Dynamics of two and three component Fermi mixtures

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Cold Quantum Matter (EuroQUAM)
Fermix 2009 Meeting
5th June 2009
Trento, Italy

Funding: ESF (EuroQUAM, EURYI), Academy of Finland

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- Three-component mixtures, attractive interactions: a BdG study in a trap (preliminary)
- The repulsive Hubbard model in a one-dimensional lattice: exact hopping modulation dynamics
- RF-spectroscopy in a 1D system: signatures of the FFLO state
- Fermi condesates as sensors

- **Three-component mixtures, attractive interactions: a BdG study in a trap (preliminary)**
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Three-component system in a harmonic trap

- Two-component systems in harmonic traps described by Bogoliubov – deGennes (BdG) equations (e.g. Griffin, Törmä, Machida, Levin, Randeria, Stringari groups)
- BdG equations for 3 components (1, 2, 3): to allow detailed description of trapping effects
- 1+2 interacting, 2+3 interacting, 1+3 not (to simplify)
- Expansion into harmonic trap eigenstates
- Self-consistent equations
- Hartree fields ignored

The mean-field Hamiltonian:

$$H_{\mathrm{MF}} = \sum_{\mathbf{k}\mathbf{k}'} \left[\sum_{\sigma} \epsilon_{\sigma\mathbf{k}\mathbf{k}'} c^{\dagger}_{\sigma\mathbf{k}'} c_{\sigma\mathbf{k}} + \frac{1}{2} \sum_{\sigma\neq\sigma'} \left(J_{\sigma\sigma'\mathbf{k}\mathbf{k}'} c^{\dagger}_{\sigma'\mathbf{k}'} c_{\sigma'\mathbf{k}} + J_{\sigma'\sigma\mathbf{k}\mathbf{k}'} c^{\dagger}_{\sigma\mathbf{k}'} c_{\sigma\mathbf{k}} \right) \right.$$
$$\left. + \frac{1}{2} \sum_{\sigma\neq\sigma'} \left(F^{*}_{\sigma\sigma'\mathbf{k}\mathbf{k}'} c^{\dagger}_{\sigma\mathbf{k}'} c^{\dagger}_{\sigma\mathbf{k}} + F_{\sigma\sigma'\mathbf{k}\mathbf{k}'} c_{\sigma\mathbf{k}'} c_{\sigma\mathbf{k}} \right) \right] + C.$$

k: trap quantum numbers

Here $\epsilon_{\sigma\mathbf{k}\mathbf{k}'}$ is the single particle energy, $F_{\sigma\sigma'\mathbf{k}\mathbf{k}'}$ integrated pairing potential $J_{\sigma\sigma'\mathbf{k}\mathbf{k}'}$ integrated Hartree potential and C a constant shift in the energy.

O. Nummi, J. Kinnunen, P.Törmä, in preparation

MF-Hamiltonian in matrix form

$$H_{\mathrm{MF}} = \mathbf{c}^{\dagger}_{\mathbf{k}'} \mathbf{H}_{\mathbf{k}\mathbf{k}'} \mathbf{c}_{\mathbf{k}} + C', \quad \mathbf{c}^{\mathrm{T}}_{\mathbf{k}} = [c_{a\mathbf{k}}, c^{\dagger}_{b\mathbf{k}}, c_{c\mathbf{k}}]$$

Bogoliubov transformation into quasiparticles

$$\begin{pmatrix} \gamma_{a\mathbf{k}} \\ \gamma^{\dagger}_{b\mathbf{k}} \\ \gamma_{c\mathbf{k}} \end{pmatrix} = \mathrm{B}^{\mathbf{k}} \begin{pmatrix} c_{a\mathbf{k}} \\ c^{\dagger}_{b\mathbf{k}} \\ c_{c\mathbf{k}} \end{pmatrix}$$

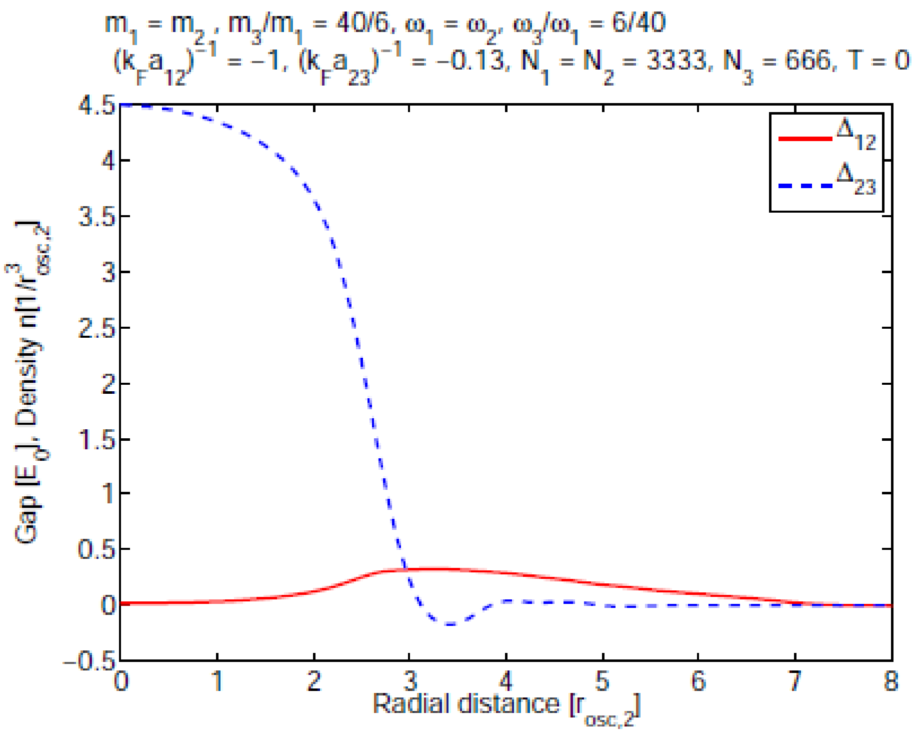
$\mathrm{B}^{\mathbf{k}}$ is unitary

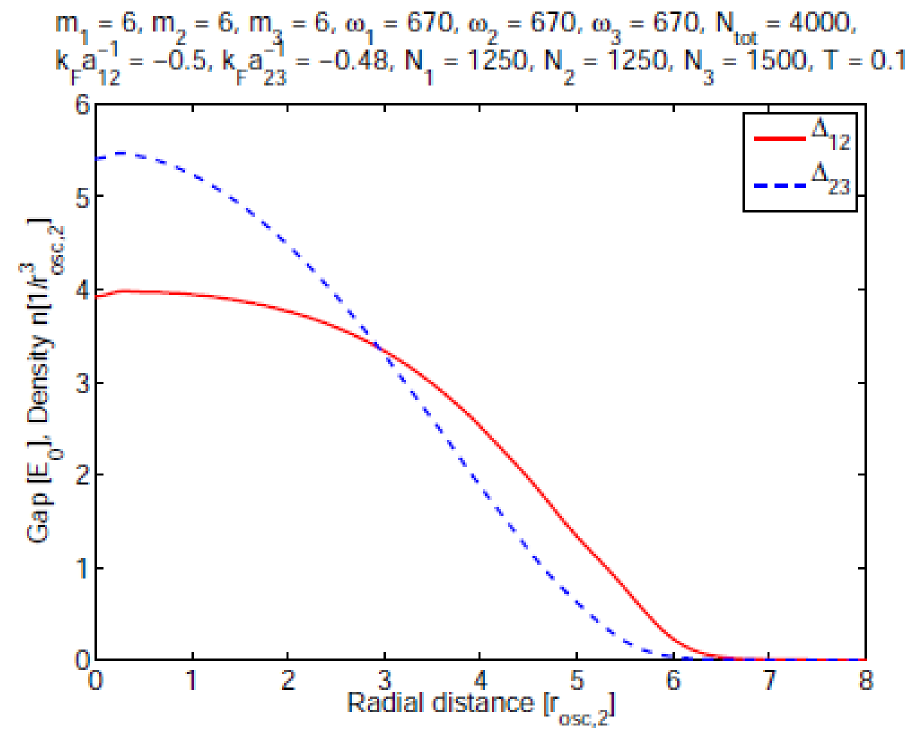
Expand in harmonic trap eigenstates

$$\Psi_\sigma(\mathbf{r}) = \sum_{nlm} R^\sigma_{nl}(r) Y_{lm}(\mathbf{\Omega}) c_{nlm\sigma}$$

Separate different l-quantum numbers and get self-consistent equations for gaps Δ_{12}, Δ_{23} and densities n_1, n_2, n_3

