Chandrasekhar-Clogston limit and phase separation in Fermi mixtures at unitarity

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In collaboration with:
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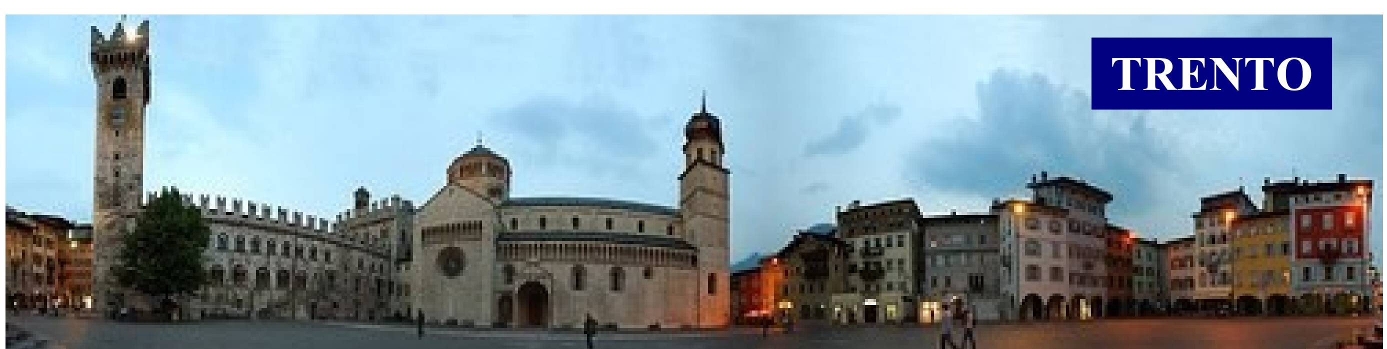
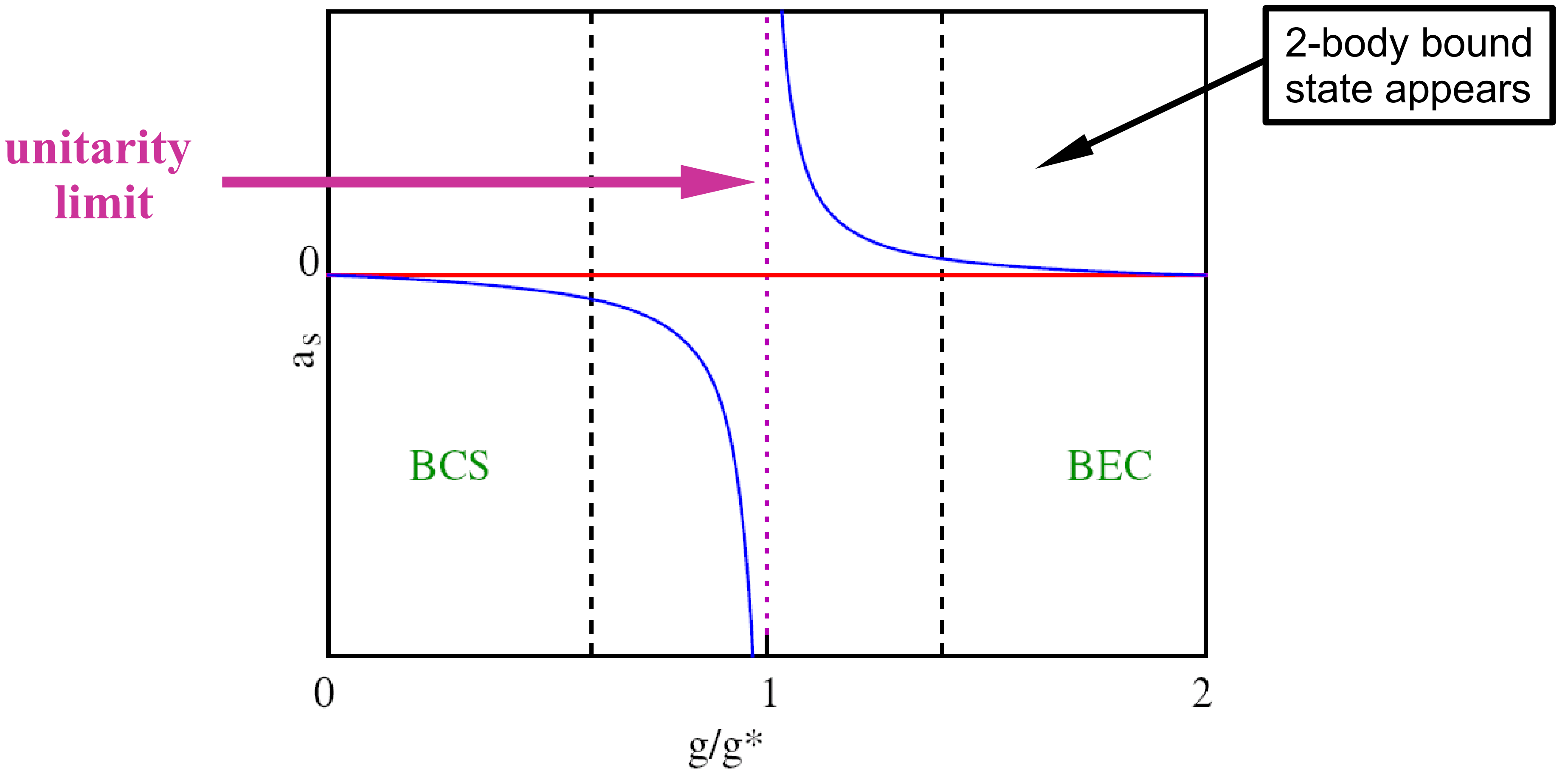


