BEC Meeting

2-3 May 2006

Cold Atoms in Optical Lattices

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1. how to detect BCS transition in Fermi gases ?

- *Umklapp collisions and oscillations of a trapped Fermi gas*, **PRL 2004**
- *Sound propagation and oscillations of a superfluid Fermi gas in a 1D optical lattice*, **PRA 2005**

2. two-body problem in optical lattices

- *Formation of molecules near a Feshbach resonance in 1D optical lattice*, **PRL 2005**
- *Two-body problem in periodic potentials*, **PRA 2006**

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2. two-body problem in optical lattices

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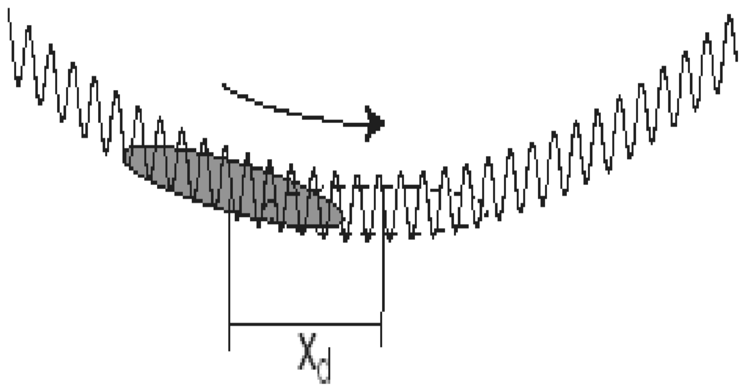
Superfluid Fermi gas in a 1D optical lattice, **PRL 2005**

1D tight optical lattice + weak harmonic trapping



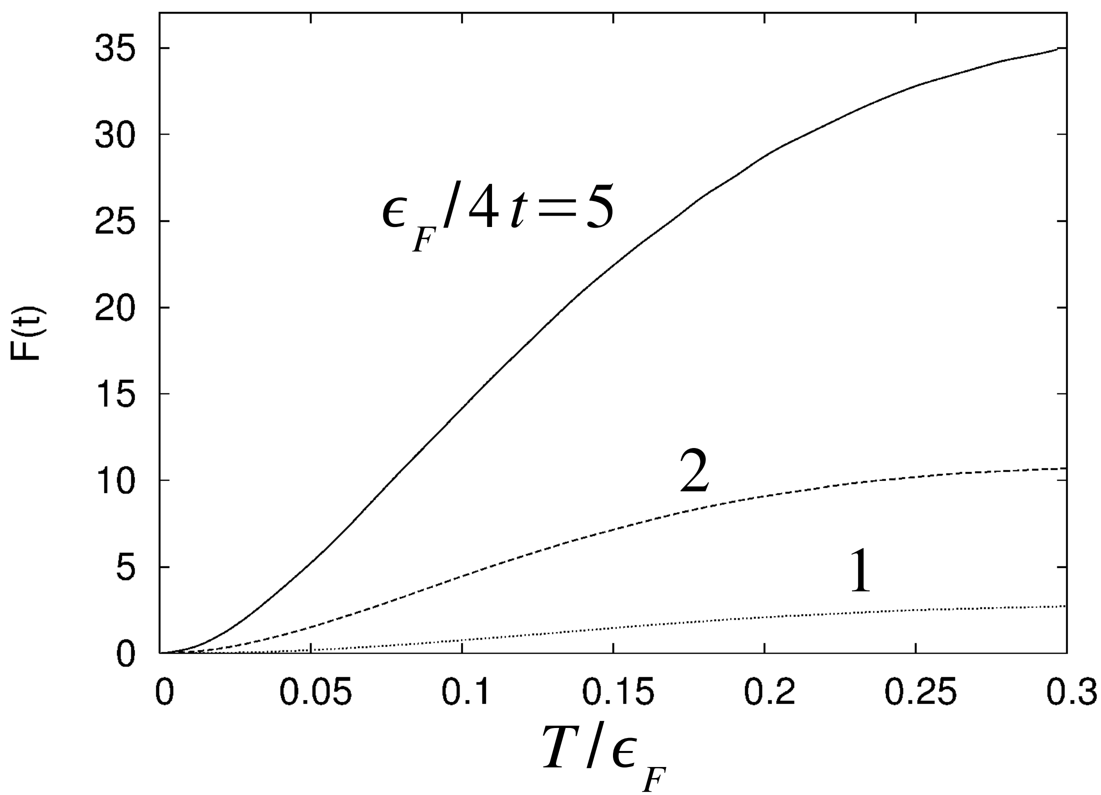
Kohn's theorem does not apply

Question: what happens in interacting Fermi gases ?

collisional regime: oscillations **killed** by umklapp collisions

Relaxation rate $1/\tau_{uk}$ from quantum Boltzmann equation

$$\frac{1}{\tau_{uk}} = C^2 \frac{a^2}{d^2} \frac{2t}{\hbar} F\left(\frac{T}{\epsilon_F}, \frac{\epsilon_F}{4t}\right)$$



PRL 2004

superfluid phase: **persistent Josephson oscillations**

why ? phonons protected by energy gap

Can write **hydrodynamic equations** as for BEC.