Example gap profiles



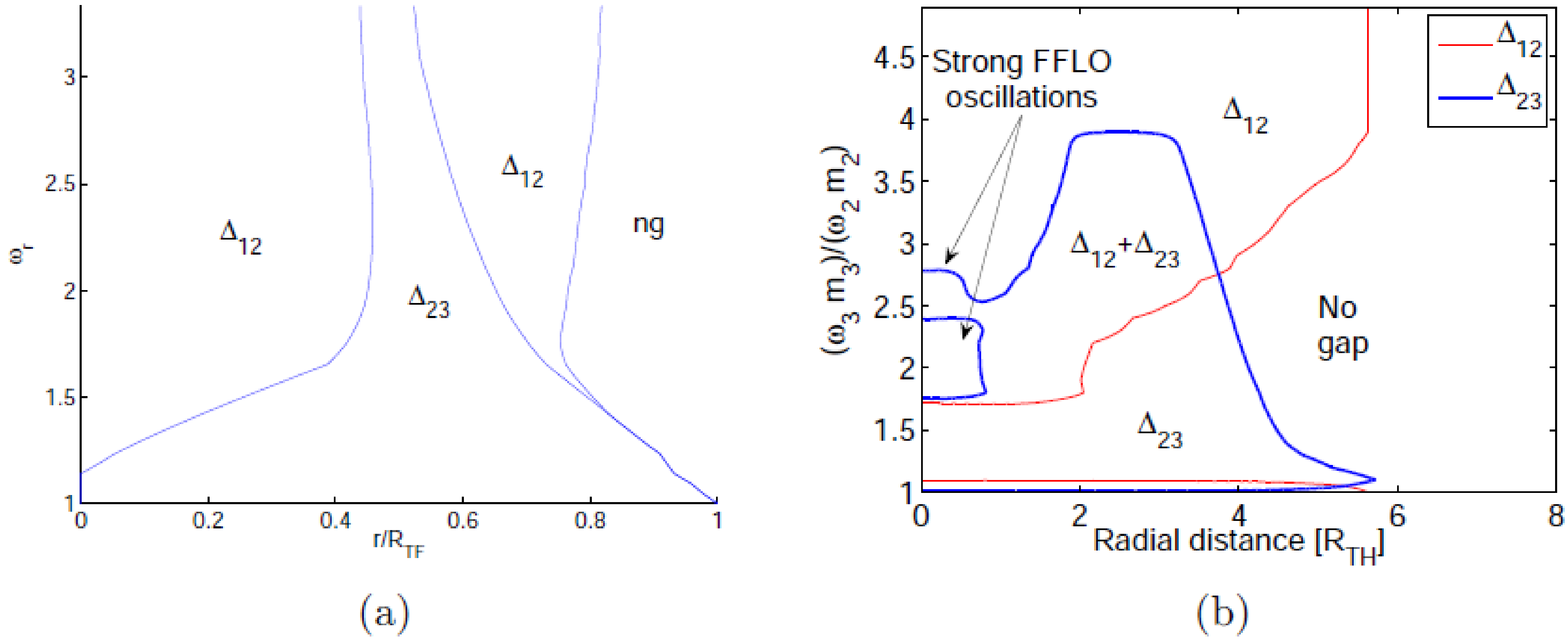


A smooth transition between two gaps and FFLO-type oscillations

Two gaps occurring at the same place

O. Nummi, J. Kinnunen, P.Törmä, in preparation

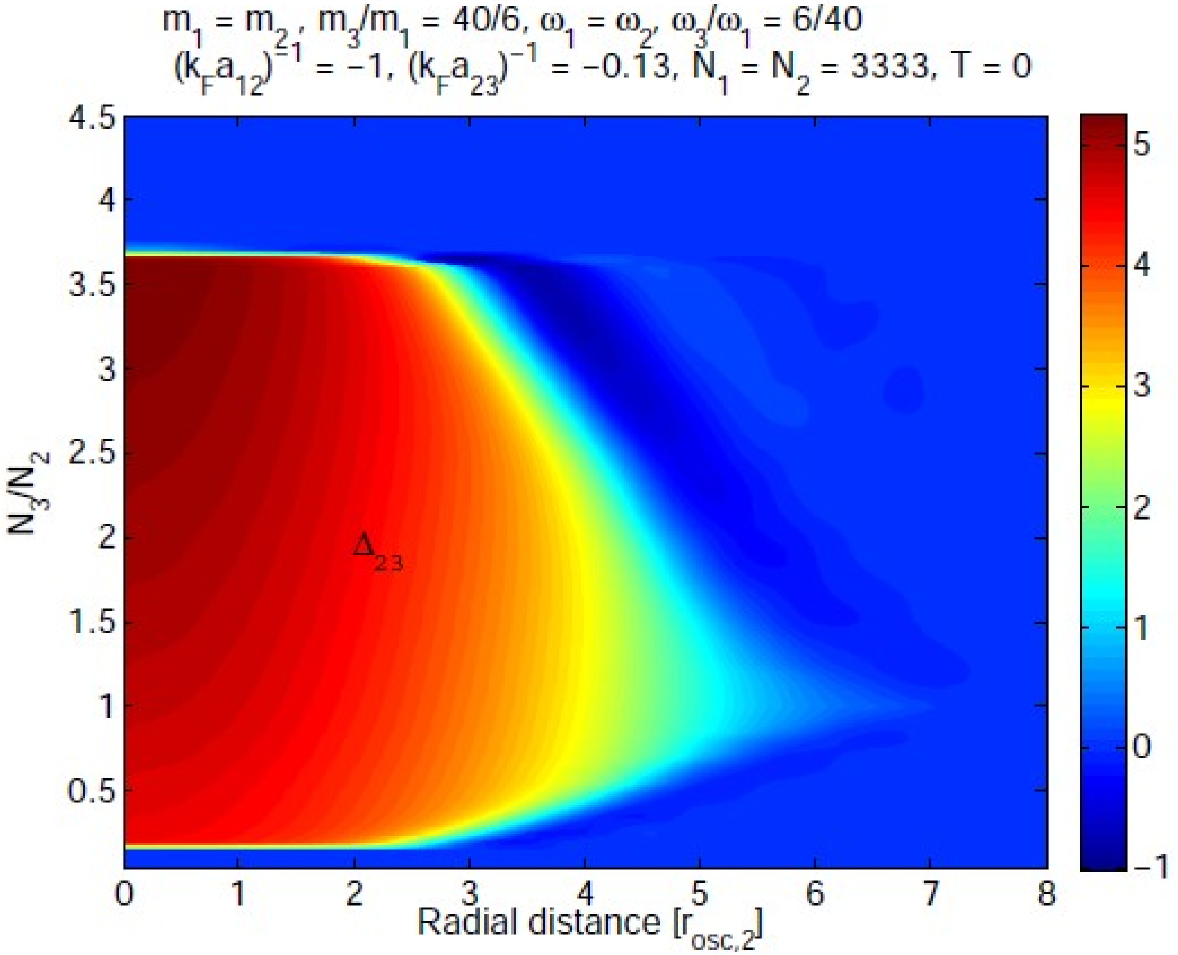
Phase diagram: trap frequency ratio



(Left) Local density approximation (LDA) [1]
(Right) BdG results. Narrow superfluid regions do not exist in BdG unlike in LDA. BdG predicts clear coexistence, LDA not.

[1] T. Paananen, P. Törmä, J.-P. Martikainen, PRA 2007

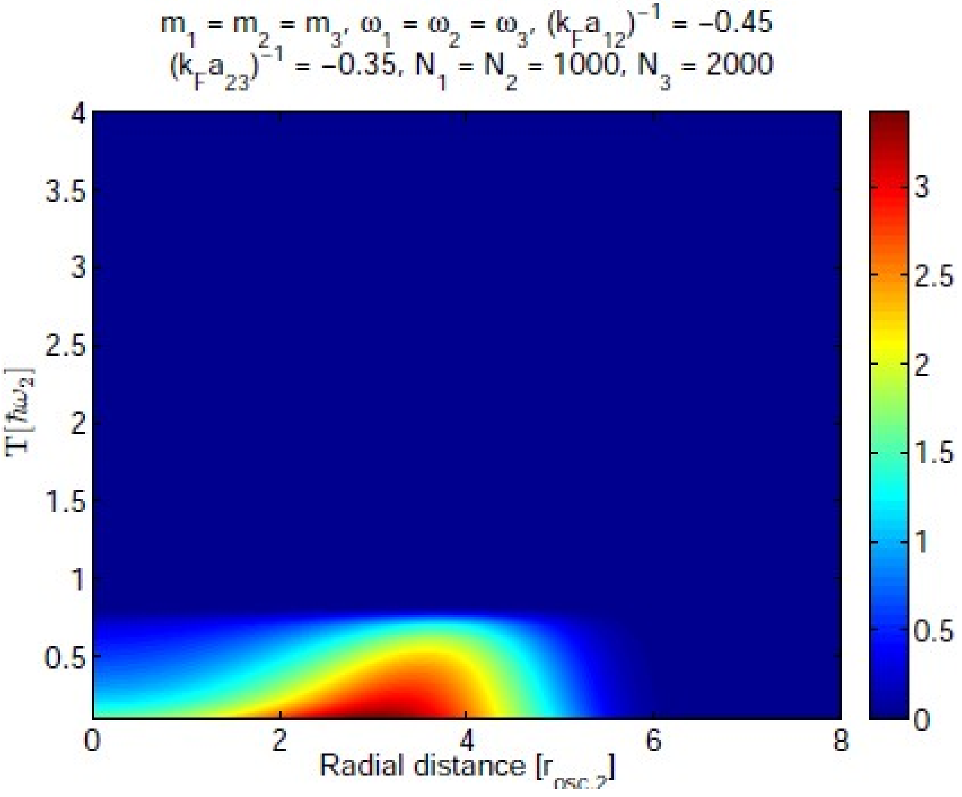
Phase diagram: polarization



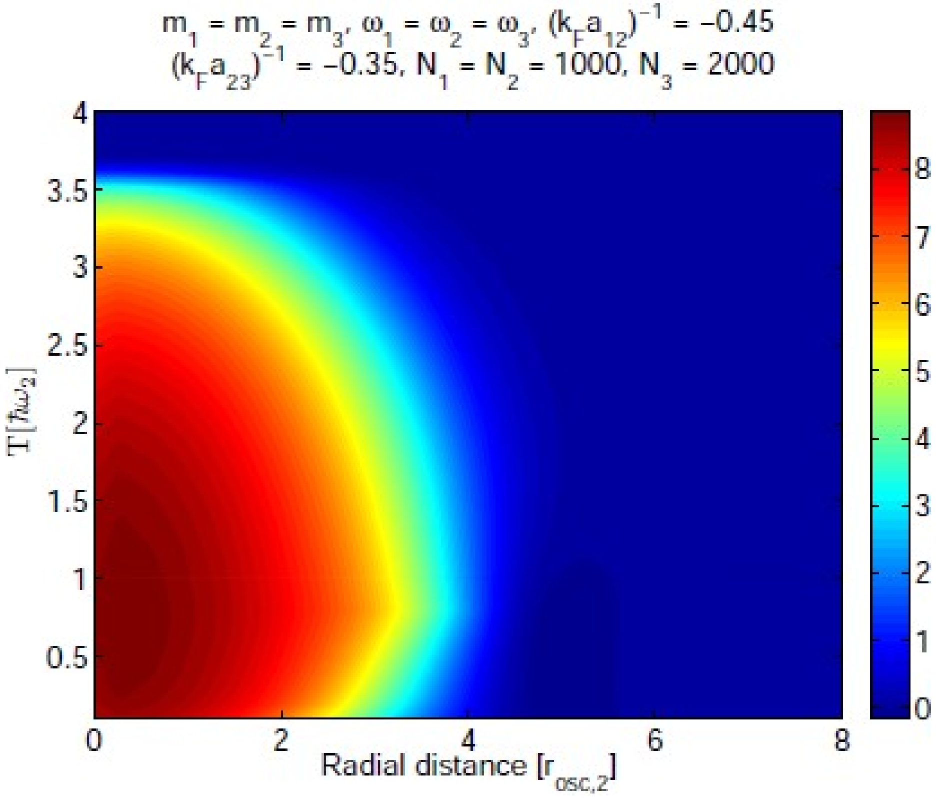
Δ_{23}

Note the FFLO-oscillations (the deep blue)

Phase diagram: temperature



Δ_{12}



Δ_{23}

Disappearance of Δ_{12} as temperature increases and corresponding increase of Δ_{23}

Three component BdG study: summary

- Narrow spatial regions of superfluids in the trap, predicted by LDA, disappear in the BdG treatment
- BdG predicts coexistence of two superfluids over large spatial regions and parameter ranges, which LDA never does. Reasons? Finite size effects? The trap stabilizes the superfluids? Others???
- Three-component mixtures non-trivial even at the mean field level
- These are preliminary, unpublished results

- Three-component mixtures, attractive interactions: a BdG study in a trap (preliminary)
- **The repulsive Hubbard model in a one-dimensional lattice: exact hopping modulation dynamics**
- RF-spectroscopy in a 1D system: signatures of the FFLO state
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Motivation:

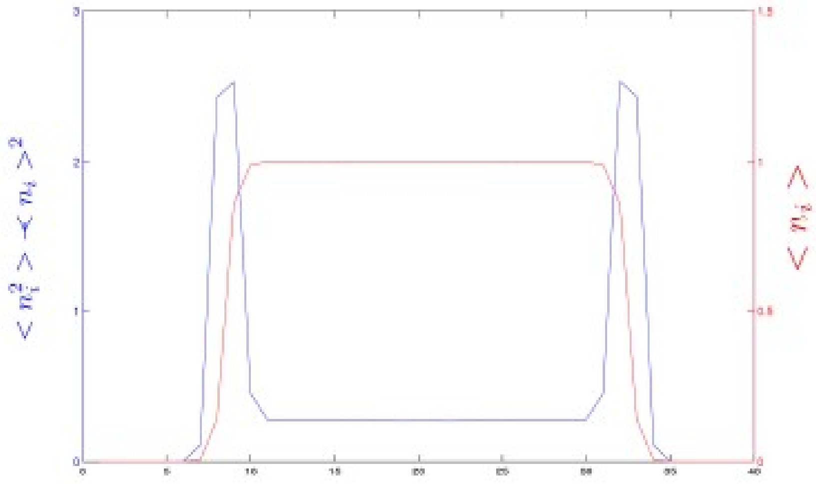
- Recent modulation experiments in ETH [R. Jördens, N. Strohmaier, K. Gunter, H. Moritz, T. Esslinger, Nature 2008; 2009 arXiv by Esslinger and Demler groups], and description of high T_c superconductors.

What we have done:

- Exact numerical simulation (TEBD code) of the modulation of a 1D Hubbard chain both in presence of harmonic confinement and for open boundary conditions;
- interpretation of the results in terms of Bethe ansatz equations;
- connection between double occupancy spectrum and energy spectrum.

