided $\gamma >> 1$!

Simple Picture for Change in Filling Factor



$$H = -J\sum_{\langle i,j\rangle}\hat{a}_i^{\dagger}\hat{a}_j + \sum_i \frac{1}{2}m\omega^2\left(\frac{\lambda}{2}\right)^2 i^2\hat{n}_i + \frac{1}{2}U\sum_i \hat{n}_i(\hat{n}_i - 1)$$

Averaging over the Different 1D Gases



Atom number in potential tube i,j

$$N_{i,j} = N_{0,0}\left(1-\frac{5}{2\pi}\frac{N}{N_{0,0}}\left(i^2+j^2\right)\right)^{3/2}$$

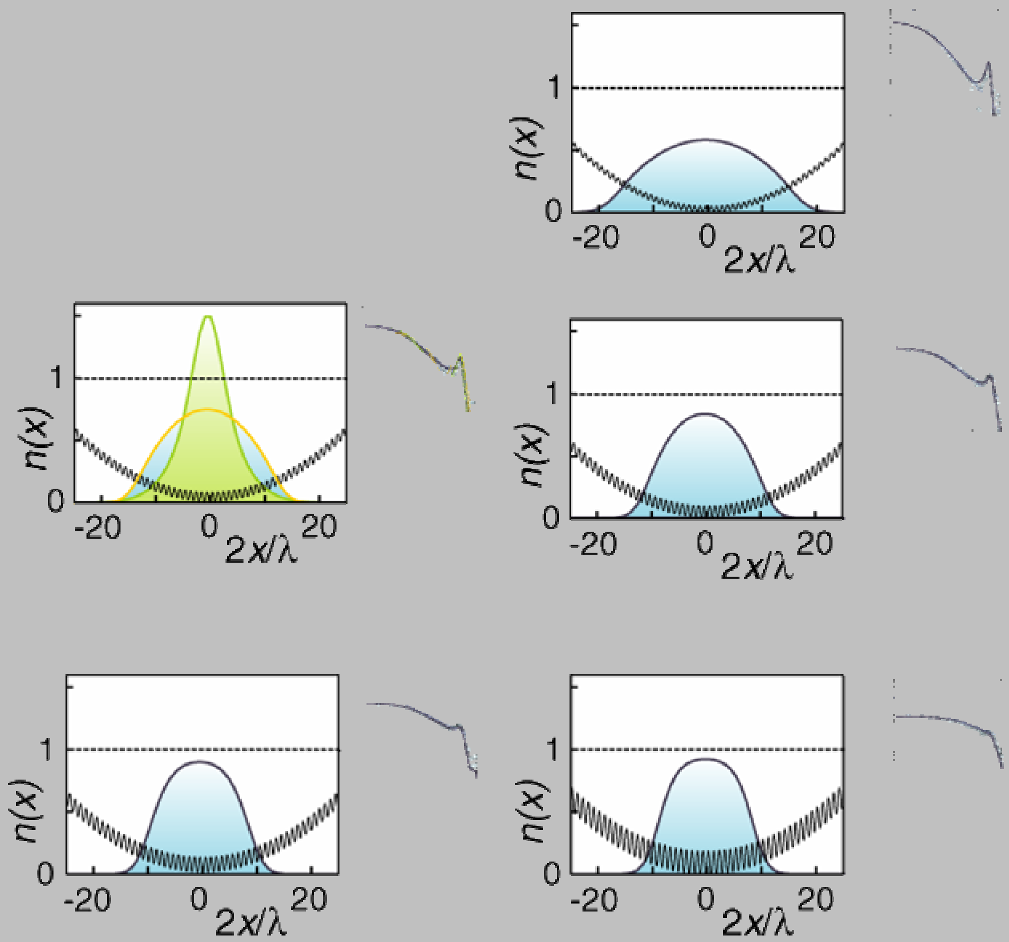
Probability for finding tube with M atoms

$$P(M) = \frac{2}{3}\frac{1}{N_{0,0}^{2/3}M^{1/3}}, \quad M \le N_{0,0}$$

$N_{0,0}$ atom number in central tube !

We use this probability distribution to average over the momentum distributions of different tubes.