Ingredients: effective mass and compressibility (from BCS theory)

$$\frac{1}{m_s} = \frac{\int \frac{\partial^2 \epsilon}{\partial k_z^2} n_0(\boldsymbol{k})\, d\boldsymbol{k}}{\int n_0(\boldsymbol{k})\, d\boldsymbol{k}}$$

Quasi-momentum distribution

$$n_0(\boldsymbol{k}) = 2\,\Theta(\epsilon_F - \epsilon(\boldsymbol{k}))$$

eff. mass is **density dependent**. Recover BEC limit for $\epsilon_F \ll 4t$

BEC: $n_0(\boldsymbol{k}) = n\,\delta(\boldsymbol{k}) \rightarrow 1/m_s^b = 1/m^*$

PRA 2005

- study of **dynamical instability**
- application to **trapped** gases

Bound states in 1D optical lattice

$$V_{ext}(z) = s\, E_R \sin^2(\pi z/d)$$

no confinement in x-y directions

s laser intensity, $E_R = \dfrac{\hbar^2 \pi^2}{2 m d^2}$ recoil energy, d lattice spacing (0.1-1 μm)

interparticle interactions modeled via *s-wave* pseudopotential

$$\hat{U}\,\psi(\vec{r}) = g\,\delta(\vec{r})\,\partial_r \left(r\,\psi(\vec{r}) \right)$$

$\vec{r} = \vec{r}_1 - \vec{r}_2$ relative distance $\qquad g = \dfrac{4\pi\hbar^2 a}{m}$ → s-wave scattering length

e.g. harmonic potential $V_{ext}(r)=\frac{1}{2}m\omega^2 r^2$

COM separates

$$V_{ext}(\vec{r}_1)+V_{ext}(\vec{r}_2)=V_{CM}(\vec{R})+V_r(\vec{r}) \quad \rightarrow \quad \psi=\phi_{CM}(\vec{R})\phi_r(\vec{r})$$

$$\vec{R}=\frac{\vec{r}_1+\vec{r}_2}{2}$$ center-of-mass coordinate

Binding energy given by **algebraic** equation

$$\frac{1}{g}=\frac{\partial}{\partial r}(r\,G_E(\vec{r}))_{r=0} \qquad G_E(\vec{r})=-\sum_n \frac{\phi_n(\vec{r})\phi_n(0)}{E_n-E}$$

Green function for **relative** motion

what about optical lattices ? **e.g.** 1D lattice

!!

$$V_{opt}(z_1)+V_{opt}(z_2)\neq V_{CM}(Z)+V_r(z)$$

c.o.m. and relative motion coupled

Solution via 2-particle Green's function

$$\psi(\vec{r},Z)=\int dZ'\,G_E(\vec{r},Z;\vec{0},Z')\,g\frac{\partial}{\partial r'}(r'\psi(\vec{r}',Z'))_{r'=0}$$

Binding energy given by **integral** equation in COM variable

$$Y(Z)=g\int dZ'\,K_E(Z,Z')\,Y(Z')$$

regular kernel

$K_E(Z, Z')$ **invariant** under $Z \to Z + d$ → quasi-momentum Q is <u>conserved</u>