F. Massel, M.J. Leskinen, P. Törmä, arXiv:0904.4815

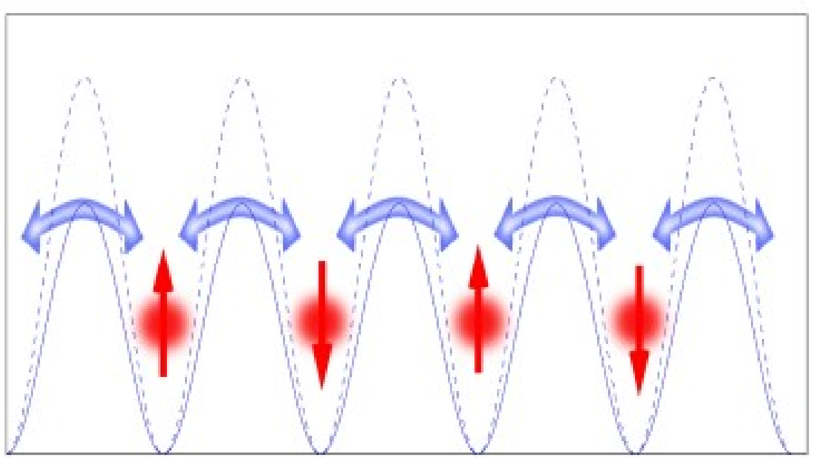
c.f. R. Sensarma, D. Pekker, M.D. Lukin, E. Demler, arXiv:0902.2586
(3D, mean-field, linear response)



Ground state

Hubbard Hamiltonian with parabolic confining potential:

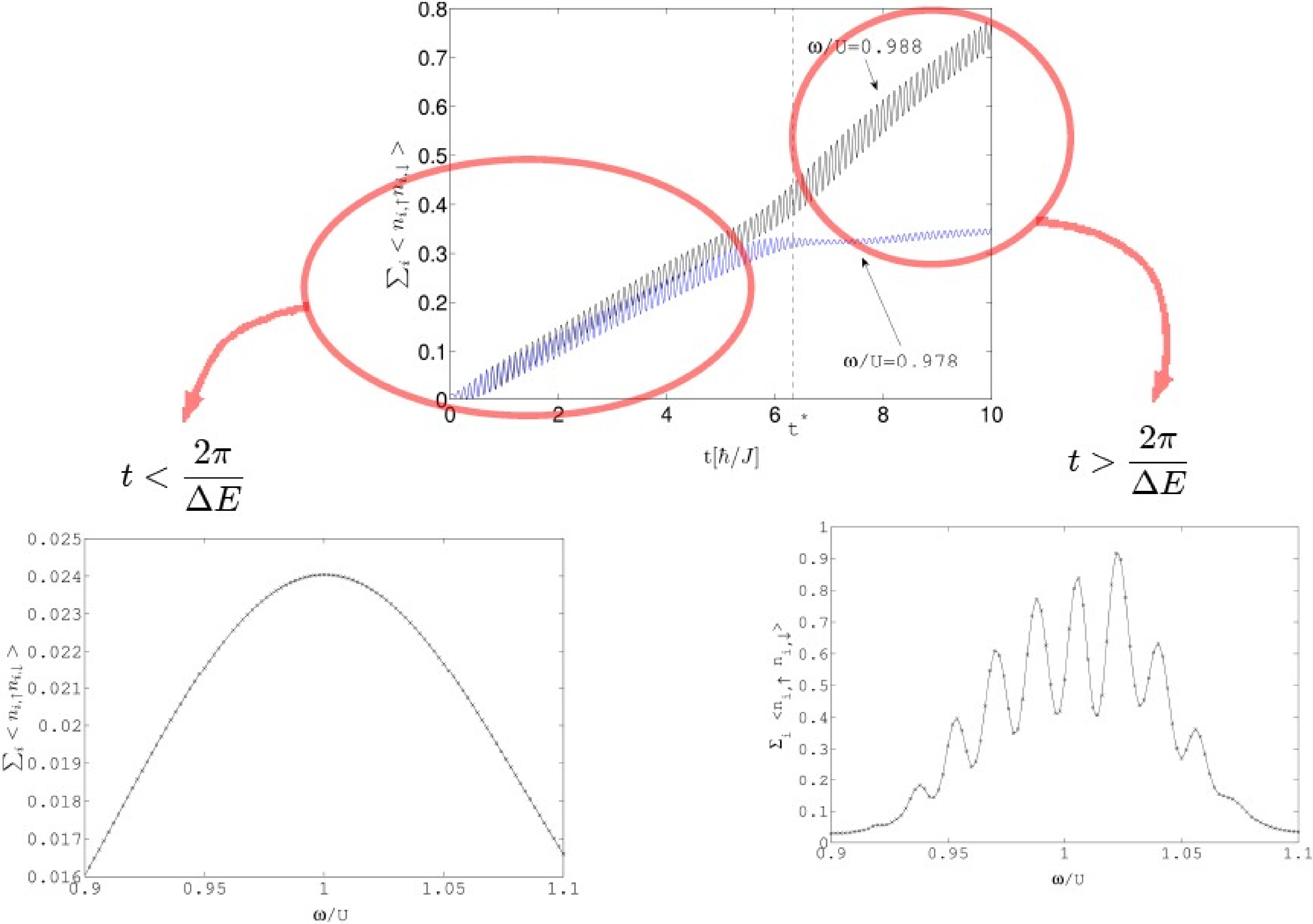
$$
\begin{aligned}
H &= -J \sum_{i\,\sigma} c^{\dagger}_{i,\sigma} c_{i+1\,\sigma} + h.c. \\
&+ U \sum_i n_{i\uparrow} n_{i\downarrow} + \sum_i V_i (n_{i\uparrow} + n_{i\downarrow})
\end{aligned}
$$

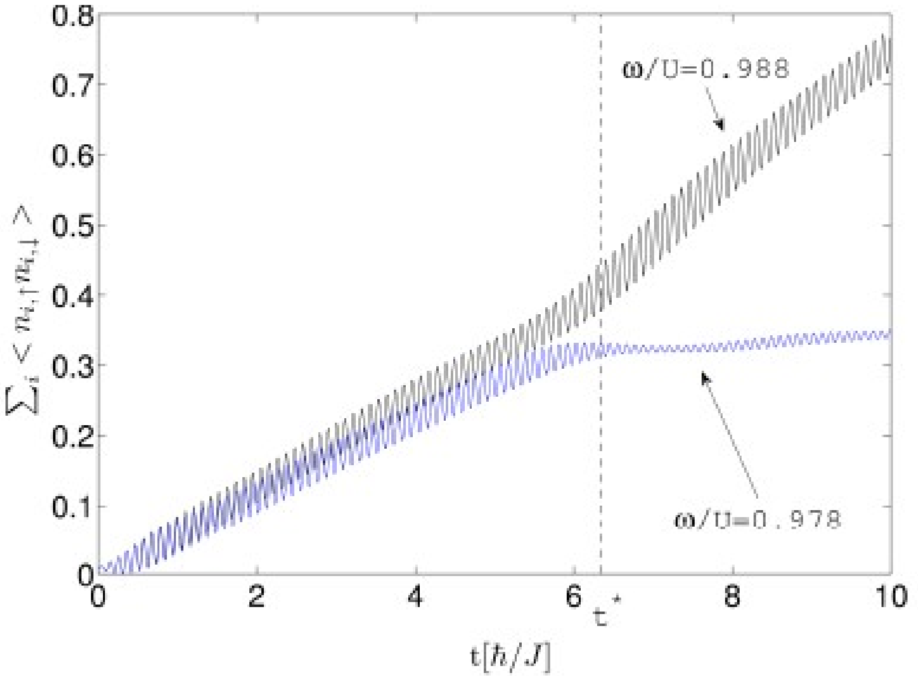


Hopping modulation:

$$J = J_0 + \delta J \sin(\omega t)$$

Monitoring double occupancy as a function of time



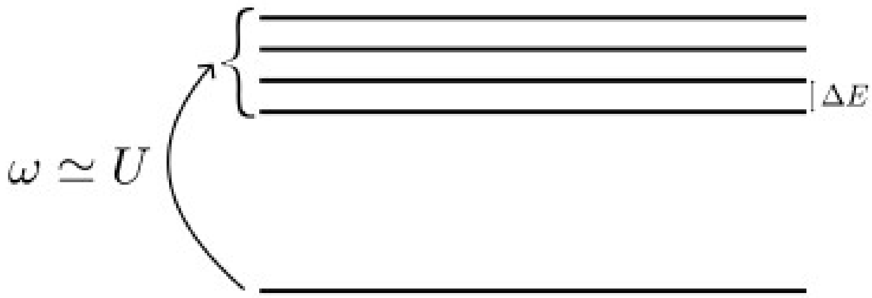


$$t < \frac{2\pi}{\Delta E}$$ Upper Hubbard band perceived as a continuum

$$P(t) \propto \delta J \frac{\sin^2\left[(U - \hbar\omega)\, t / 2\hbar\right]}{(U - \hbar\omega)^2}$$

$$t > \frac{2\pi}{\Delta E}$$ Detailed structure can be resolved

$$\sum_n P_n(t) = \left(\frac{2\pi}{\hbar}\right) |V_{n,\,gs}|^2 \rho(E_n) t \Big|_{E_n \simeq E_{gs} \pm \hbar\omega}$$



Simplified model ->

- no parabolic potential
- limit U/J = infinity

Solution ->

- Combination of Bethe-ansatz equations for open boundary conditions [H. Asakawa, M. Suzuki, J. Phys. A 1996] and infinite interaction energy [M. Ogata, H. Shiba, PRB 1990]

Bethe ansatz equations for open boundary conditions:

$$2Lk_j = 2\pi I_j - 2k_j - \sum_{\beta=1}^{M} \left[\Phi\left(2\frac{\sin(k_j) - \lambda_\beta}{u}\right) + \Phi\left(2\frac{\sin(k_j) + \lambda_\beta}{u}\right) \right]$$

$$\sum_{j=1}^{N} \left[\Phi\left(2\frac{\lambda_\alpha - \sin(k_j)}{u}\right) + \Phi\left(2\frac{\lambda_\alpha + \sin(k_j)}{u}\right) \right] = 2\pi J_\alpha + \sum_{\beta=1(\beta\neq\alpha)} \left[\Phi\left(\frac{\lambda_\alpha - \lambda_\beta}{u}\right) + \Phi\left(\frac{\lambda_\alpha + \lambda_\beta}{u}\right) \right]$$