BCS vs Bose-Einstein Condensation

The behaviour of the Fermionic s-wave scattering length is ***not continous***

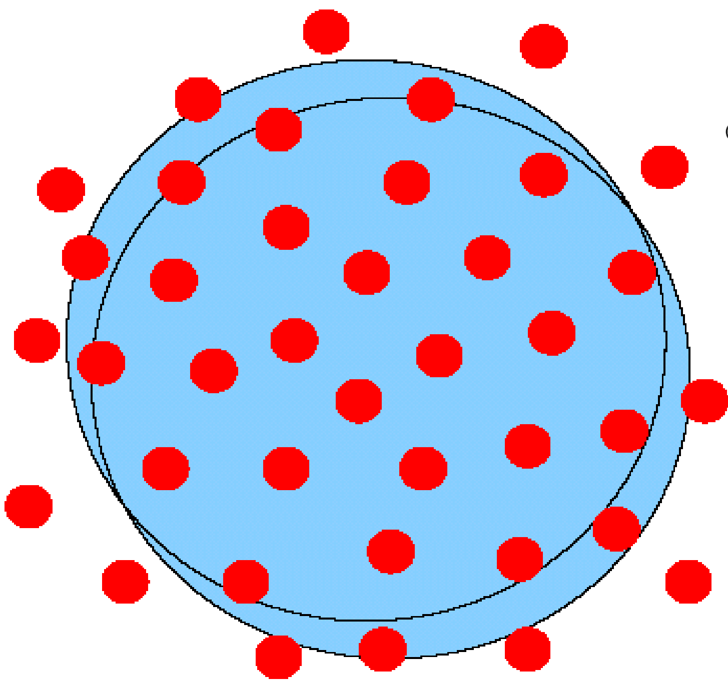


Crossover postulate: even though the scattering length changes abruptly in the many-body problem the ***crossover is smooth***
[Leggett; Nozieres/Schmitt-Rink]