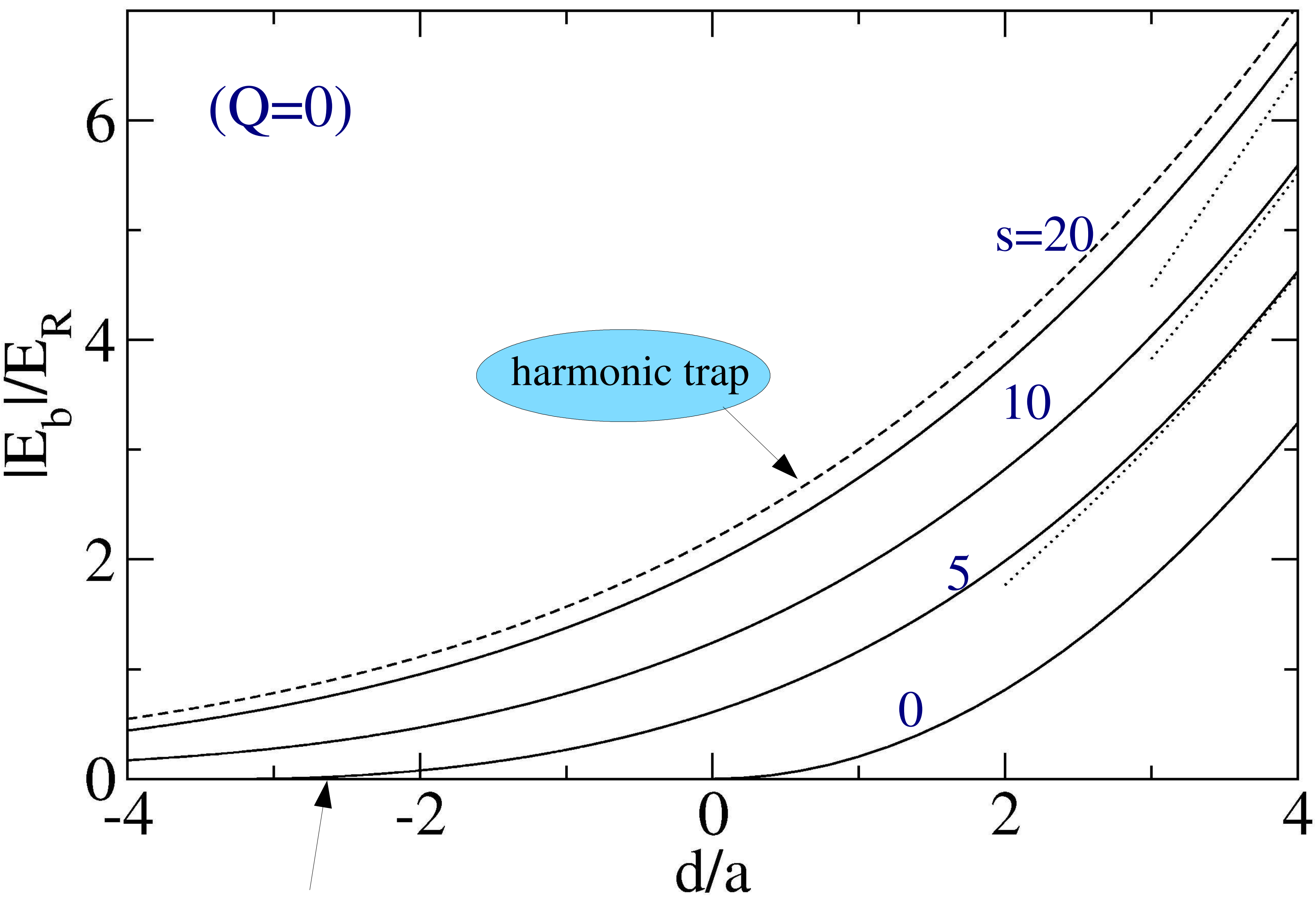
solutions are Bloch states $f_Q(Z+d) = e^{iQd} f_Q(Z)$

COM and relative motion coupled → binding energy <u>depends</u> on quasi-momentum