where

$j = 1, \ldots, N,\ \alpha = 1, \ldots, M,\ I_j\ ,\ J_\alpha \in \mathbb{N}$

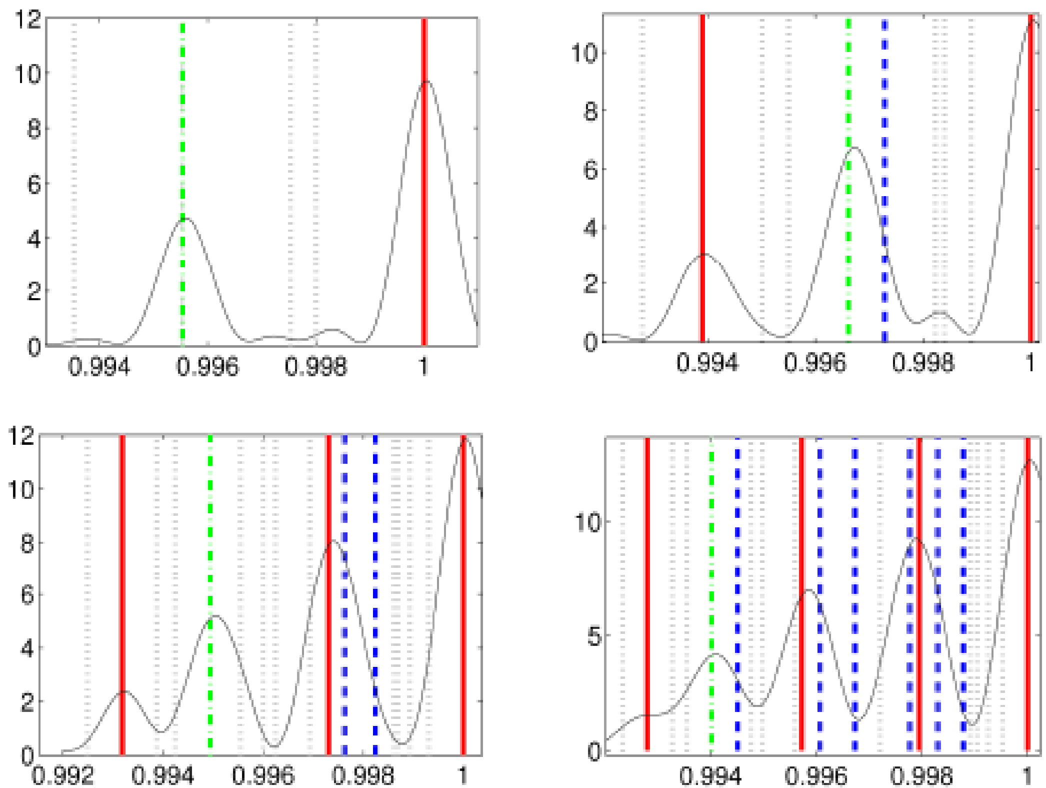
$\Phi(x) = 2\ \tan^{-1}(2\,x)$

$U/J \to \infty$

$$k_j = \frac{\pi}{L+1} j$$

$$\Delta E = -2J(\cos(k_p) - \cos(k_h)) + U$$

Free spinless fermions

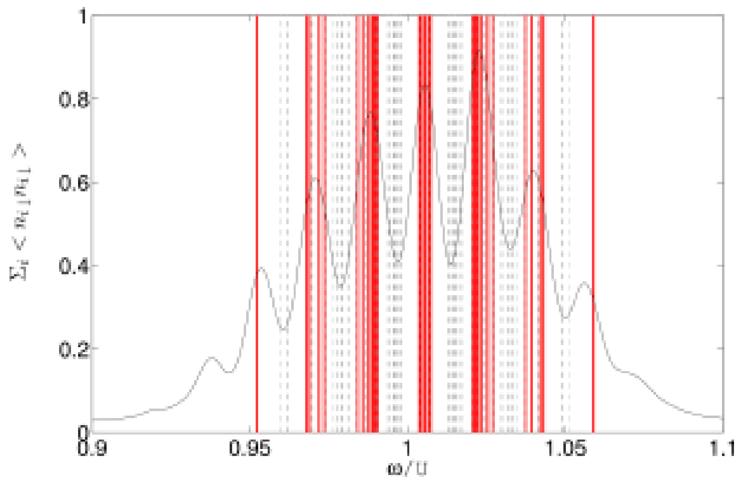


Due to selection rules many excitation energies do not correspond to any peak in the d.o. spectrum

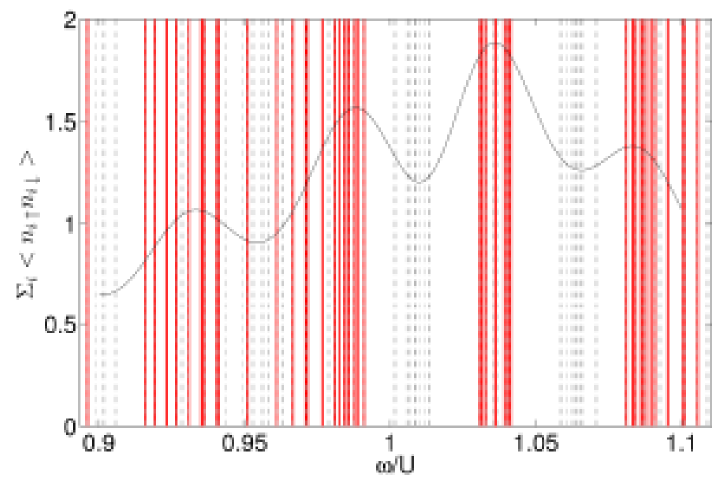
The same approach for the confined system:
According to [A.M. Rey, G. Pupillo, C.W. Clark, C.J. Williams, PRA 2005], the spectrum of a spinless fermion in presence of lattice + parabolic confinement is given by:

$$E_i - E_0 = 2\sqrt{J\Omega}(i+1/2) - \frac{\Omega}{32}\left[(2i+1)^2 + 1 - \frac{(2i+1)^3 + 3(2i+1)}{32\sqrt{J/\Omega}}\right] \quad i < i_c,$$

$$E_{i=2r} \simeq E_{i=2r-1} \simeq \Omega r^2 + \frac{2J}{(2r)^2 - 1} \quad i > i_c$$



$$\frac{U}{J} = 60$$



$$\frac{U}{J} = 20$$

Hopping modulation simulations in 1D repulsive Hubbard model

- Fine features in the spectrum appear after long enough modulation times
- The peak structure explained using Bethe ansatz; selection rules present
- Discreteness of the spectrum here due to 1) Finite system 2) Trap potential
- Implications to 3D: the AFM gap could be observed by modulation experiments after long enough modulation times (for present values about 1 s, but would be shorter for smaller U/J^2)

F. Massel, M.J. Leskinen, P. Törmä, arXiv:0904.4815

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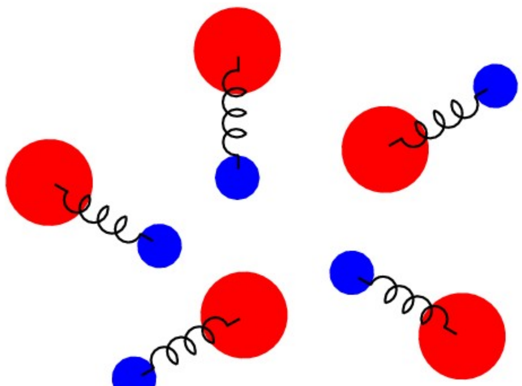
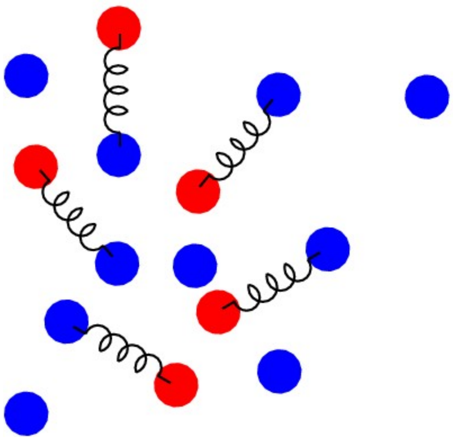
Imbalanced/Polarized Fermi gases





Polarization

$$P = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$$

Pairing between particles with unequal mass or unequal total number

Related to, e.g., high energy physics (colour superconductivity of quarks)

Experiments:

M.W.Zwierlein, A.Schirotzek, C.H.Schunck, W.Ketterle, Science 2006

G.B.Partridge, W.Li,R.I.Kamar, Y.Liao, R.G.Hulet, Science 2006

G.B.Partridge, W.Li,Y.Liao, R.G.Hulet, M.Haque, H.Stoof, PRL 2006

M.W.Zwierlein, C.H.Schunck, A.Schirotzek, W.Ketterle, Nature 2006

C.H.Schunck, Y.Shin, A.Schirotzek, M.W. Zwierlein, W.Ketterle, Science 2007

Y.Shin, C.H.Schunck, A.Schirotzek, W.Ketterle, Nature 2008

?, ?, ?, M. Zwierlein, arXiv 2009

COULD ONE OBSERVE THE FFLO STATE IN ULTRACOLD GASES?

- FFLO (Fulde, Ferrel, Larkin, Ovchinnikov) state
  - Finite polazation P and superfluidity simultaneously (also at T=0)
  - Non-uniform order parameter $\Delta e^{i2\mathbf{q}\cdot\mathbf{r}}$ $\Delta \sin(2\,q\,r)$
- Observations under debate
  - H.A. Radovan, N.A. Fortune, T.P. Murphy, S.T. Hannahs, E.C. Palm, S.W. Tozer, D. Hall, Nature 2003
  - A. Bianchi, R. Movshovich, C. Capan, P.G. Pagliuso, J.L. Sarrao, PRL 2003
  - K. Kakuyanagi, M. Saitoh, K. Kumagai, S. Takashima, M. Nohara, H. Takagi, Y. Matsuda, PRL 2005
  - V.F. Correa, T.P. Murphy, C. Martin, K.M. Purcell, E.C. Palm, G.M. Schmiedeshoff, J.C. Cooley, S.W. Tozer, PRL 2007
- The parameter window for existence of this phase is exceedingly small for particles in free space, in 3D
  - Exceedingly small for particles in free space, in 3D. See e.g. D.L. Sheehy, L. Radzihovsky, PRL 2006
  - Enhanced in optical lattices, T.K. Koponen, T. Paananen, J.-P. Martikainen, P. Törmä, PRL 2007
  - … and expecially in 1D

Mean-field study of 1D Fermi gas: BdG formalism

The 1D (attractive) Fermi gas in optical lattices can be studied at mean-field level by the following (Hubbard) model:

$$H_{mf} = -t \sum_{i,\sigma} \left(\hat{c}^{\dagger}_{i\sigma} \hat{c}^{\dagger}_{i+1\sigma} + h.c \right) + \sum_{i} \left(\Delta_i \, \hat{c}^{\dagger}_{i\uparrow} \hat{c}^{\dagger}_{i\downarrow} + h.c \right) + \sum_{i\sigma} \left(V^{\text{ext}}_i - \mu_\sigma \right) \hat{n}_{i\sigma}$$

where the pairing is defined as

$$\Delta_i = -U < \hat{c}^{\dagger}_{i\downarrow} \hat{c}^{\dagger}_{i\uparrow} >$$

The Hamiltonian can be diagonalized using Bogoliubov transformation

$$\hat{c}^{\dagger}_{i\sigma} = \sum_{\alpha} \left(u_{\alpha i\sigma} \, \hat{\gamma}_{\alpha\sigma} - \sigma \; v^{\star}_{\alpha i\sigma} \, \hat{\gamma}^{\dagger}_{\alpha\bar{\sigma}} \right)$$

