Theory of radio-frequency spectroscopy of ultracold Fermi atoms

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"FerMix 2009 Meeting"
Trento (Italy), 3-5 June 2009

References:

[1] A. Perali, P. Pieri, and G.C. Strinati,
Phys. Rev. Lett. **100**, 010402 (2008):
"Competition between final state and pairing gap effects in the radio-frequency spectra of ultracold Fermi atoms" [below T_c]

[2] P. Pieri, A. Perali, and G.C. Strinati,
preprint at http://arxiv.org/abs/0811.0770:
"Enhanced paraconductivity-like fluctuations in the radio frequency spectra of ultracold Fermi atoms" [above T_c]

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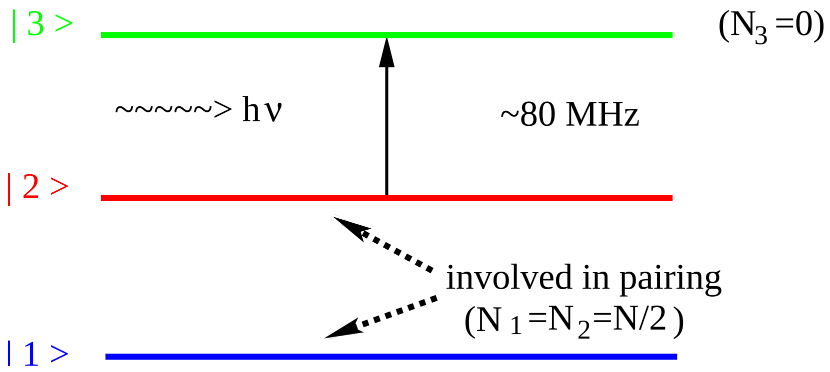
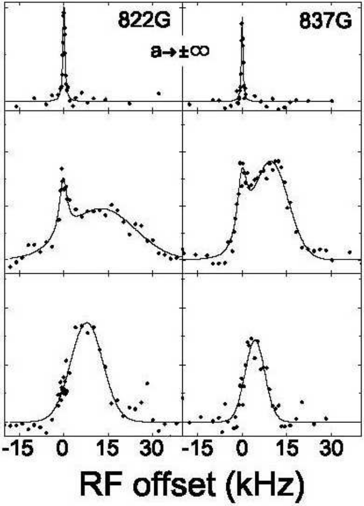


Original Innsbruck-Grimm data (^{6}Li) :

From Fig.1 of C. Chin *et al.*, Science **305**, 1128 (2004):



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2) To what extent final-state effects affect the RF spectra ?

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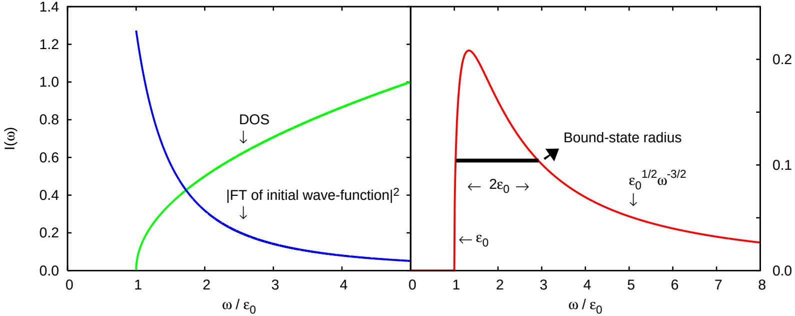

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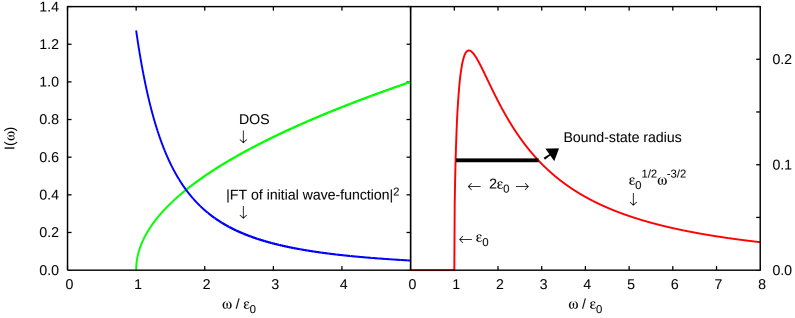

$\implies$ extract binding energy from threshold & bound-state radius from width of half-maximum

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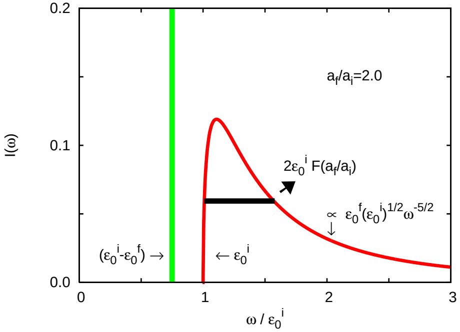

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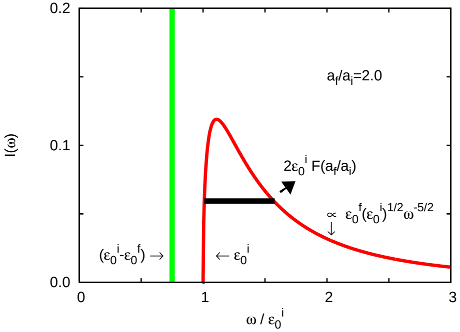

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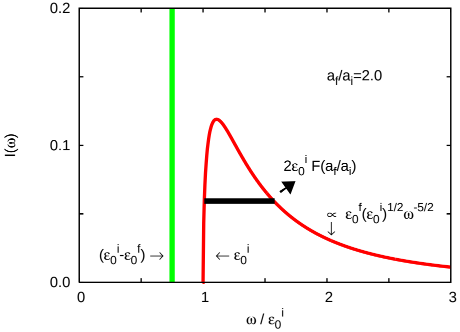

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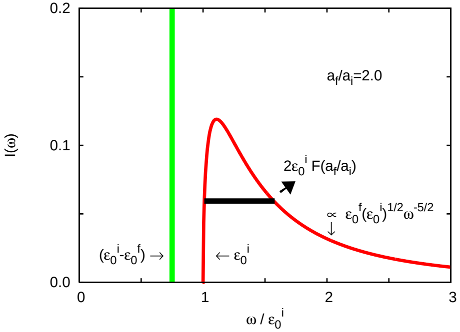