For given Q, integral equation has been solved

- **numerically** *PRL 2005*
- **analytically (tight-binding)** *PRA 2006*

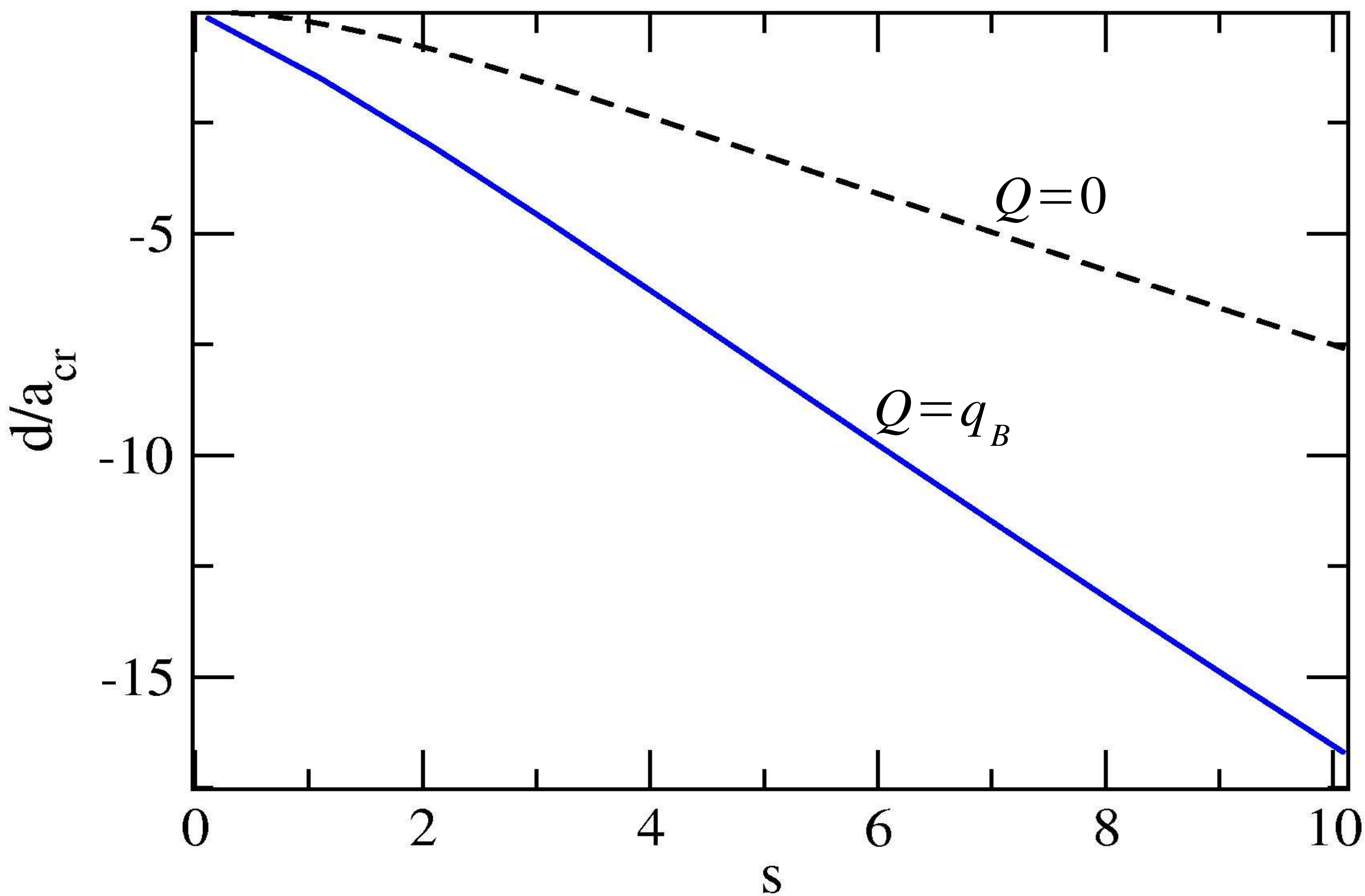
Binding energy versus inverse scattering length