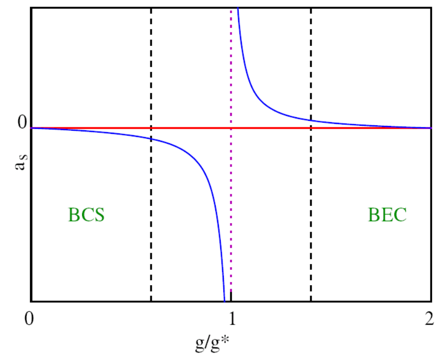
BCS vs Bose-Einstein Condensation

Weak Coupling: $k_F|a_s| \ll 1$
Ovelapping Cooper Pairs

$$\xi_b = \xi_0 = \frac{\hbar v_F}{\pi \Delta_0} \gg k_F^{-1}$$

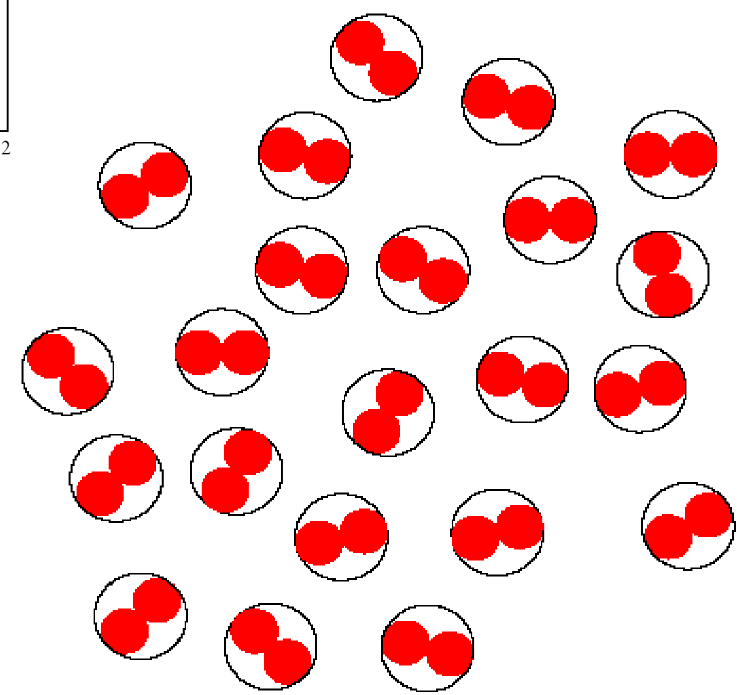


$T^* = T_c^{(BCS)}$



Strong Coupling: $k_F a_s \ll 1$
(Ideal) gas of molecules

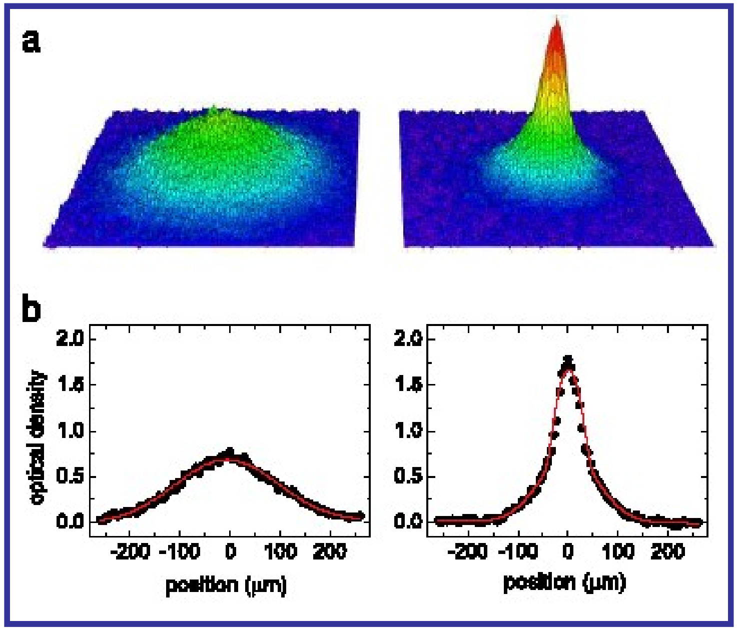
$$\xi_b \sim a_s \ll k_F^{-1} \quad E_b = \frac{\hbar^2}{m a_s^2}$$



$T^* \gg T_c^{(BEC)}$

Note on finite T: Except for very weak coupling (BCS) pairs form and condense at different temperature, ***T**** and T_c