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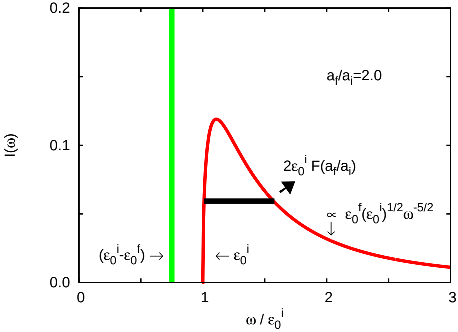

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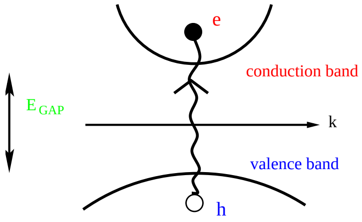

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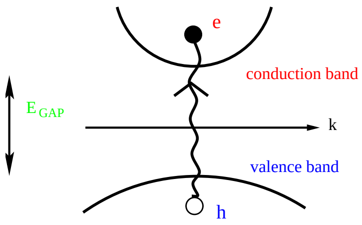

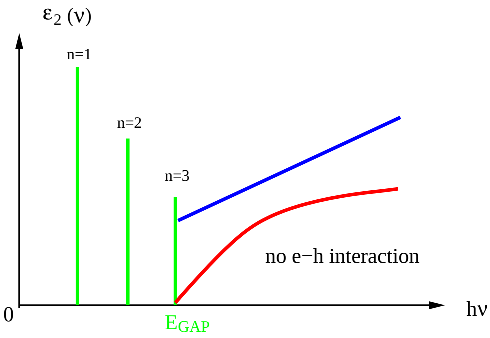

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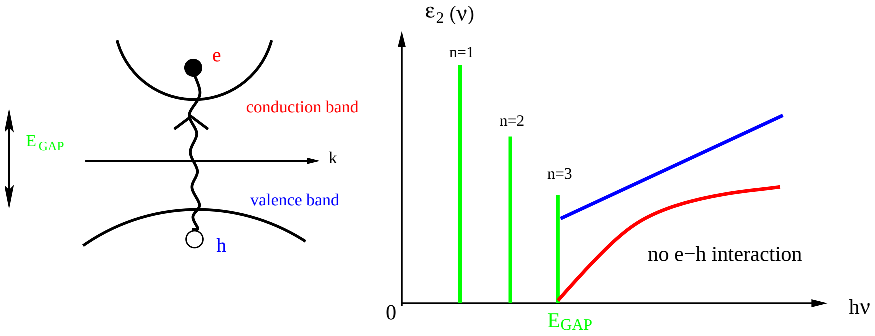
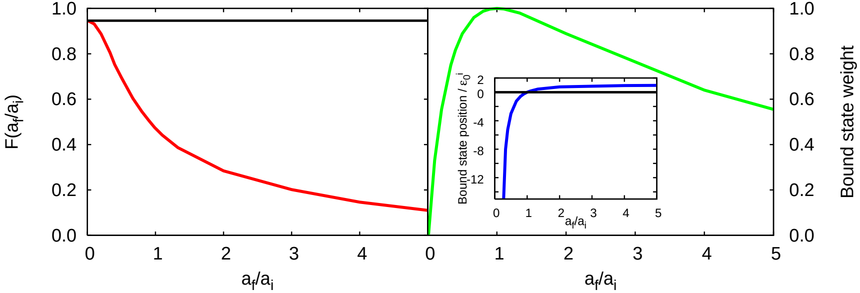


$\Longrightarrow$ competition between finite-gap ($\longrightarrow$) and excitonic ($\longleftarrow$) effects !

Characteristics of molecular spectra:



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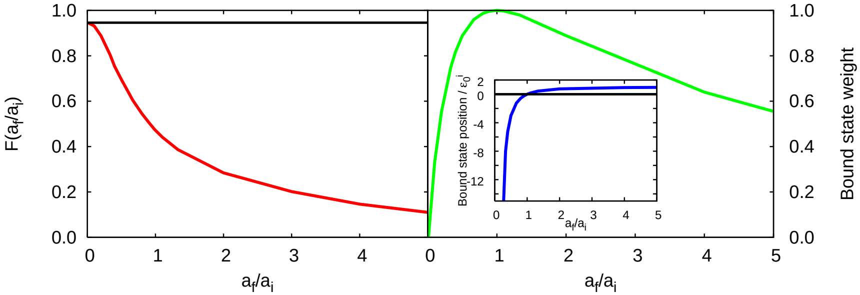

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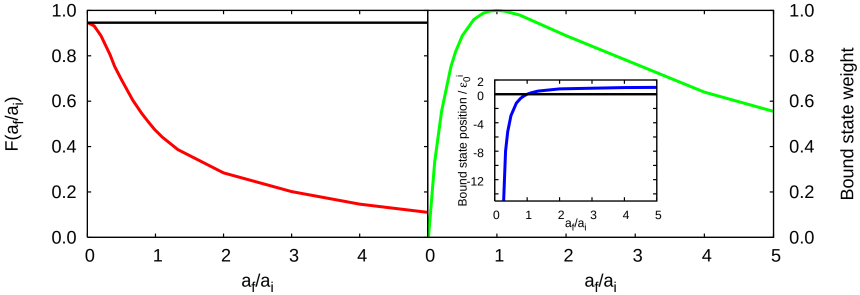

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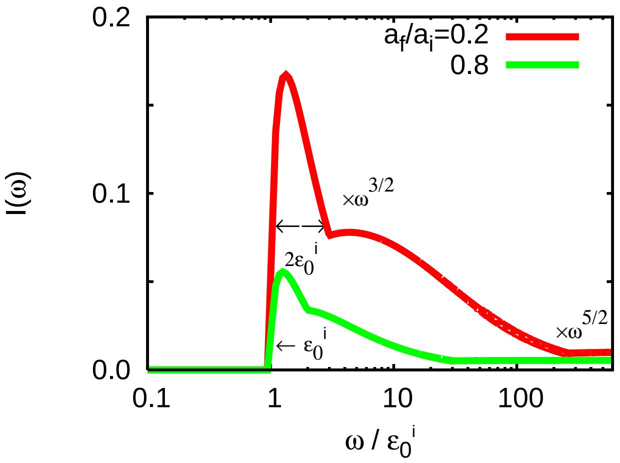

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$$\circ \quad -1.0 \qquad \circ \quad 0.0 \qquad \circ \quad 1.0 \qquad \longrightarrow \quad (\mathbf{k_F\, a_i})^{-1}$$

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$\Rightarrow$ different "many-body diagrams" are expected to be important in the two temperature regimes !