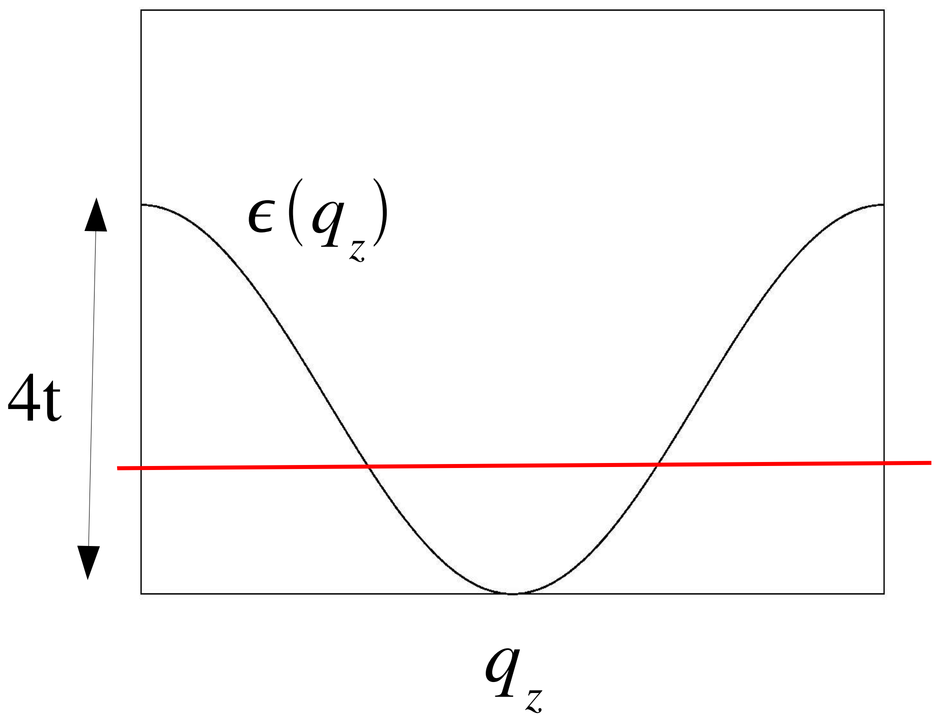
dots: lattice as perturbation

d/a_{cr} binding energy vanishes

Critical value of scattering length



Regimes in tight 1D optical lattices



$|E_b| \ll 4t$

anisotropic 3D regime

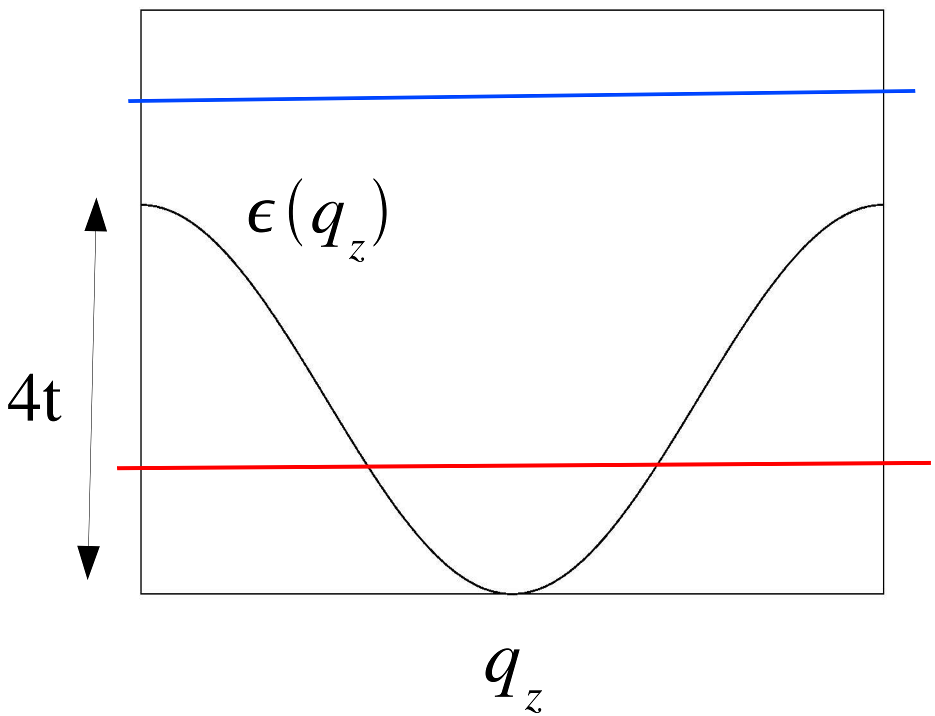
3D regime

- bound state spread over <u>many</u> lattice sites
- binding energy takes universal form

$$|E_b| \sim \frac{\hbar^2}{m^* C}\left(\frac{1}{a} - \frac{1}{a_{cr}}\right)^2$$

$g \rightarrow gC$ renormalization of coupling constant

Regimes in tight 1D optical lattices



$\epsilon_g \gg |E_b| \gg 4t$ **quasi-2D regime**

$\Uparrow$ *dimensional crossover*

$|E_b| \ll 4t$ **anisotropic 3D regime**

3D regime

- bound state spread over <u>many</u> lattice sites
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$$|E_b| \sim \frac{\hbar^2}{m^* C}\left(\frac{1}{a} - \frac{1}{a_{cr}}\right)^2$$

$g \to gC$ renormalization of coupling constant

quasi-2D regime

- tunneling is **irrelevant**: effective harmonic oscillator ω_0
- binding energy approaches **asymptotic value**