Change in Densities



$V_0 = 4.6\ E_r$

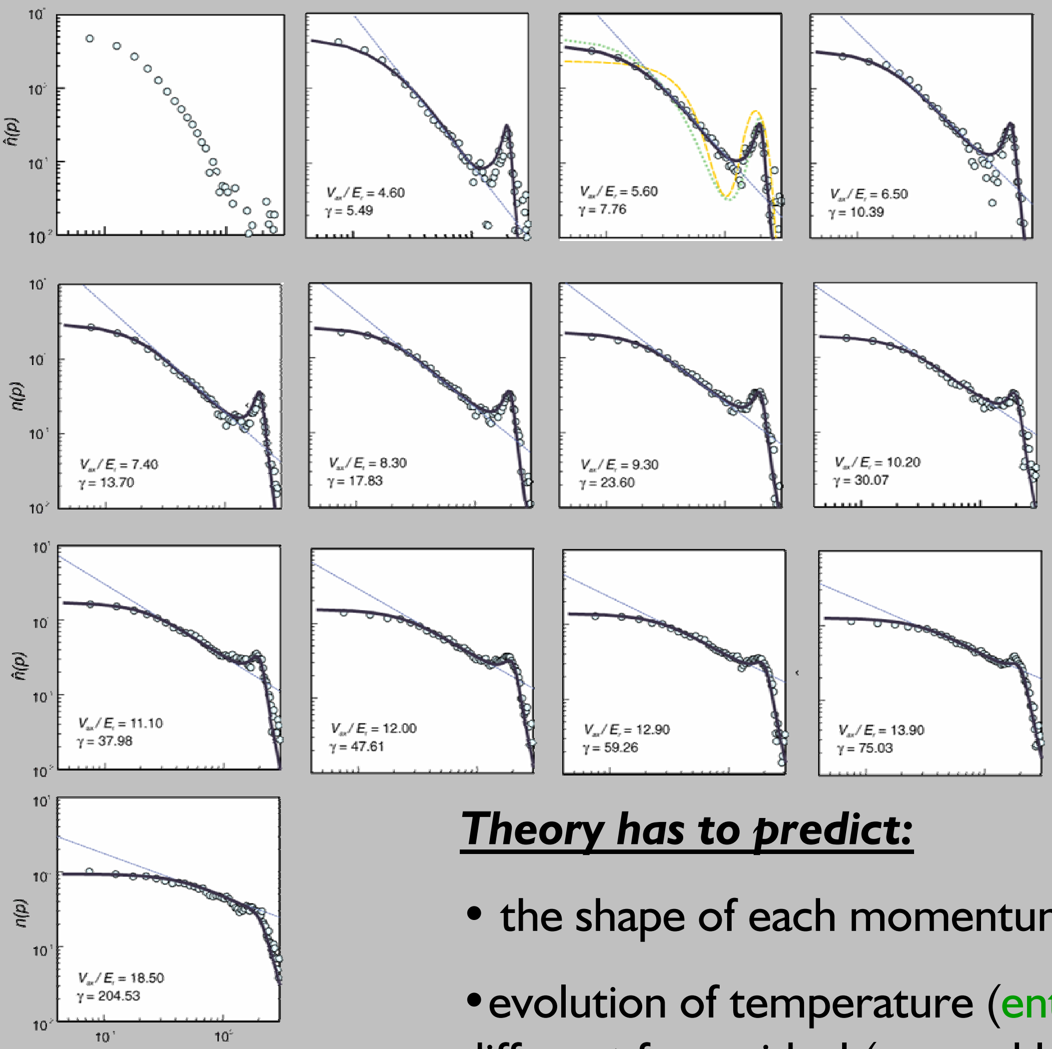
$V_0 = 7.4\ E_r$

$V_0 = 9.3\ E_r$

$V_0 = 12\ E_r$

$V_0 = 18.5\ E_r$

Comparison Theory-Experiment (All Series)



Only two fit parameters for whole series!

$$k_B T_0 / J \approx 0.5$$

$$N_{0,0} = 18$$

Theory has to predict:

- the shape of each momentum profile
- evolution of temperature (entropy) which is very different for an ideal (or weakly interacting) Bose and Fermi gas

Conclusion & Outlook
- Fermionization -

1D Quantum gases

- ***We have been able to enter the Tonks-Girardeau regime in a 2D array of one-dimensional quantum gases***

- ***Increase in effective mass good way to increase interactions***

- ***For Fermionization to be applicable it is however important to work at low filling factors***

- ***We observe excellent agreement with the theory based on a fermionization approach***

→ ***First quantitative comparison of momentum distribution with theory***

- ***Good agreement has allowed us to determine temperature of the quantum gases***

- ***Next: Use PA to probe two-body correlations in gas***

D.M. Gangardt & G.V. Shlyapnikov PRL 90, 150402 (2003)