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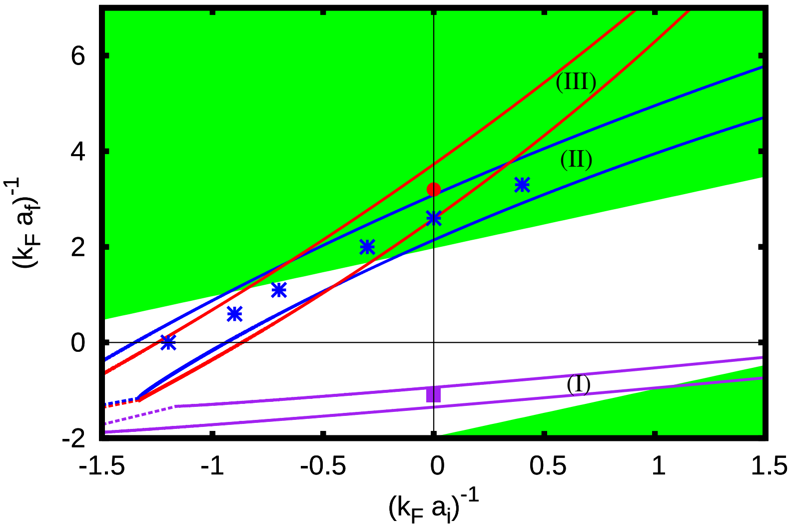
Group (year)	a_i	a_f	N_0	N'
Törma (2004)	yes	no	yes	no
Griffin (2005)	yes	no	yes	no
Levin (2005)	yes	no	yes	no
Bruun & Stoof (2008)	yes	no	no	yes
Yu & Baym (2006)	yes	yes	yes	no
Strinati (2008)	yes	yes	yes	no
Mueller (2008)	yes	yes	yes	no
Levin (2009)	yes	yes	yes	no
Strinati (2009)	yes	yes	no	yes

Experimental coupling plane for ^{6}Li:



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- Bohr frequency $\omega_{32} = \varepsilon_3 - \varepsilon_2$ between "bare" atomic levels 3 and 2
- Two chemical potentials:
  $\mu \leftrightarrow$ common to spins "1" and "2" $(N_1 = N_2)$
  $\mu_3 \leftrightarrow$ spin "3" $(N_3 = 0)$

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$\frac{dN_3(t)}{dt}$ as induced by the perturbing Hamiltonian:

$$H'(t) = \gamma \int d\mathbf{r}\, e^{i(\mathbf{q}_{RF}\cdot\mathbf{r} - \omega_{RF} t)}\, \psi_3^\dagger(\mathbf{r})\psi_2(\mathbf{r}) + h.c.$$

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$\frac{dN_3(t)}{dt}$ is related to the current operator:

$$\begin{aligned} I(t) &= i[H'(t), N_3] \\ &= -i\gamma \int d\mathbf{r}\, e^{i(\mathbf{q}_{RF}\cdot\mathbf{r} - \omega_{RF} t)}\, \psi_3^\dagger(\mathbf{r})\psi_2(\mathbf{r}) + h.c. \end{aligned}$$

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the RF spectrum is given by

$$I(\omega_{th}) = -2\gamma^2 \int d\mathbf{r}\, d\mathbf{r}' \, \mathrm{Im}\{\Pi^R(\mathbf{r},\mathbf{r}';\omega_{th})\}$$

where $\omega_{th} = \omega_{RF} + \mu - \mu_3$ is a "theoretical" detuning frequency.

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A quite difficult part of the whole story !
($\leftrightarrow$ sometimes recourse to Padé approximants)

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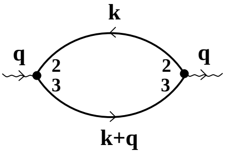
- $\boxed{a_i \neq 0\,,\, a_f = 0}$ $\Longrightarrow$ atom in initial state "2"
  correlates with its mate in "1" within the BCS approximation $\Longrightarrow$ RF spectrum is obtained from the BCS bubble

BCS & BCS-RPA diagrams below T_c:

(a)



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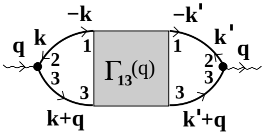
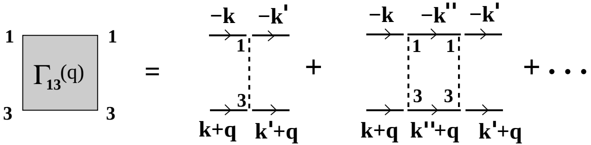
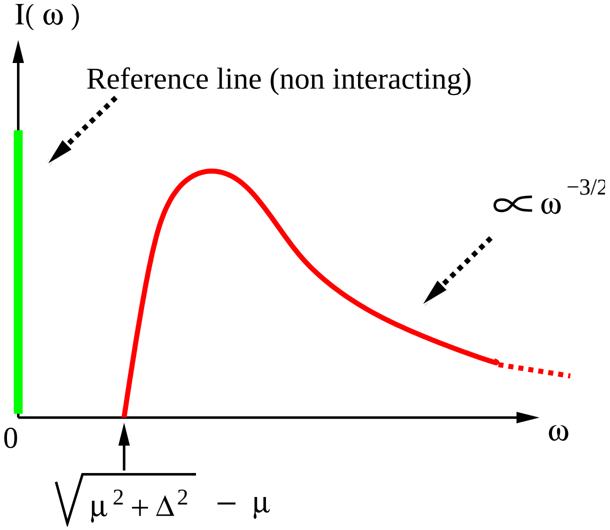




RF spectrum from BCS bubble at $T = 0$:



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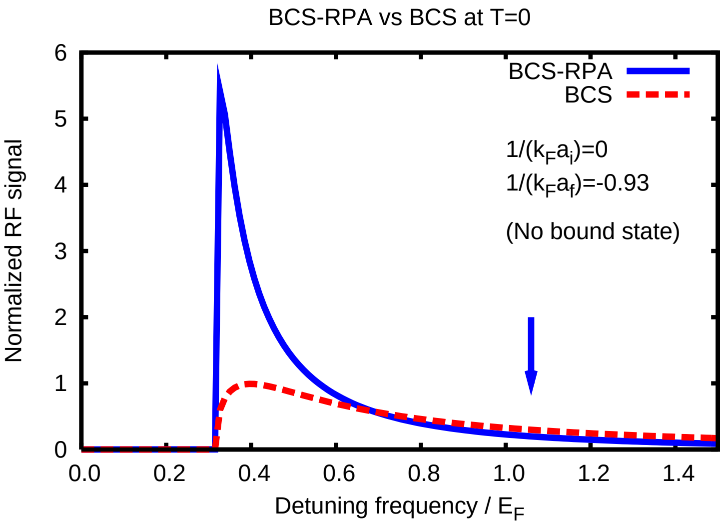

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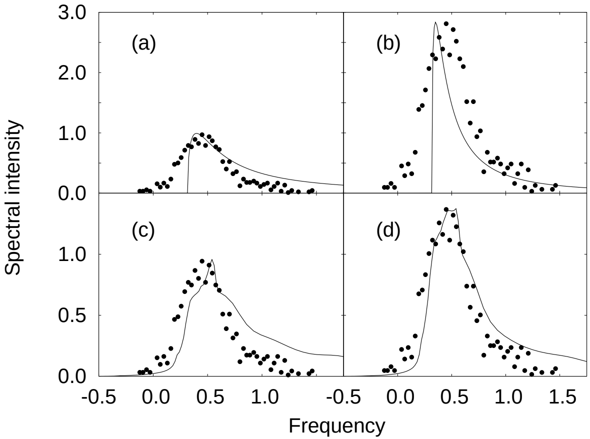
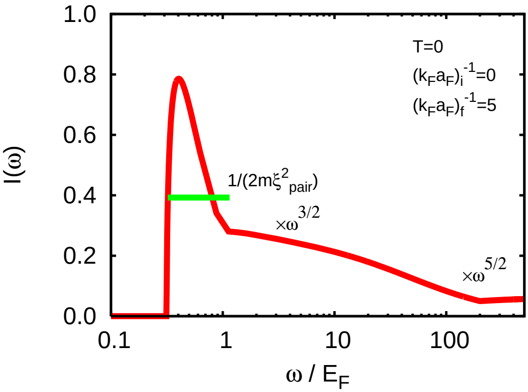


$$(k_F a_i)^{-1} = 0 \qquad (k_F a_f)^{-1} = -1.32 \qquad T \lesssim 0.5\, T_c$$

[Exp. data: Fig.2(d) of PRL **99**, 090403 (2007)]

When a_f is quite different from a_i:



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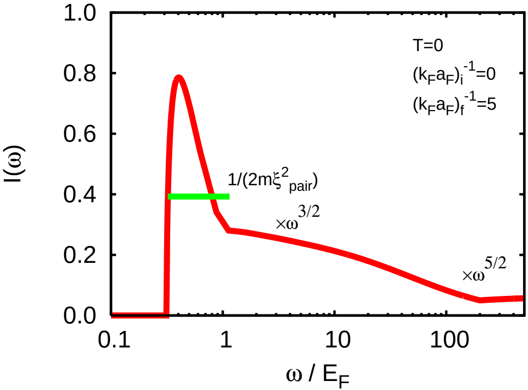

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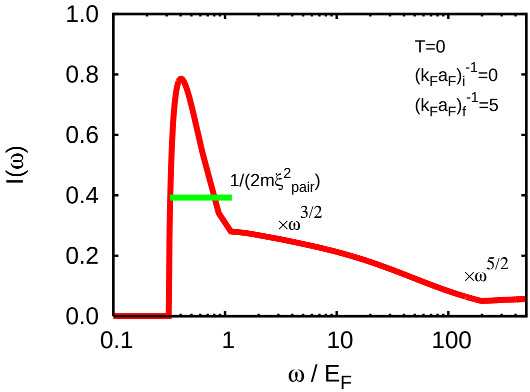

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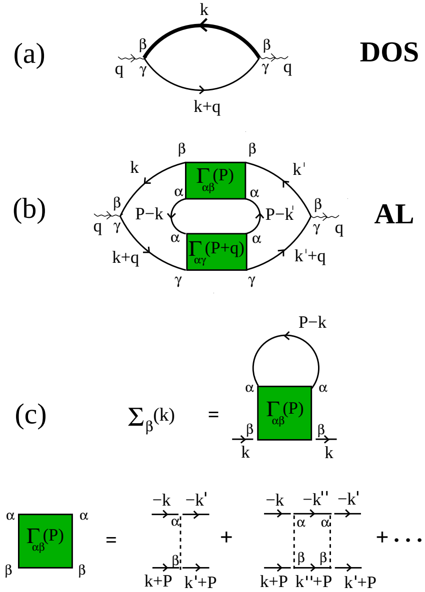
DOS & AL diagrams above T_c:



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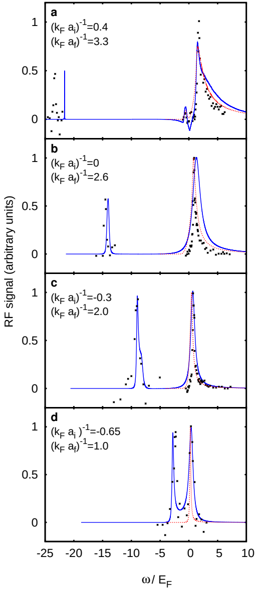

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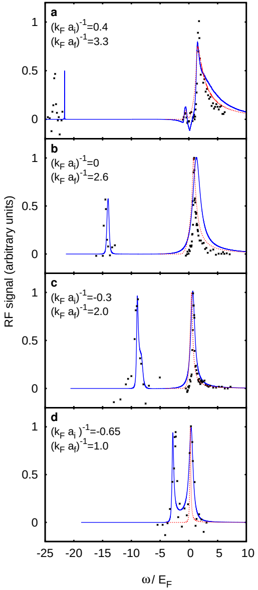

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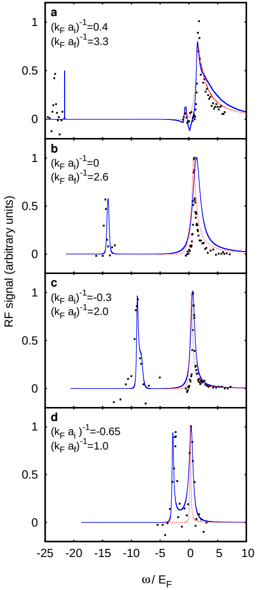