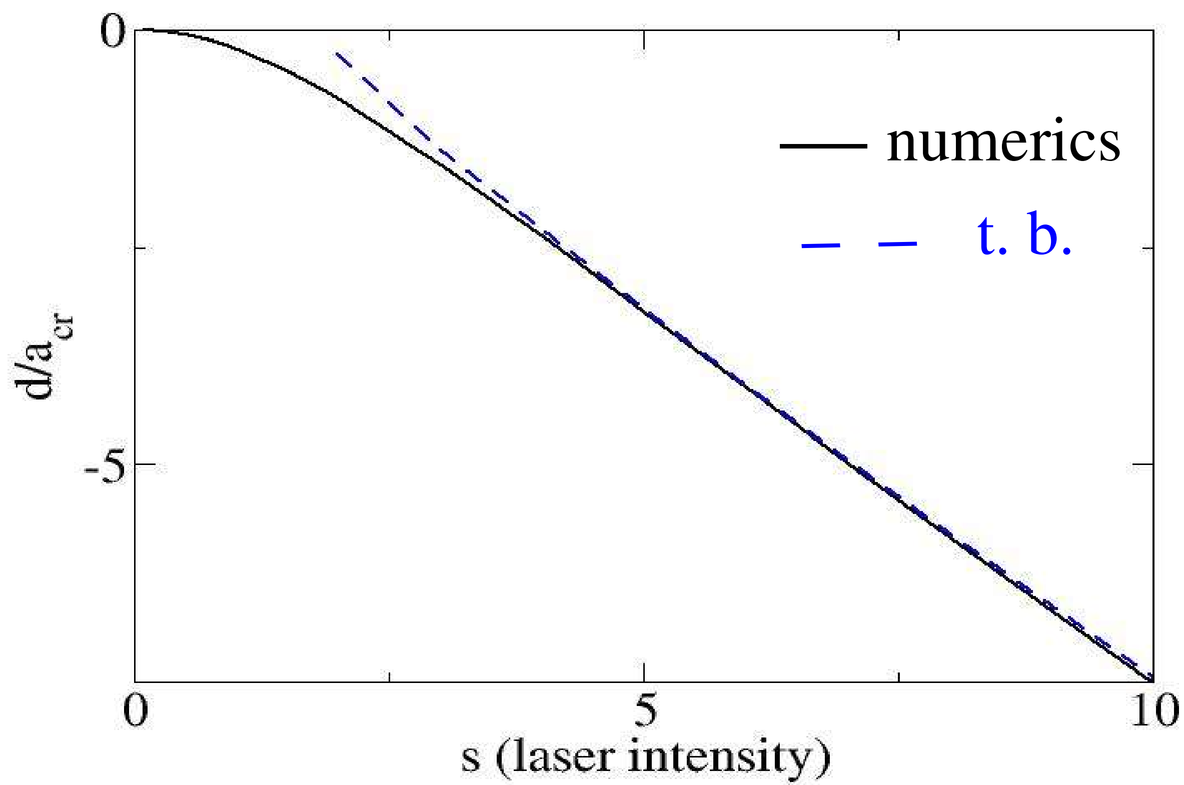
$$|E_b| = \lambda \hbar \omega_0 \exp\left(-\sqrt{\frac{\pi}{2}}\frac{\sigma}{|a|}\right)$$

$\lambda \to$ 0.29

$\sigma \to$ harmonic oscillator length $(\hbar / m\omega_0)^{1/2}$

yields formula for a_{cr}

$$\frac{d}{a_{cr}} = -C \ln\left(\frac{\lambda \hbar \omega_0}{2t}\right)$$



BCS transition in 1D optical lattice

QUESTION: how is T_c modified by 1D lattice ?

$$\frac{1}{g_{eff}} = P \int \frac{d^3 q}{(2\pi)^3} \frac{1}{\xi_{\mathbf{q}}} \frac{1}{\exp(\xi_{\mathbf{q}}/T_c^0)+1}$$

$$\xi_{\mathbf{q}} = \frac{\hbar^2 \mathbf{q}_p^2}{2m} + \epsilon(q_z) - \mu$$ dispersion relation with $\mu = \epsilon_F$

g_{eff} **effective** coupling constant for Cooper pairs

related to 2-body **scattering amplitude**

$$\frac{1}{g_{eff}} = \frac{m}{4\pi\hbar^2} \Re\left[\frac{1}{f_{sc}(E=2\mu)}\right]$$

Scattering amplitude

$$f_{sc}(E)=a\int dZ\,\varphi_E(0,Z)\,\partial_r\left(r\,\psi(\vec{r},Z)\right)_{r=0}$$

$$\varphi_E(\vec{r},Z)=e^{i\vec{q}_p\cdot\vec{r}_p}\,\phi_{q_z}(z_1)\,\phi_{-q_z}(z_2)$$

incident 2-particle state

$\phi_{q_z}(z)$ **Bloch states**

$$E=\frac{\hbar^2 q_p^2}{m}+2\epsilon(q_z)$$

energy

$$Y(Z)=\varphi_E(0,Z)+g\int dZ'\,K_E(Z,Z')\,Y(Z')$$

$$Y(Z)\equiv\partial_r\left(r\,\psi(\vec{r},Z)\right)_{r=0}$$

Tight-binding solution

ansatz: $Y(Z) \sim A \sum_j w^2(Z - j d)$

$$w(z) = \frac{1}{\pi^{(1/4)} \sigma^{(1/2)}} \exp\left[\frac{-z^2}{2\sigma^2}\right]$$