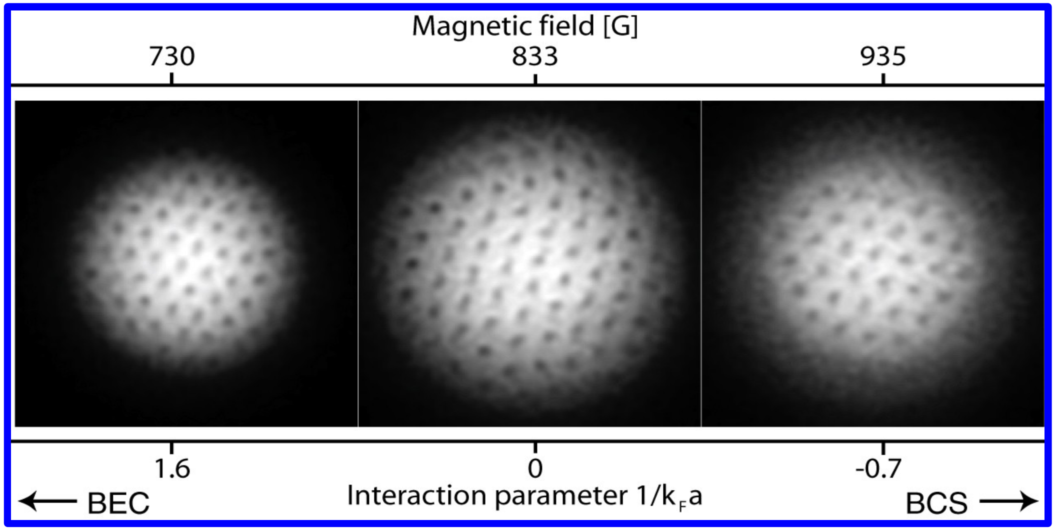
Superfluid fermions



Molecular Bose-Einstein condensation from a fermionic gas

[JILA, Innsbruck, MIT, ENS, RICE, 2003]

Vortex lattice on the BCS-BEC crossover [MIT, 2005]



Superfluid fermions at unitarity

- The only scales at unitarity are the Fermi energy and the temperature.
- The thermodynamic properties have an "universal" form.

In particular at *T=0*

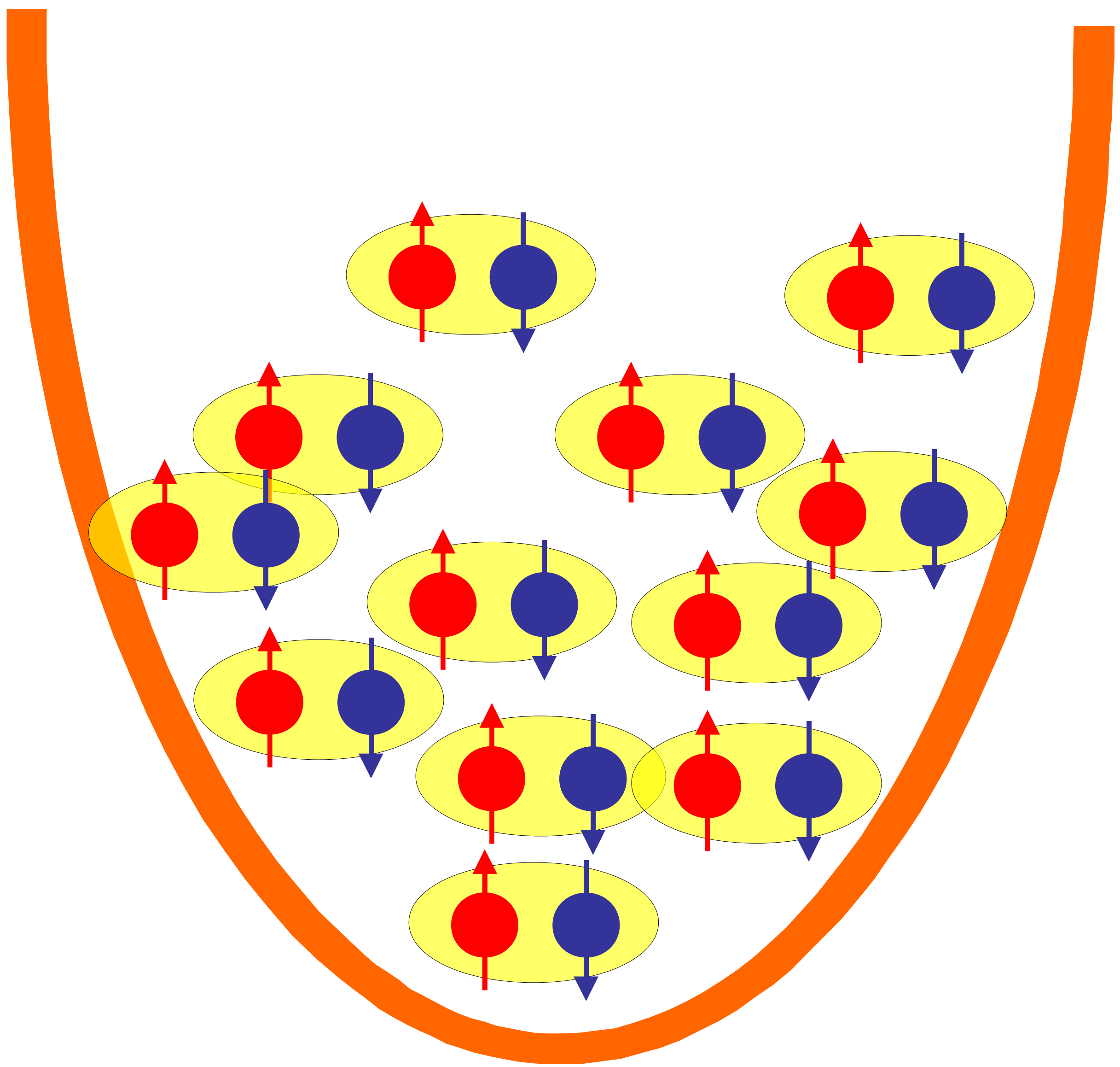
energy density, pressure, chemical potential are ***proportional*** to the ones of an ideal Fermi gas with a density equal to the superfluid one.