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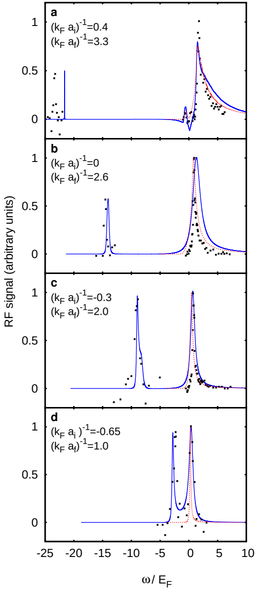

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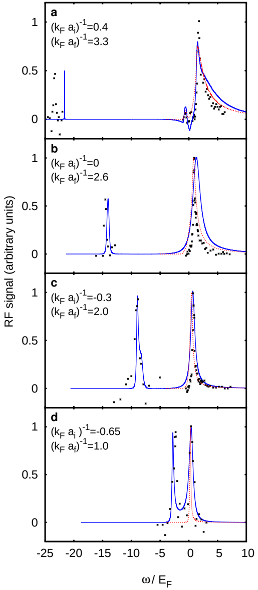

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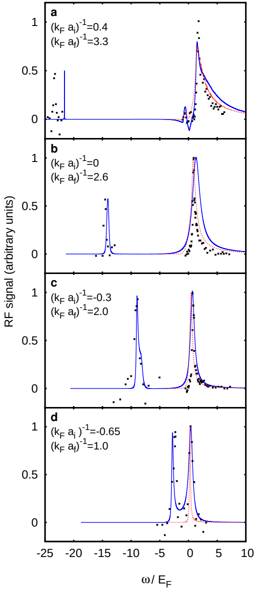

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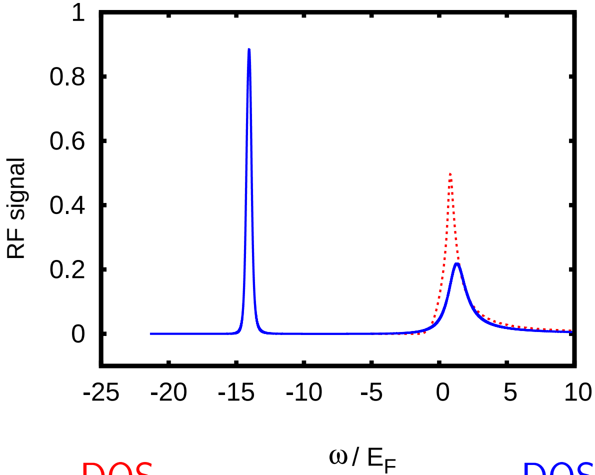
[Exp. data from Fig.4 of Nature **454**, 739 (2008)]

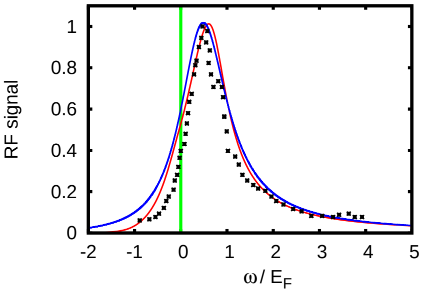
Comparison between DOS and DOS+AL on an absolute scale:



- - - DOS ——— DOS + AL

$\frac{1}{k_F a_i} = 0.0$ $\frac{1}{k_F a_f} = 2.6$ $T \approx T_c$

Further comparison with data ($T \approx T^*$):



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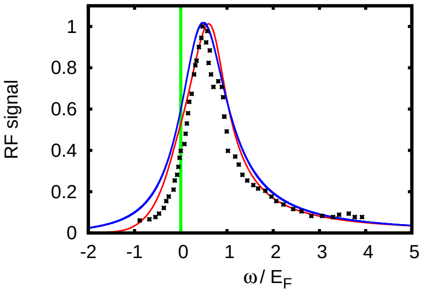

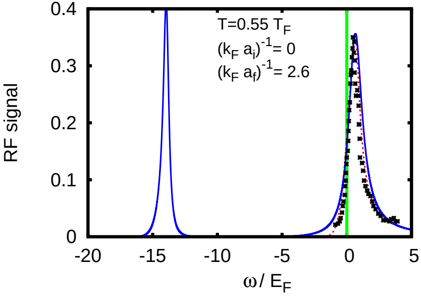

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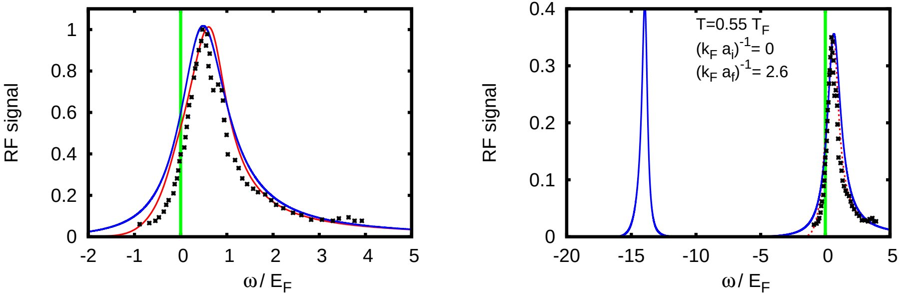

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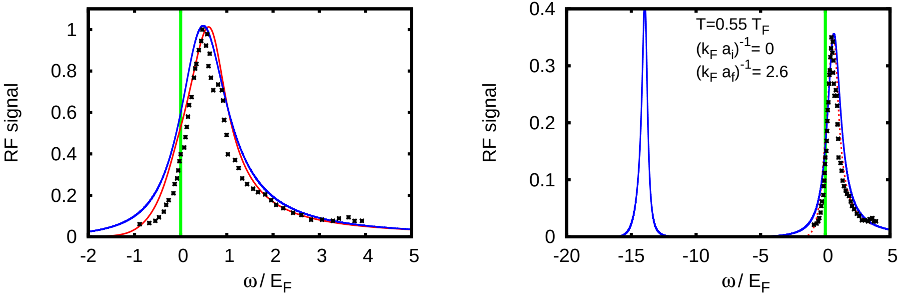