Wannier state

$$f_{sc}(E) = \frac{a C}{1 - a/a_{cr} + (a/\sqrt{2\pi}\sigma)\beta(E)}$$

$\beta(E)$ describes **dimensional crossover**

β =i arccos$(1 - E/4t)$ $(E<8t)$

$\beta = -\log[E(1+\sqrt{1-8t/E})^2/8t] + i\pi$ $(E>8t)$

anisotropic 3D regime

($E \ll 8t$)

$$f_{sc}(E) = \frac{C}{1/a - 1/a_{cr} + iC\sqrt{E m^{*}}/\hbar}$$

shift of Feshbach resonance

$$\frac{1}{g_{eff}} = \frac{m}{4\pi\hbar^2 C}\left(\frac{1}{a} - \frac{1}{a_{cr}}\right)$$

density **independent**

$$|a| \ll |a_{cr}| \rightarrow g_{eff} \simeq Cg$$

e.g. dilute BEC in optical lattices

lattice enters through **effective mass** and **coupling constant** quasi-2D regime

($E \gg 8t$)

$$f_{sc}(E) = \frac{C}{\frac{1}{a} + \frac{1}{\sqrt{2\pi}\sigma}\left[\ln\frac{\lambda\hbar\omega}{E} + i\pi\right]}$$

$\lambda = 0.29$

$\mu > 4t$

$$\frac{1}{g_{eff}} = \frac{m}{4\pi\hbar^2 C}\left(\frac{1}{a} + \frac{1}{\sqrt{2\pi}\sigma}\ln\frac{\lambda\hbar\omega}{2\mu}\frac{4}{(1+\sqrt{1-4t/\mu})^2}\right)$$

density **dependent**

change in behaviour occurs **exactly** at $\mu = 4t$

Results for transition temperature

$\epsilon_F < 4t$

$$T_c^0 = \frac{2\gamma}{\pi} e^{-F(\epsilon_F/4t)} \epsilon_F \exp\left[\frac{-\pi}{2 q_{zF} C}\left(\frac{1}{|a|} - \frac{1}{|a_{cr}|}\right)\right]$$

3D regime
$\epsilon_F \ll 4t$

$$T_c^0 = 0.61\, \epsilon_F \exp\left[\frac{-\pi}{2 q_{zF} C}\left(\frac{1}{|a|} - \frac{1}{|a_{cr}|}\right)\right]$$

$\epsilon_F > 4t$

$$T_c^0 = \frac{\gamma}{2\pi}\left(1 + \sqrt{1 - \frac{4t}{\epsilon_F}}\right)\sqrt{\epsilon_F 4t} \exp\left[-\sqrt{\frac{2}{\pi}}\left(\frac{1}{|a|} - \frac{1}{|a_{cr}|}\right)\right]$$