The universal parameter (via Montecarlo & Experiments)

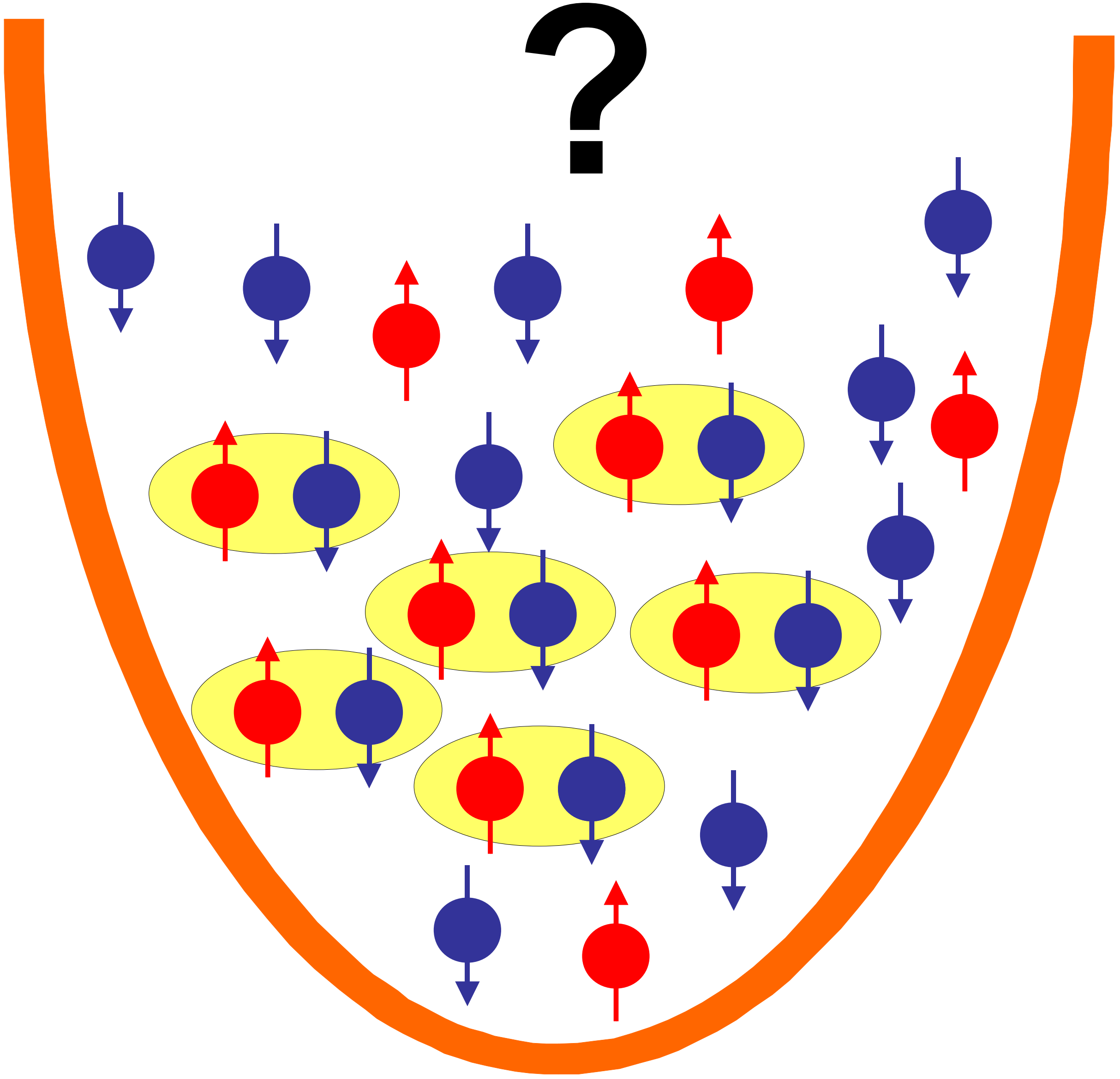
$$\xi_S \simeq 0.42$$

$$\frac{E_S}{N_S} = 2\xi_S \frac{3}{5}\frac{\hbar^2}{2m}(6\pi^2 n_S)^{2/3} \equiv 2\epsilon_S(n_S)$$