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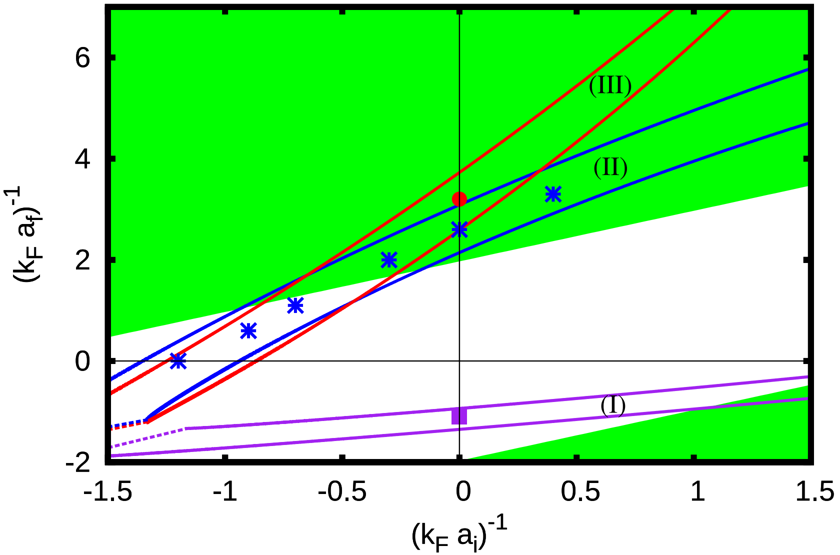
$\Longrightarrow$ do not forget about the presence of the bound state with DOS+AL !

We are here $(*) \swarrow$:



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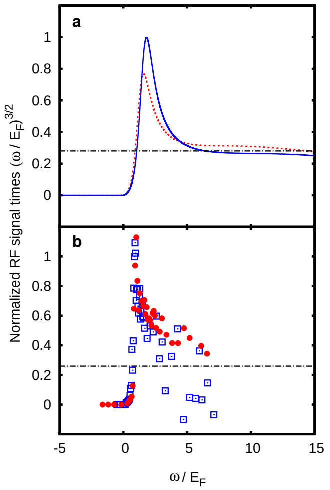
An example: $(k_F a_i)^{-1} = 0$ and $T \approx T^*$



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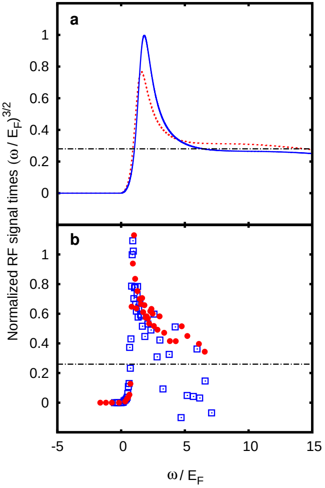

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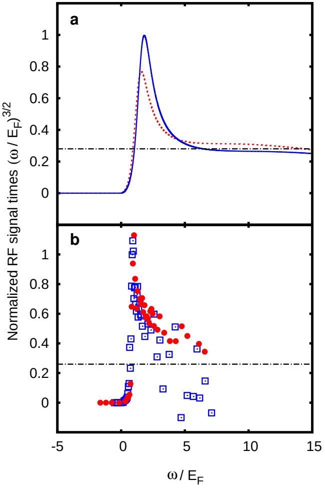

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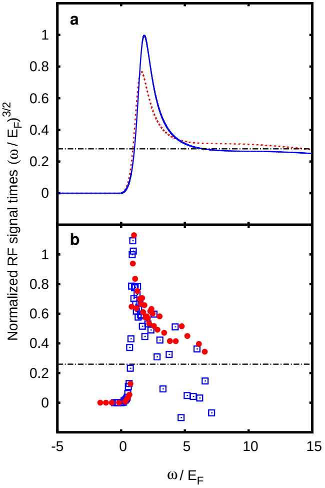

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theoret. value $\frac{\Delta_\infty}{E_F} = 0.73$

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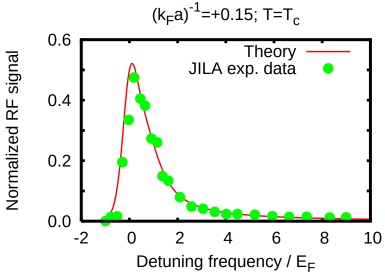

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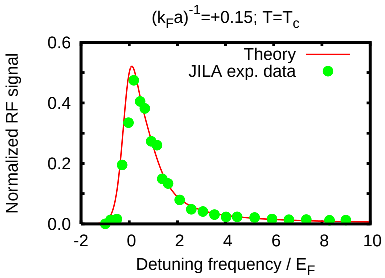

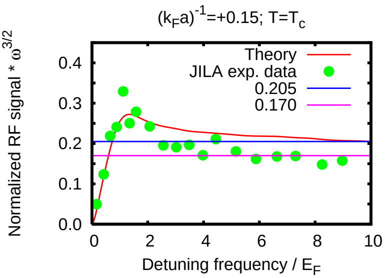

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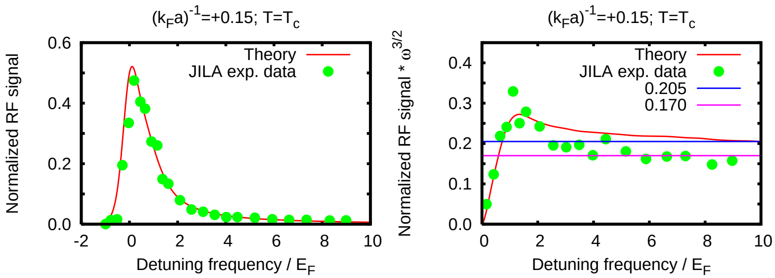

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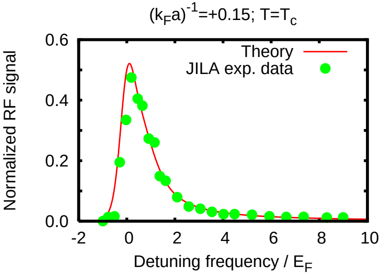

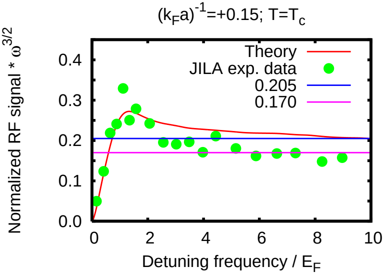

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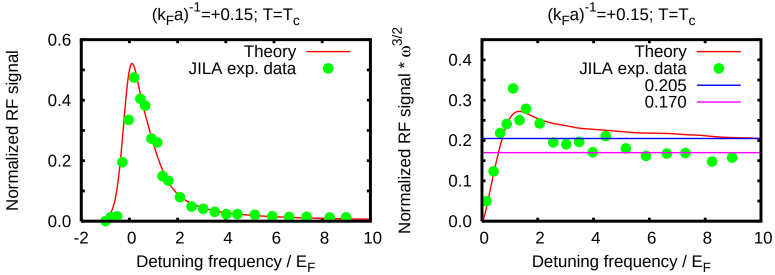