The Hamiltonian then can be expressed in the following compact form

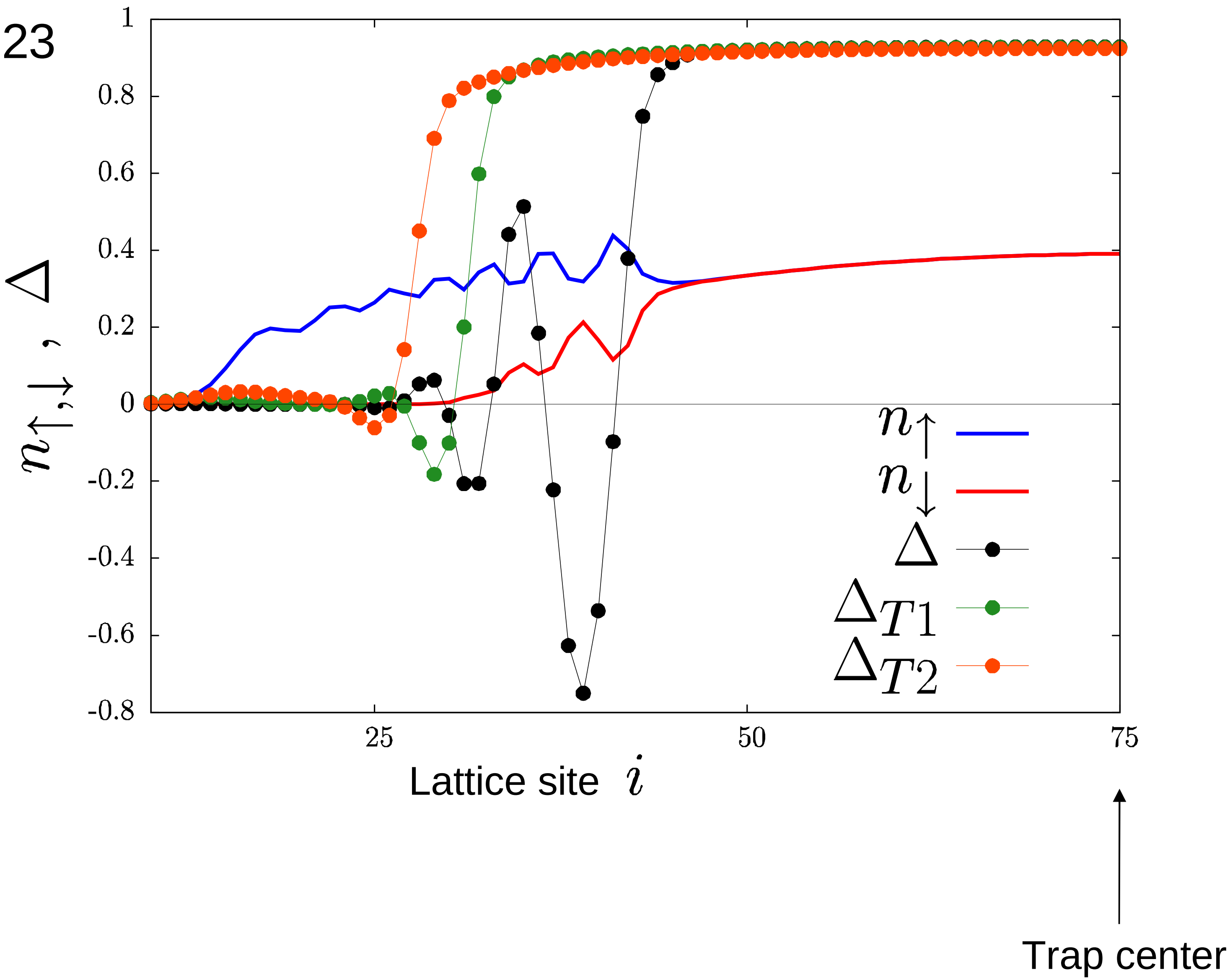
$$\sum_{j=1}^{N} \begin{pmatrix} H^{\sigma}_{ij} & \Delta_{ij} \\ \Delta^{*}_{ij} & -H^{\bar{\sigma}}_{ij} \end{pmatrix} \begin{pmatrix} u_{\alpha j\sigma} \\ v_{\alpha j\bar{\sigma}} \end{pmatrix} = E_{\alpha\sigma} \begin{pmatrix} u_{\alpha i\sigma} \\ v_{\alpha i\bar{\sigma}} \end{pmatrix}$$

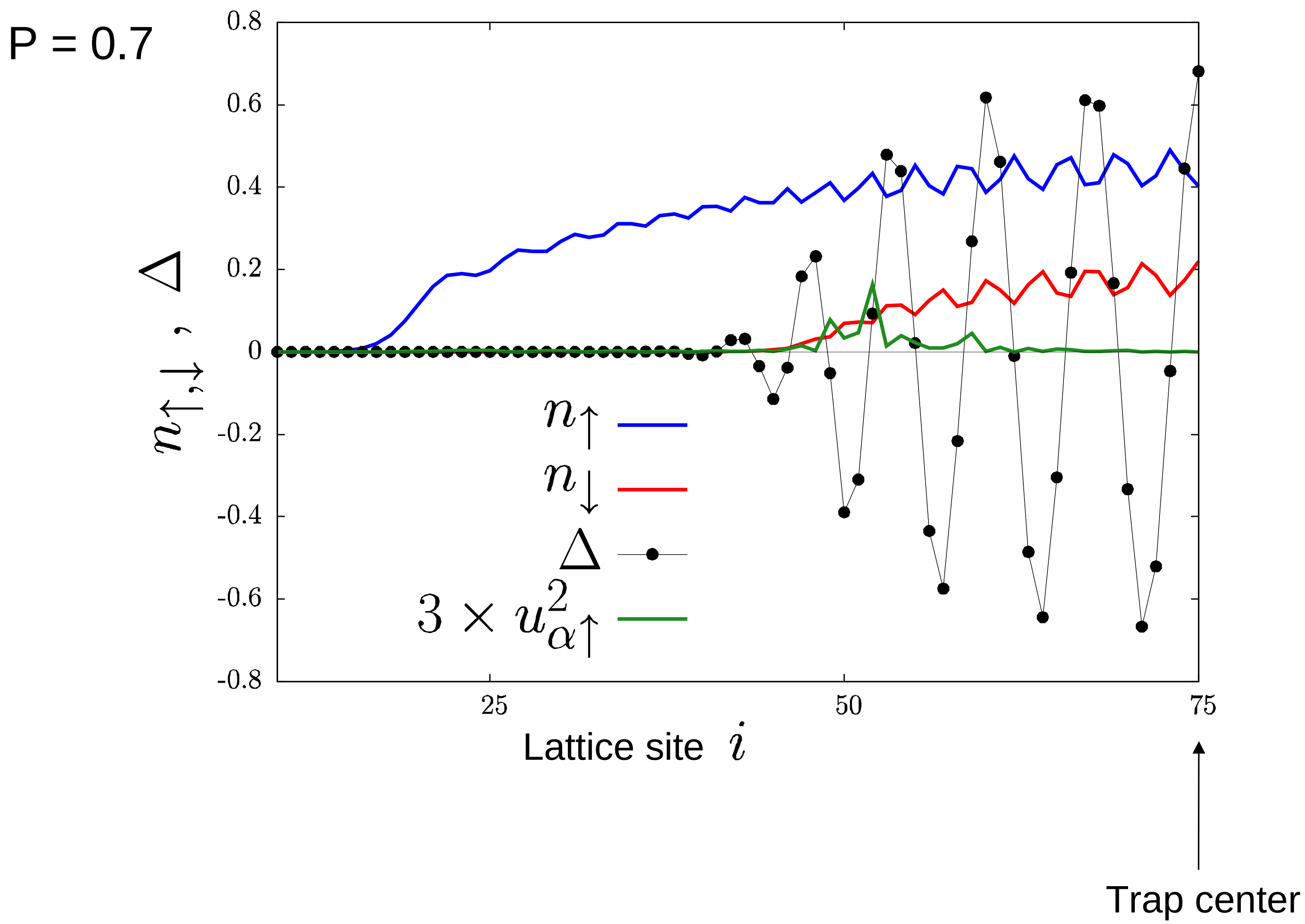
$$H^{\sigma}_{ij} = -t \;\delta_{i,i\pm 1} + \left(V^{\text{ext}}_i - \mu_\sigma \right) \delta_{ij}$$

M.R. Bakhtiari, M.J. Leskinen, P. Törmä, PRL 2008

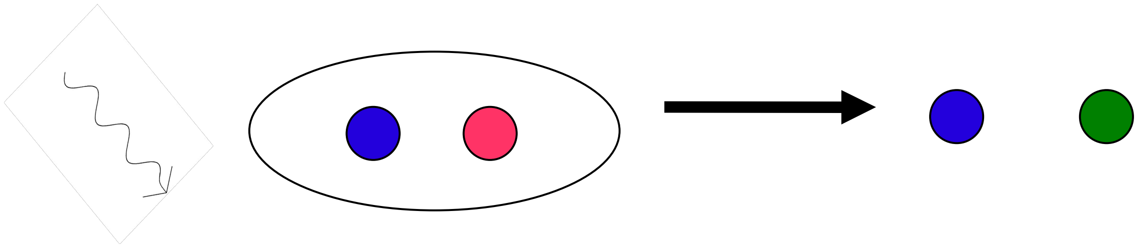
Densities and order parameter for an imbalanced gas in 1D lattice combined with external harmonic trapping

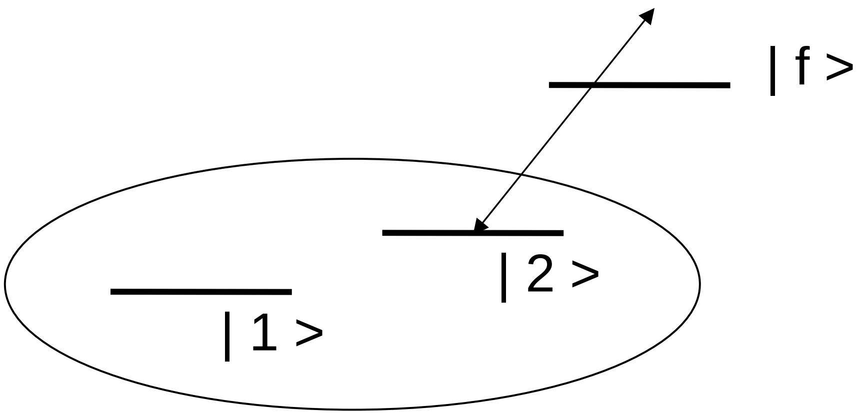
P = 0.23




P = 0.7
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
25
50
75
$n_{\uparrow,\downarrow}$, Δ
Lattice site i
$n_\uparrow$
$n_\downarrow$
Δ
$3 \times u^2_{\alpha\uparrow}$
Trap center

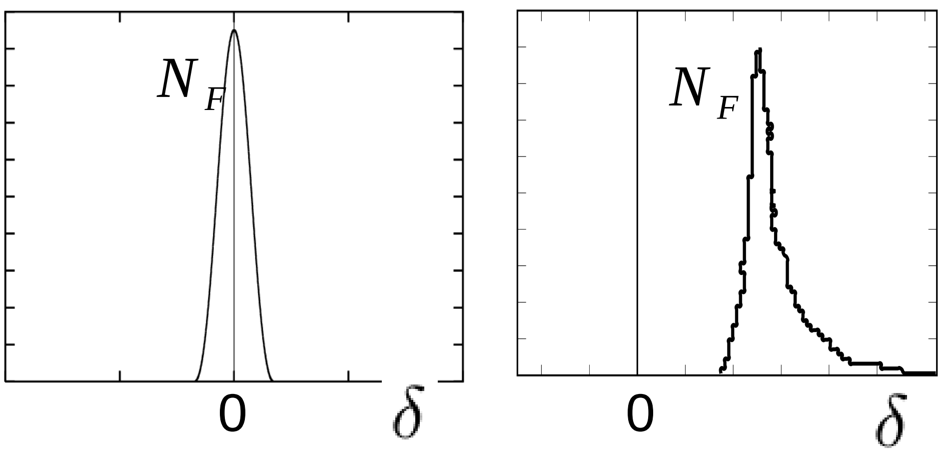
RF-spectroscopy





no interactions

|1>, |2> (and |f>) interacting



The Hamiltonian

$$H = \sum_{\sigma=1,2,f} \int d\mathbf{r}\, \Psi^\dagger_\sigma(\mathbf{r}) \left[-\frac{\nabla^2}{2m} - \mu_\sigma\right] \Psi_\sigma(\mathbf{r}) + U_{12} \int d\mathbf{r}\, \Psi^\dagger_1(\mathbf{r})\Psi^\dagger_2(\mathbf{r})\Psi_2(\mathbf{r})\Psi_1(\mathbf{r}) +$$
$$U_{1f} \int d\mathbf{r}\, \Psi^\dagger_1(\mathbf{r})\Psi^\dagger_f(\mathbf{r})\Psi_f(\mathbf{r})\Psi_1(\mathbf{r}) + \Omega \int d\mathbf{r}\, \Psi^\dagger_f(\mathbf{r})\Psi_2(\mathbf{r}) + \Omega \int d\mathbf{r}\, \Psi^\dagger_2(\mathbf{r})\Psi_f(\mathbf{r}) +$$
$$\frac{\delta}{2} \int d\mathbf{r}\, [\Psi^\dagger_2(\mathbf{r})\Psi_2(\mathbf{r}) - \Psi^\dagger_f(\mathbf{r})\Psi_f(\mathbf{r})]$$