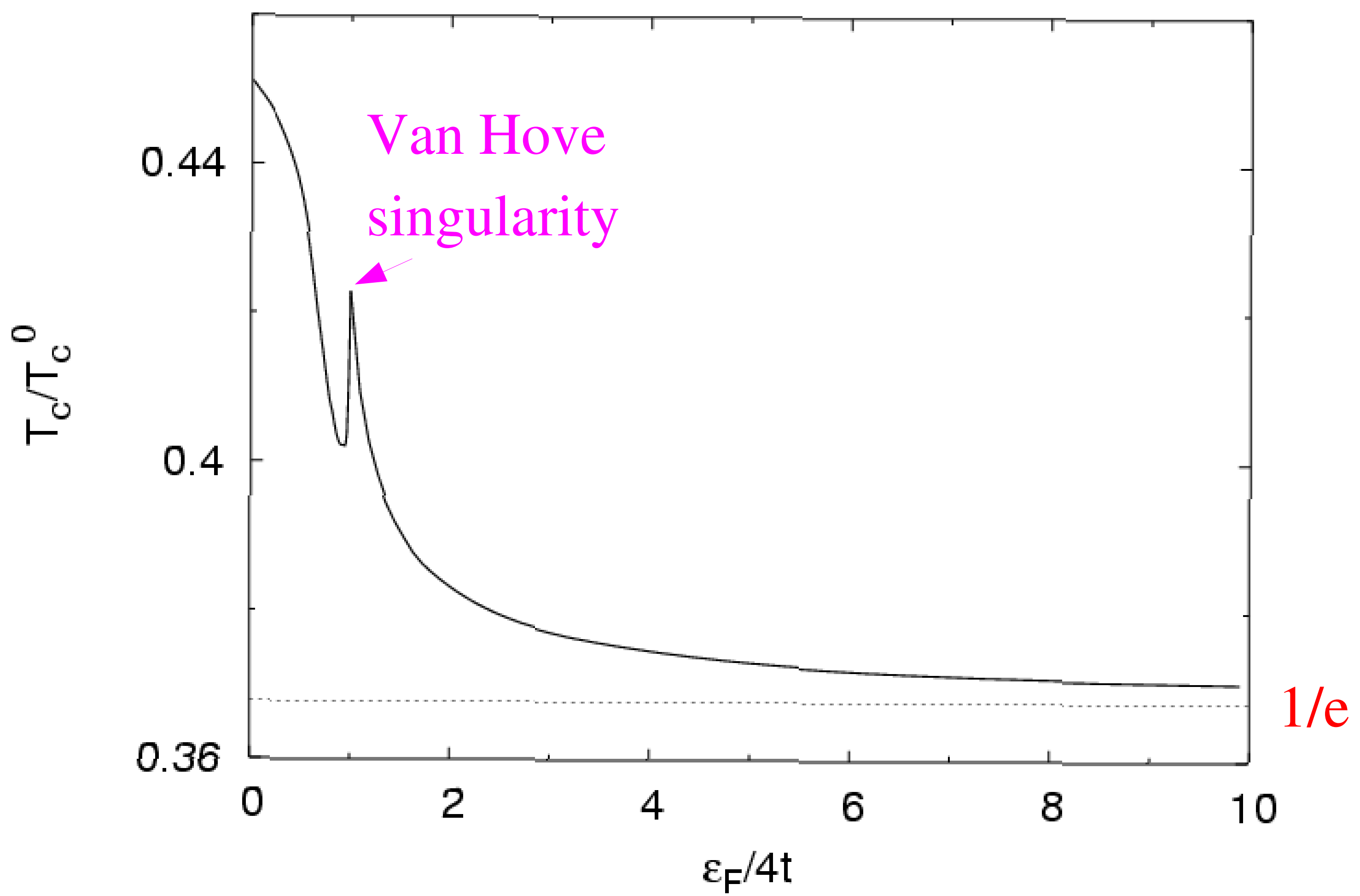
quasi 2D regime
$\epsilon_F \gg 4t$

$$T_c^0 = 0.43 \sqrt{\epsilon_F \hbar\omega} \exp\left(-\sqrt{\frac{\pi}{2}} \frac{\sigma}{|a|}\right)$$

Kosterlitz-Thouless

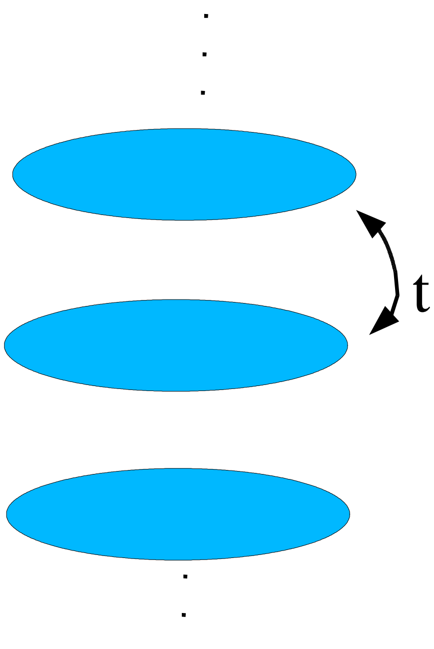
Gorkov's corrections

Gorkov's corrections reduce mean field result $T_c / T_c^0 < 1$



Superfluid-Mott Insulator QPT

- **zero** temperature
- discs are **finite size** and **tunneling is weak**



Question: what is critical tunneling rate for transition to Mott phase ?

HYDRODYNAMIC APPROACH ($N \gg 1$)

in each disc relevant variables are

$N_j = N + \Delta N_j$ number of atoms

Φ_j phase of order parameter

Quantum phase model

$$H_{QP} = \sum_j (E_c/2)\Delta N_j^2 - E_J \cos(\Phi_{j+1} - \Phi_j)$$

$$[\Delta N_j, \Phi_j] = 1$$

E_c **charging** energy E_J **Josephson** energy

$E_c < 0.81\, E_J$ superfluid

$E_c > 0.81\, E_J$ Mott insulator

Bradley-Doniach, PRB (1984)

Complete analogy with BEC: $E_c = 2\dfrac{\mu}{N}$

...but Josephson term is **different** !

BCS superfluid	weakly interacting BEC
Cooper pairs $E_J = \dfrac{t^2}{\epsilon_F} N$	single atom $E_J^B = t\, N$
$t_c \sim \dfrac{\epsilon_F}{N}$	$t_c^B \sim \dfrac{\mu}{N^2}$

Question: can this QPT be achieved with cold gases ?

Answer: only if critical tunneling rate **large** compared to atom **loss rate** due to 3-body recombination

Ultracold gases

- BEC: **NO**, unless $N \sim 1 - 5$
- BCS superfluids: **YES**, up to $N \sim 10^3 - 10^4$

3-body losses quenched by Fermi statistics

Conclusions

- umklapp collisions in Fermi gases
- propagation of sound in superfluid Fermi gases
- two-body problem
- BCS transition temperature
- Superfluid-Mott insulator QPT in Fermi superfluids

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