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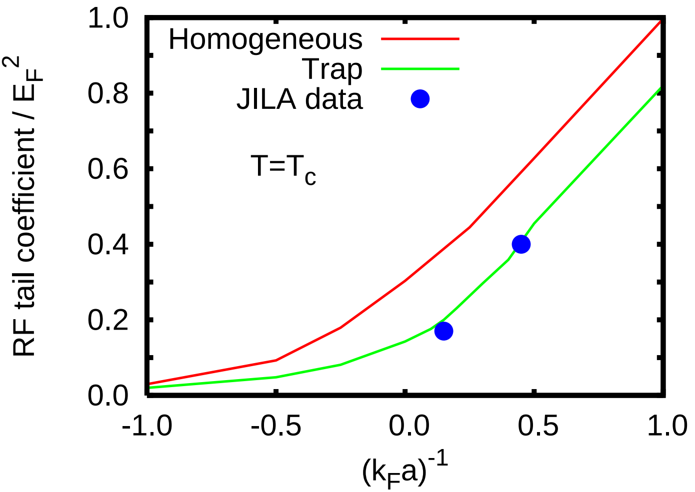
Coefficient of the tail: theory vs experiment (in a trap)



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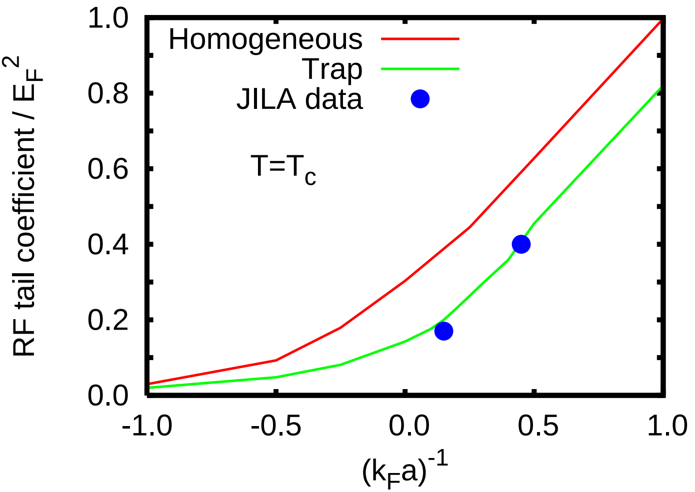

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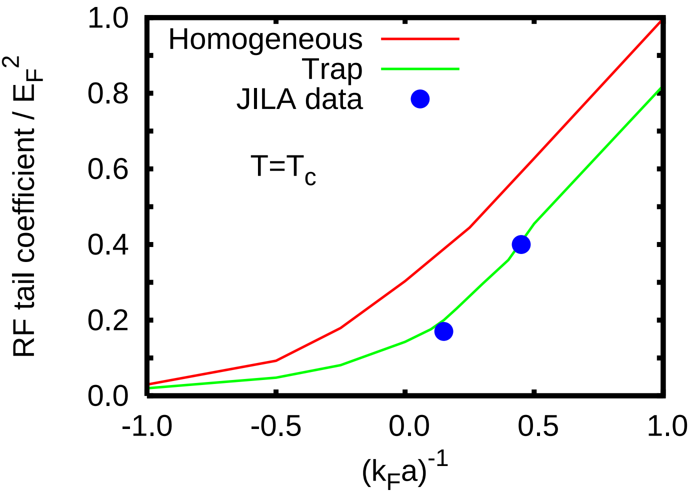

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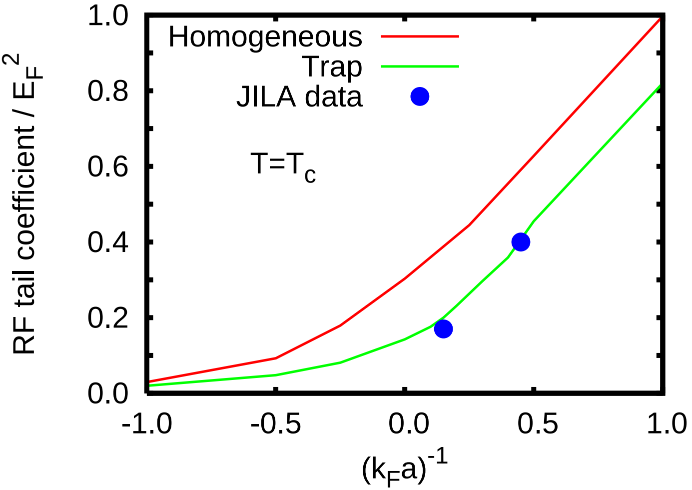

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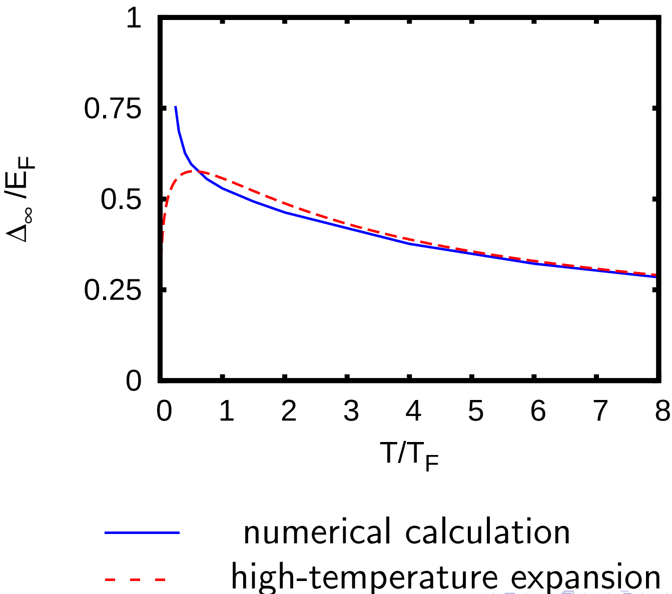
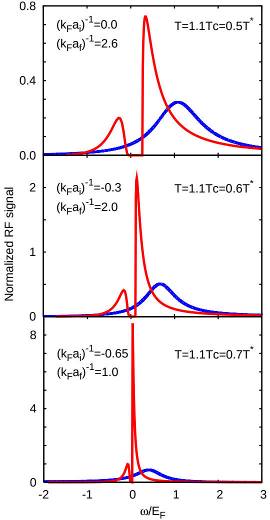
  to be compared with the value $0.8 E_F$ of the "pseudo gap" extracted from single-particle spectral function.