Linear response

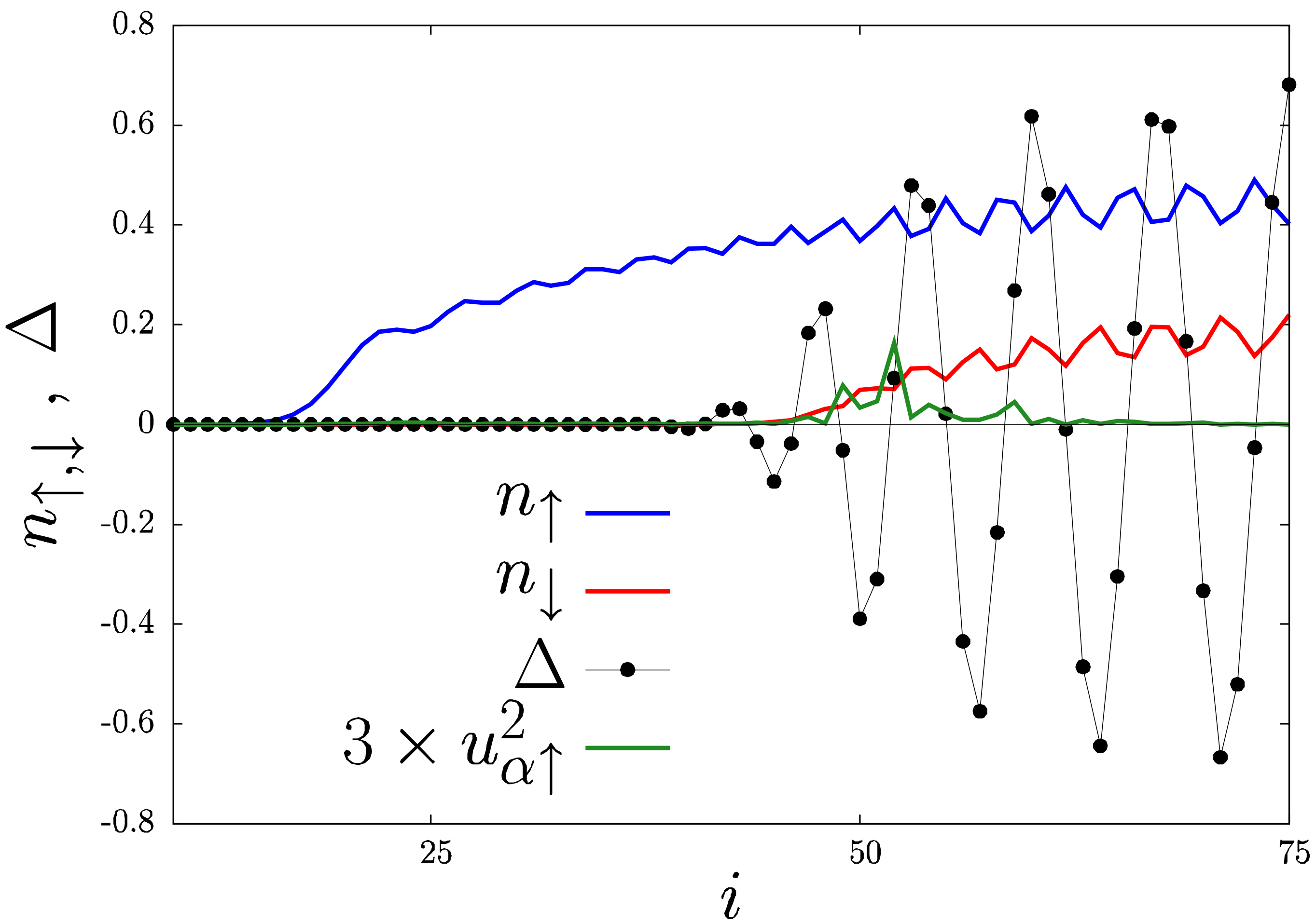
$$\chi''(\delta) = \Im[-i \int d\mathbf{r} \langle T[\psi^\dagger_2(\mathbf{r},t)\psi_f(\mathbf{r},t)\psi^\dagger_f(0,0)\psi_2(0,0)]\rangle]$$

Fermi Golden rule

$$\chi''(\delta) \propto \langle \Psi^\dagger_2 \Psi_2 \rangle \langle \Psi^\dagger_3 \Psi_3 \rangle = G_2 G_3$$

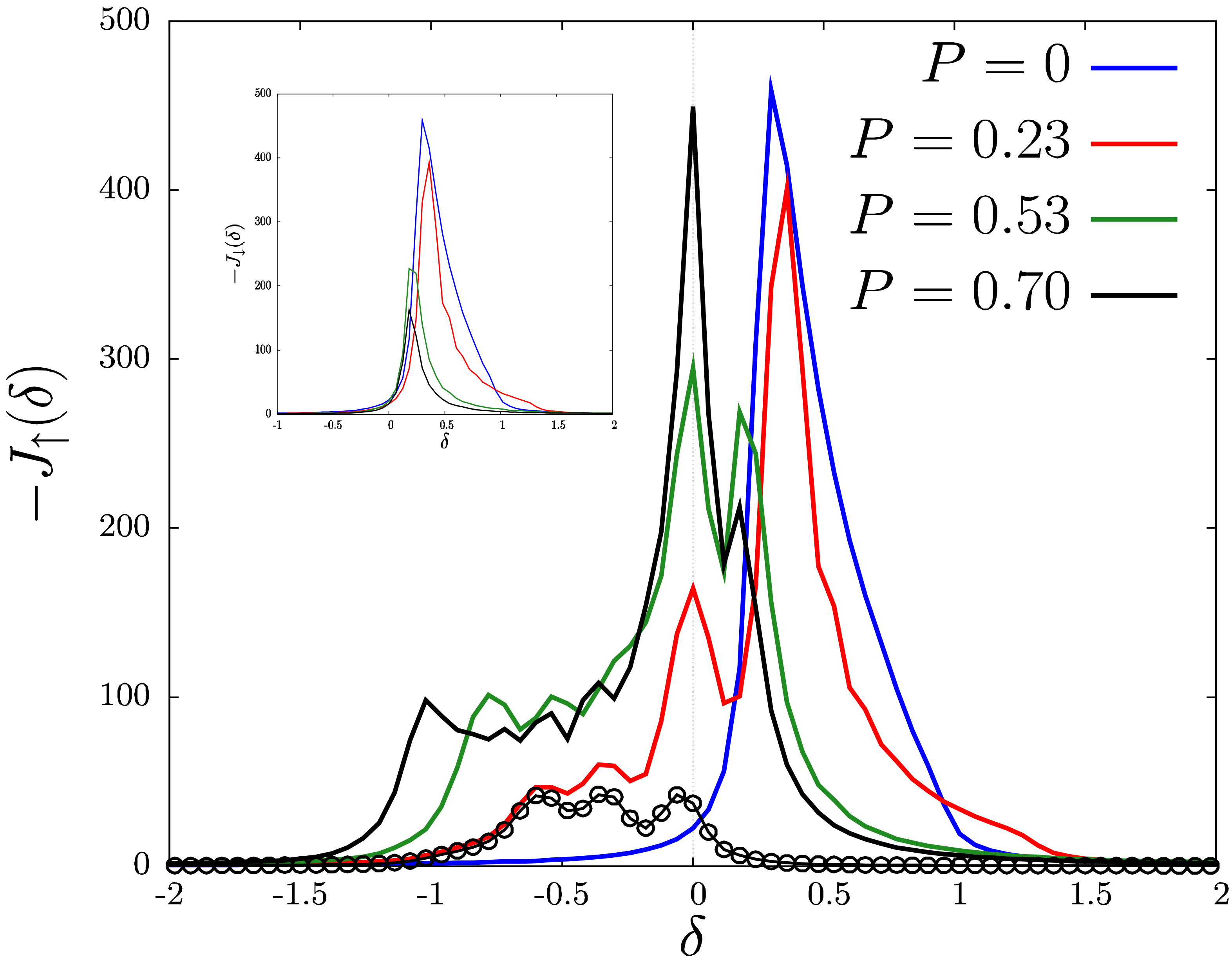
$$\delta_{th} \simeq (U_{1f} - U_{12}) n_1 + \frac{\Delta^2}{2E_F}$$