Δ_∞ vs T at unitarity:



Comparison of DOS+AL with BCS-RPA when $T_c \leq T \leq T^*$:



DOS+AL

BCS-RPA

(each curve with its own μ)

"Gedanken" experiment:

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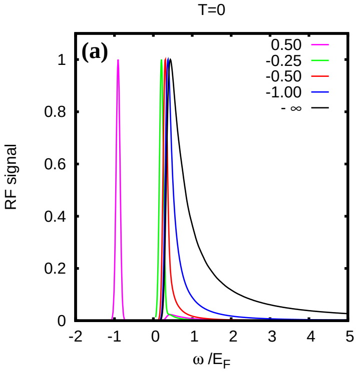
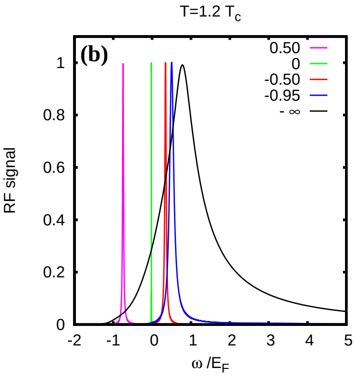
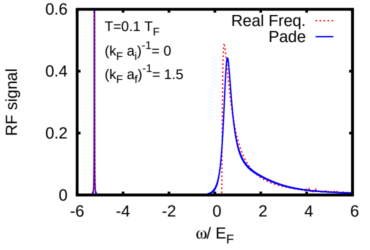


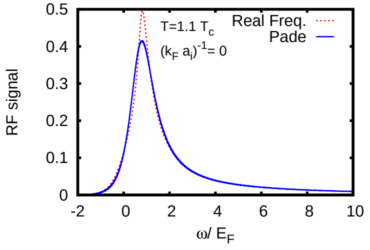


$(k_F a_i)^{-1} = 0$ $(k_F a_f)^{-1}$ varies

(a) BCS-RPA **(b)** DOS+AL

Checking Padé approximants for RF spectra both below and above T_c:





BCS-RPA for $T < T_c$ DOS for $T_c < T$

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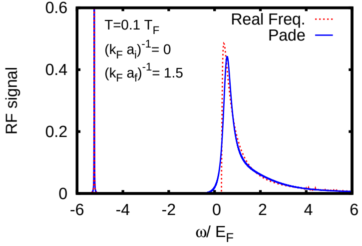

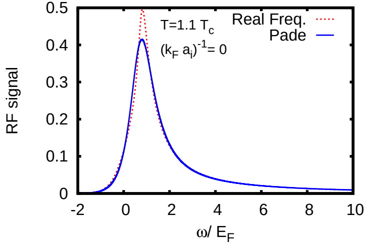

BCS-RPA for $T < T_c$ DOS for $T_c < T$

In both cases, confront with an independent calculation made directly on the real-frequency axis.

"*Dulcis in fundo*" (dessert at the end) ...

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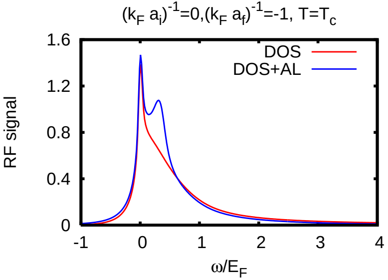

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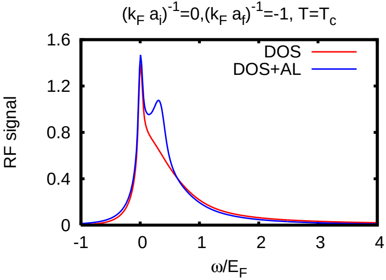

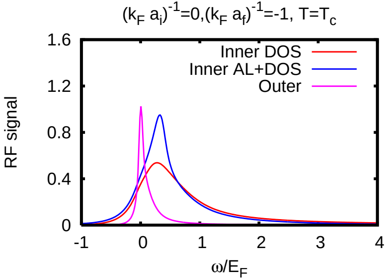

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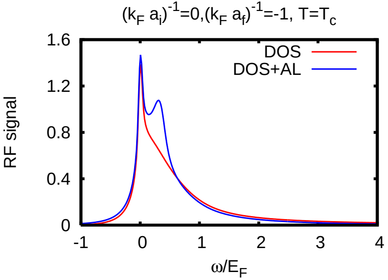
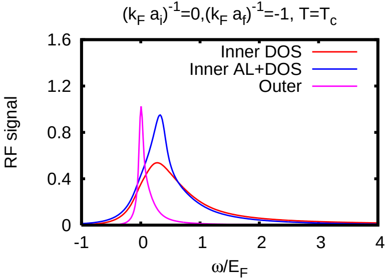
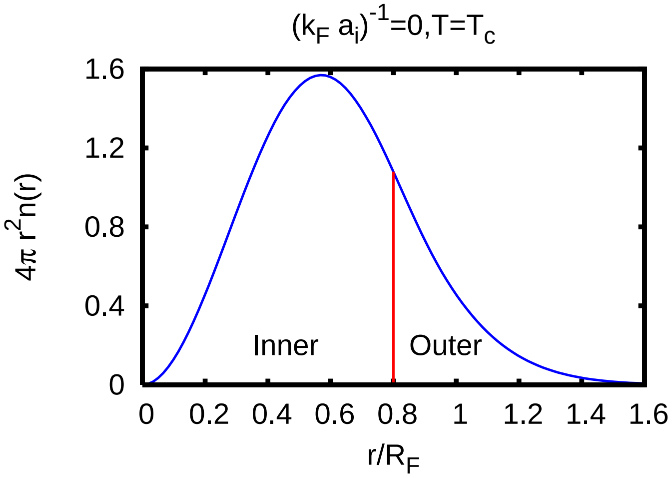




In the "inner part" of the trap final-state effects (DOS + AL) make it visible the right peak!

Boundary between the "inner" and "outer" parts of the trap:



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- ♣ Extract from RF spectra information about Tan's contact intensity.