Spectral signatures of the FFLO state in 1D optical lattices



M.R. Bakhtiari, M.J. Leskinen, P. Törmä, PRL 2008

RF-spectrum

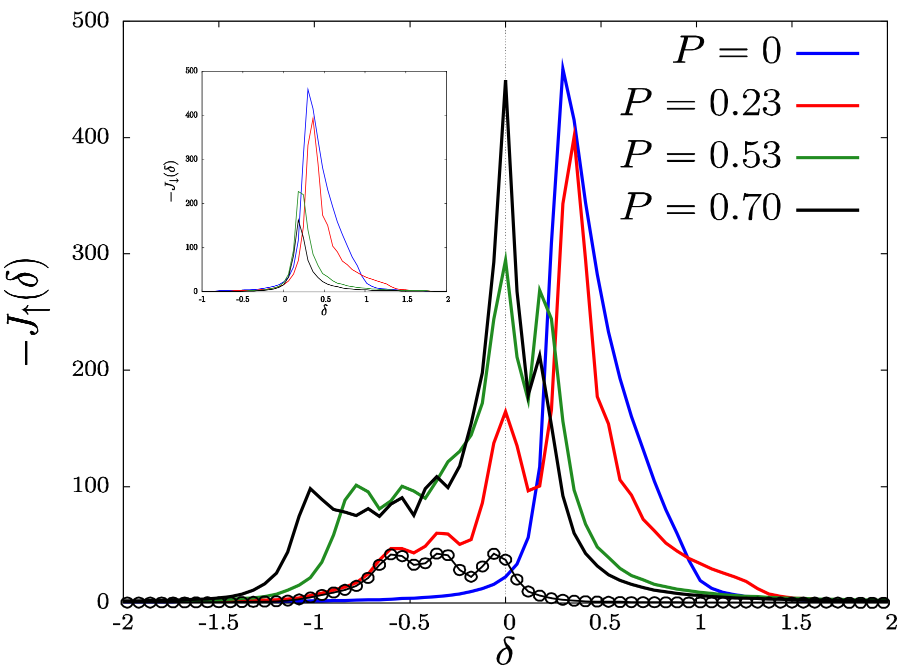


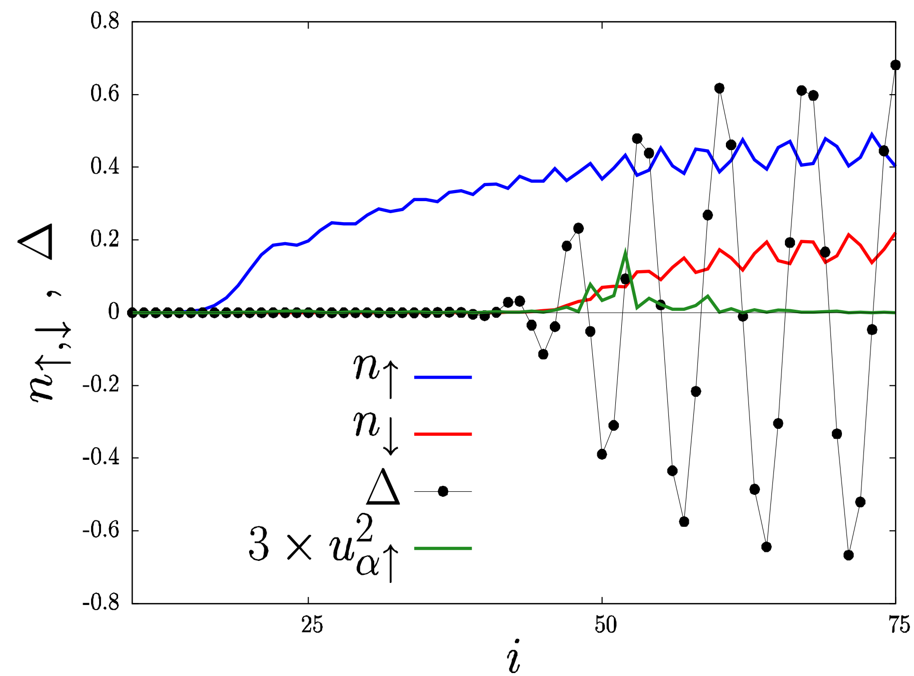
$$J_{\uparrow/\downarrow}(\delta, K) = -2\pi \sum_{\alpha=1}^{L} \Big[\Big| \sum_{i=1}^{L} v_{\alpha i\downarrow/\uparrow}\, v^{\rm non}_{Ki\uparrow\downarrow} \Big|^2 n_{\rm F}(E_{\alpha\downarrow/\uparrow})\, \delta(E_{\alpha\downarrow/\uparrow} + \epsilon_K - \delta - \mu_{\uparrow/\downarrow}) + \Big| \sum_{i=1}^{L} u_{\alpha i\uparrow/\downarrow}\, v^{\rm non}_{Ki\uparrow\downarrow} \Big|^2 n_{\rm F}(E_{\alpha\uparrow/\downarrow})\, \delta(E_{\alpha\uparrow/\downarrow} - \epsilon_K + \delta + \mu_{\uparrow/\downarrow}) \Big]. \quad (1)$$

Andreev states at the nodes of the order parameter

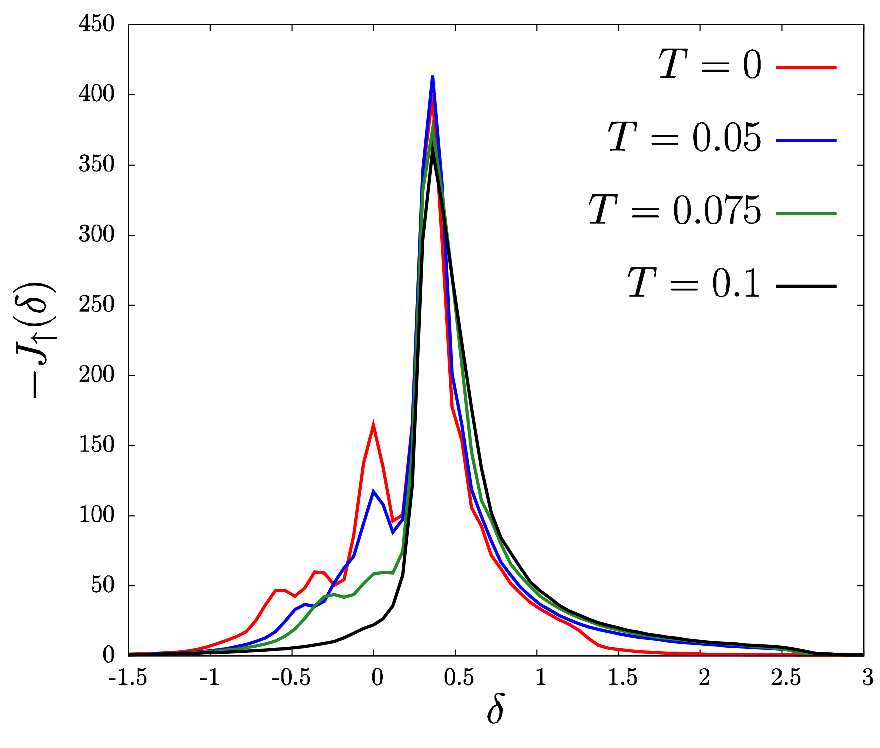
Momentum conservation in the spectroscopy

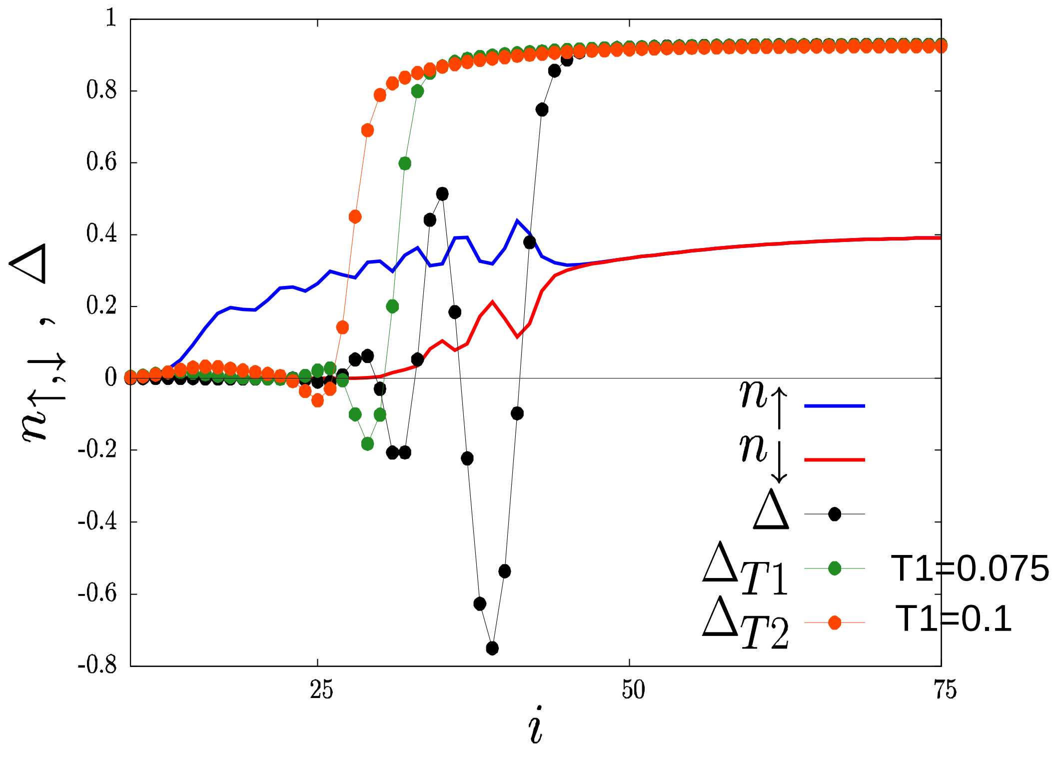
Spectra → at negative detunings



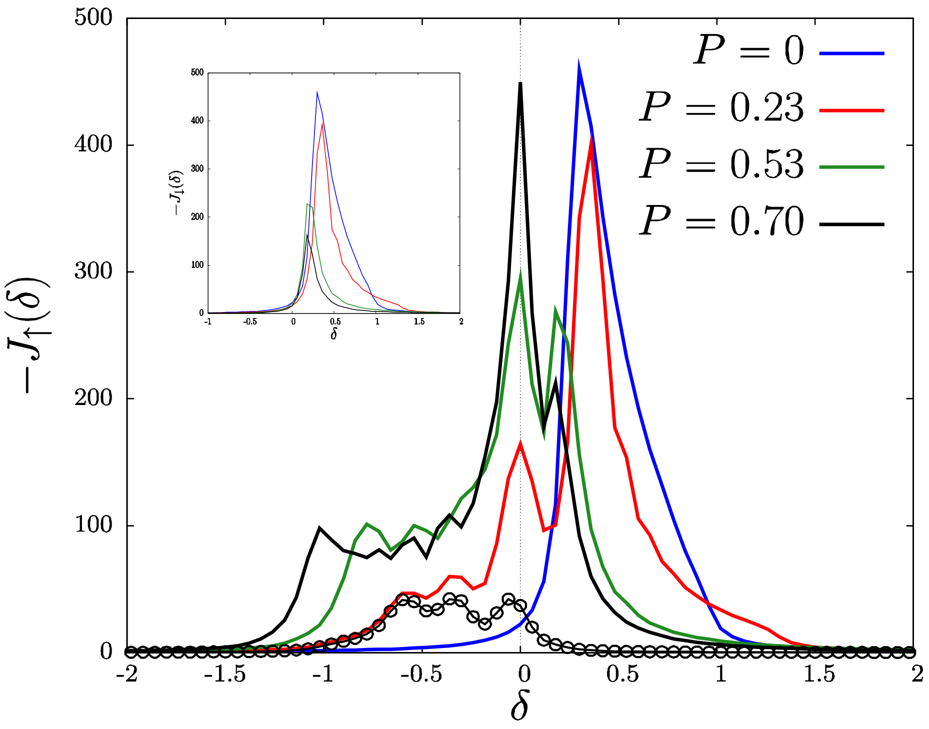


Signatures at negative detunings are related
to strongly oscillating order parameter





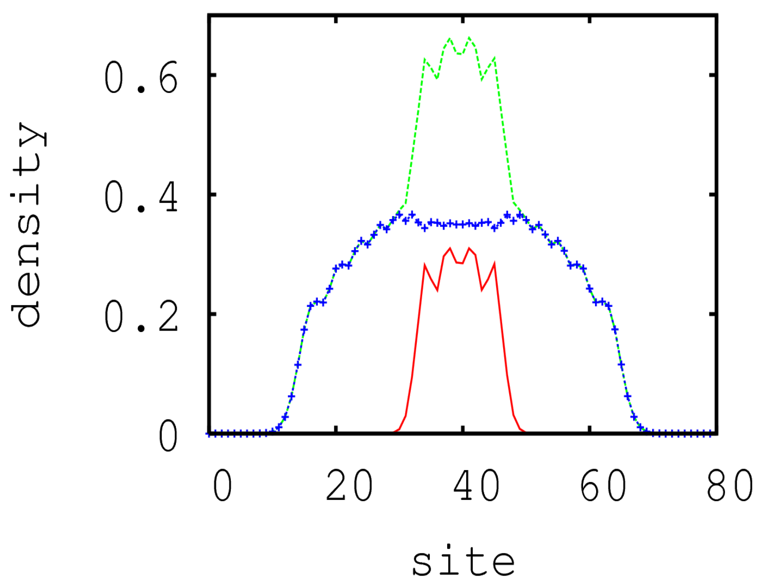
Spectral weight at the negative detunings is a direct signature of Andreev bound states and of the FFLO state

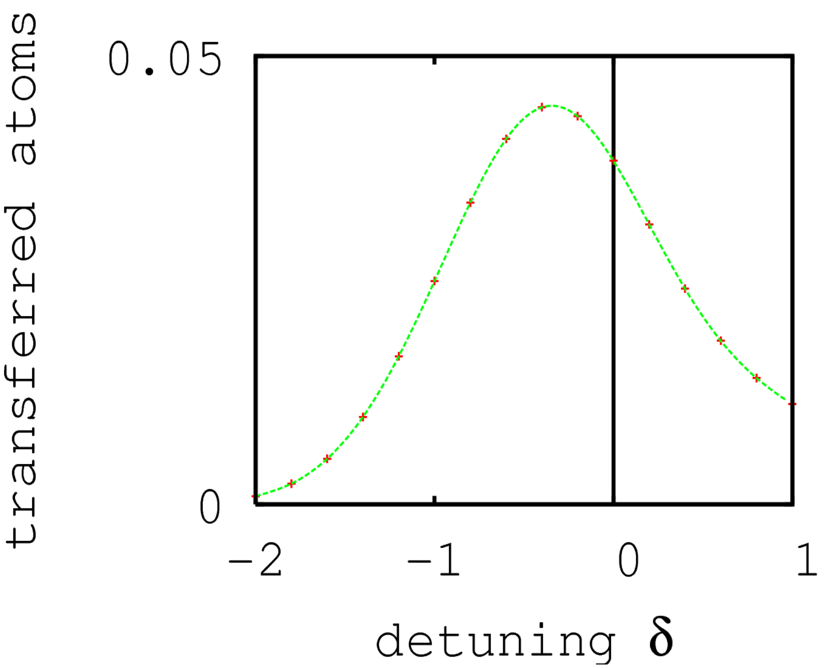


Exact numerical studies (TEBD) of the ground state and the RF spectroscopy dynamics (spectra)

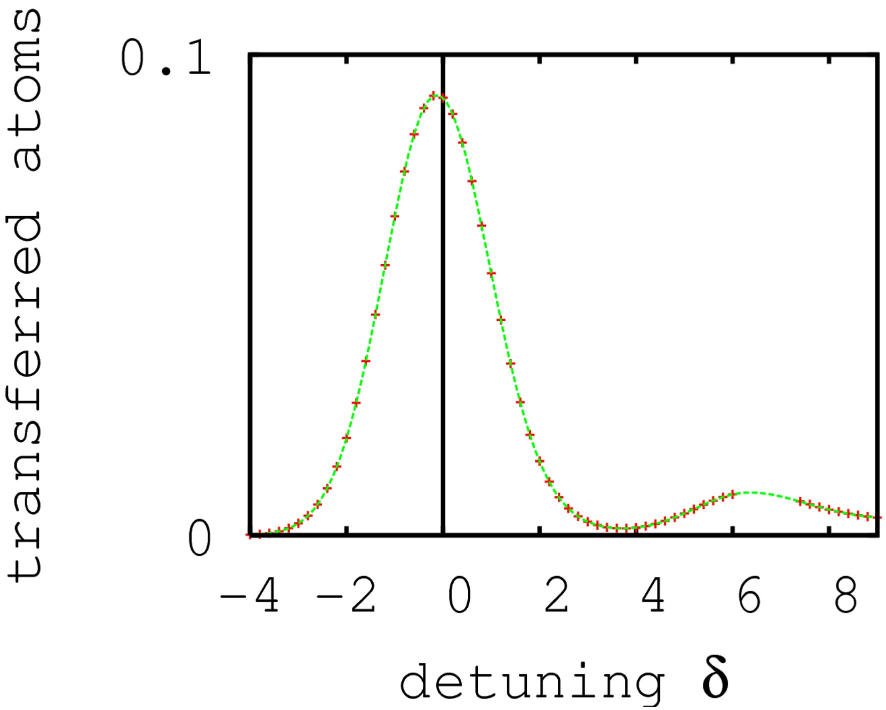
- Density profiles
  $N_1 = 4$ $N_2 = 20$
  $U = -20\,J$
  $V_{ext} \approx 0.00016$

- Spectrum
  $\Omega = 0.1$ $T = 10$
  $\chi = 80$ (Schmidt number)





c.f. R.A. Molina, J. Dukelsky, P. Schmitteckert, Comment PRL 2009
M.R. Bakhriari, M.J. Leskinen, P. Törmä, Reply PRL 2009



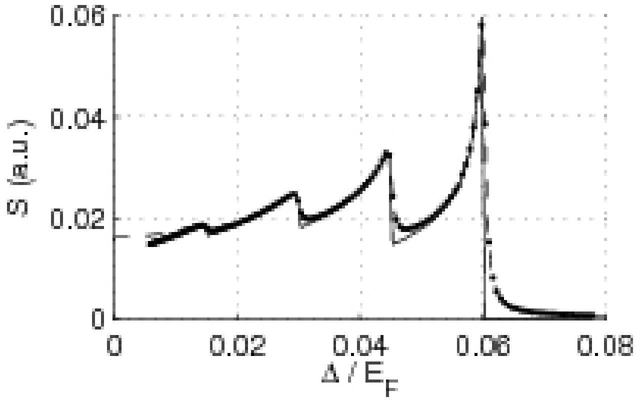
- Spectrum: U = -8 J $\Omega = 0.1$ $T = 5$
  $\chi = 80$ (Schmidt number)

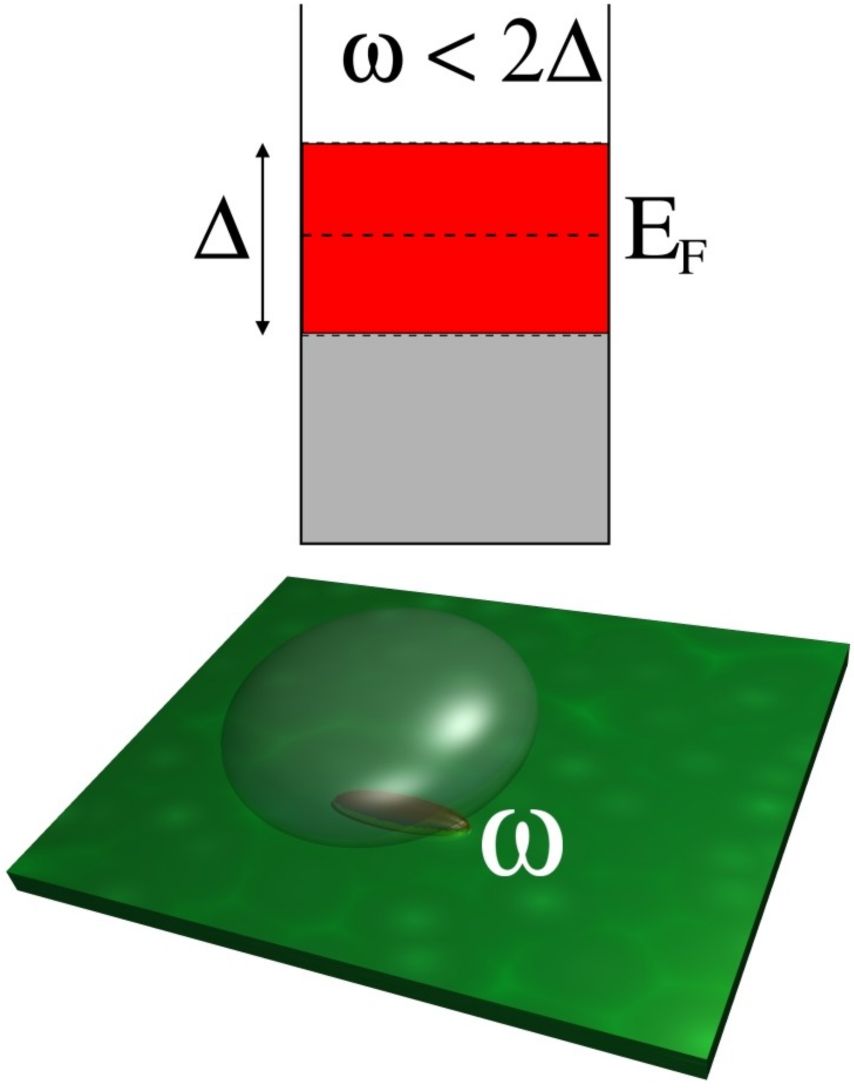
1D FFLO signatures in RF spectroscopy

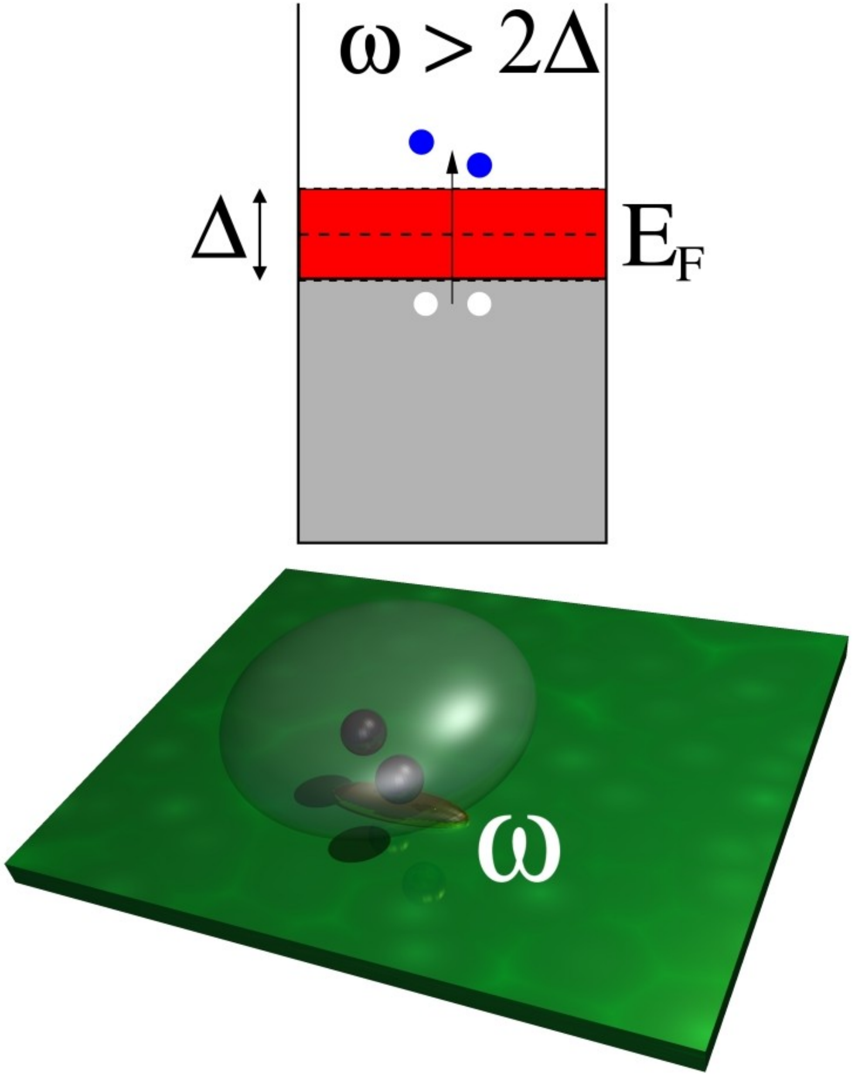
- Spectral weight at the negative detunings is a direct signature of Andreev bound states and of the FFLO state
- Predicted both by mean-field and exact numerical studies

- Three-component mixtures, attractive interactions: a BdG study in a trap (preliminary)
- The repulsive Hubbard model in a one-dimensional lattice: exact hopping modulation dynamics
- RF-spectroscopy in a 1D system: signatures of the FFLO state
- **Fermi condesates as sensors**

Fermi condensates for dynamic imaging of electromagnetic fields
T.K. Koponen, J. Pasanen, P. Törmä, PRL 2009







Summary

- Preliminary studies of three-component systems in traps: LDA and BdG produce differing results
- Exact hopping modulation dynamics in 1D lattices: two timescales; the structured double occupation spectrum accurately reveals the discrete energies of the ground state; implications for the observation of the AFM gap
- Observation of FFLO and Andreev bound states by RF-spectroscopy
- Fermi condesates as sensors: gap provides frequency selection