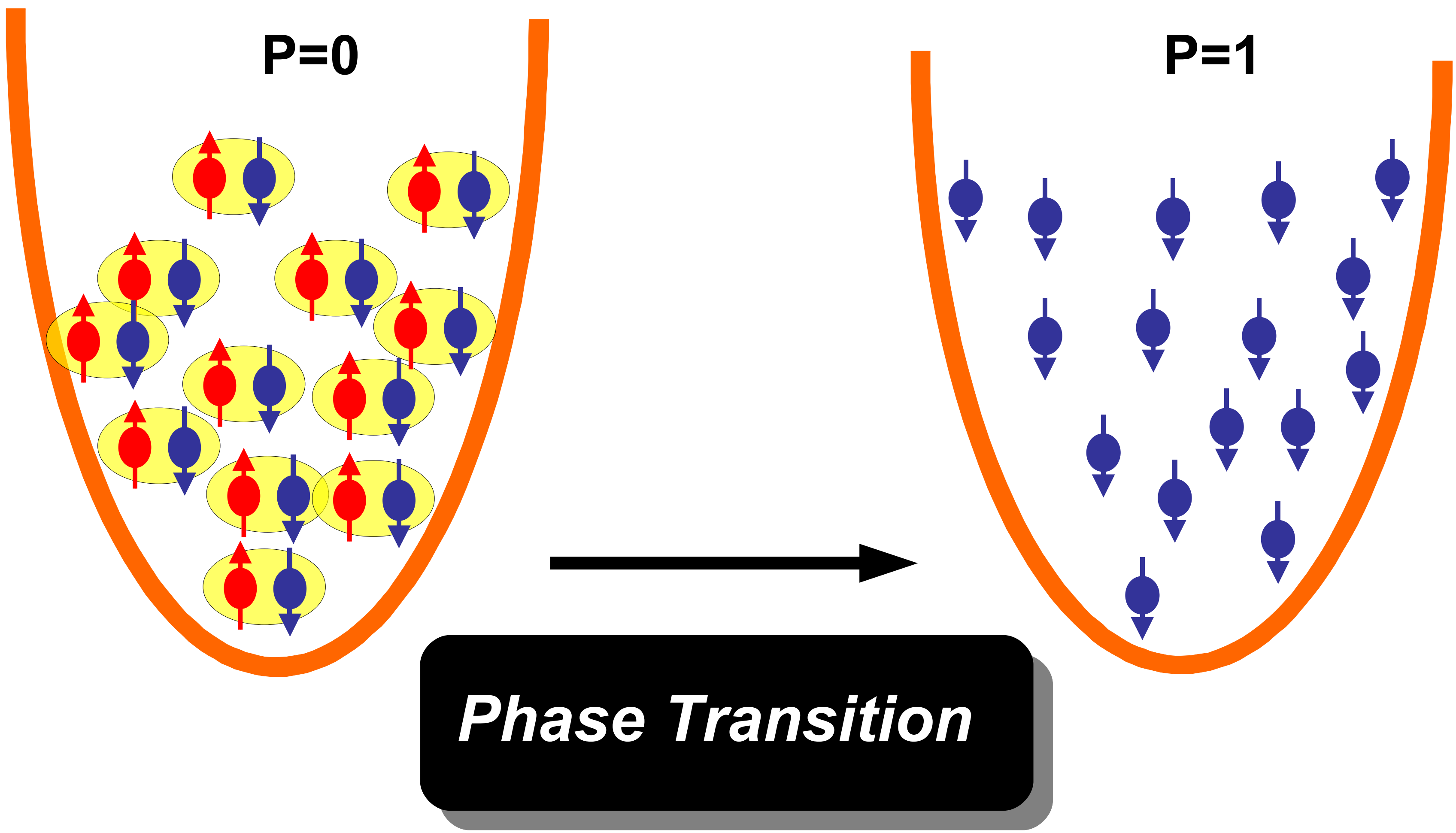
Balanced Fermi gases at unitarity



Imbalanced Fermi gases at unitarity

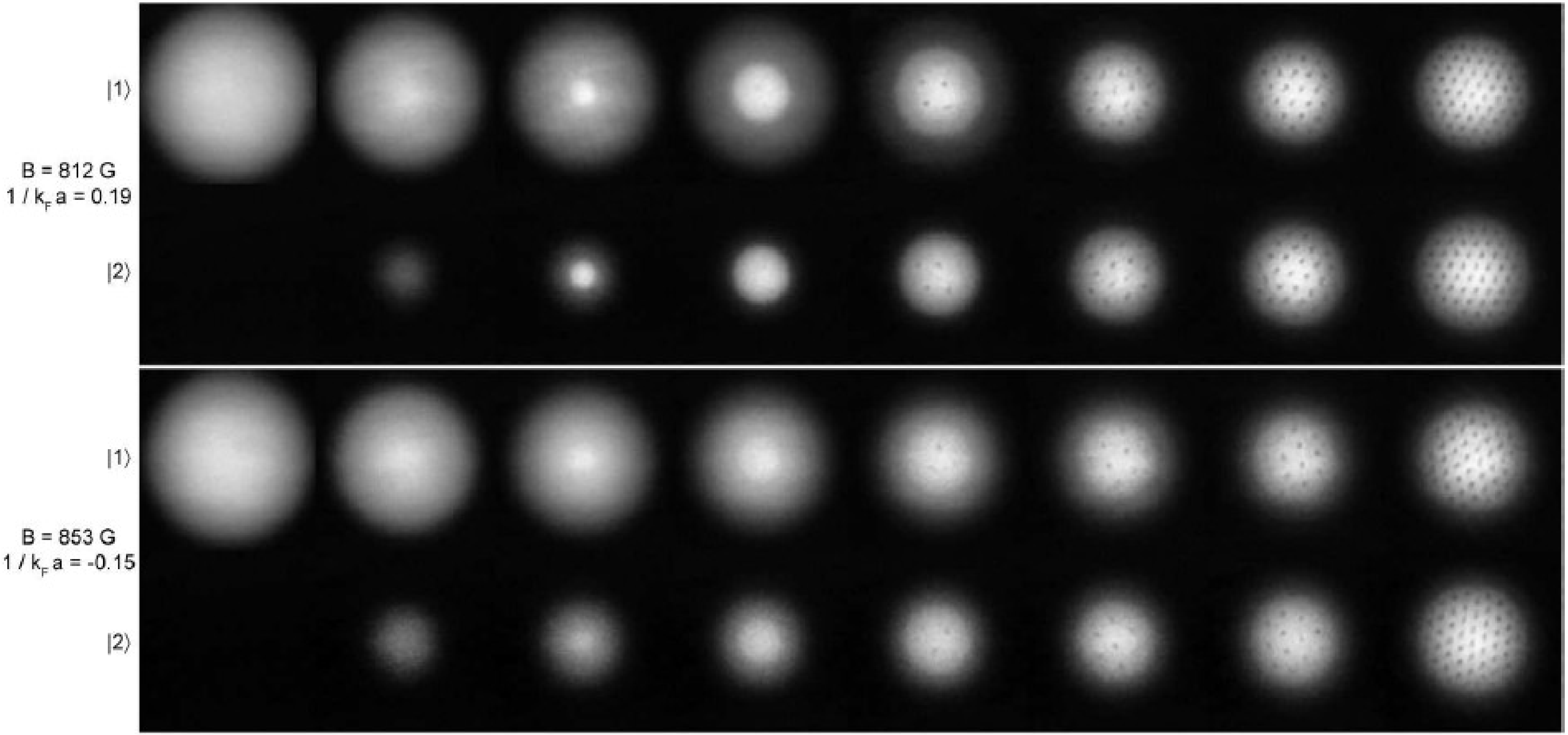


Balanced Fermi gases at unitarity



[Phase Transition to a normal phaase for large magnetic field
B. S. Chandrasekhar (1962), A. M. Clogston (1962)]

Recent Experiments on imbalanced Fermi gases at unitarity



MIT, Science **311**, 492 (2006)

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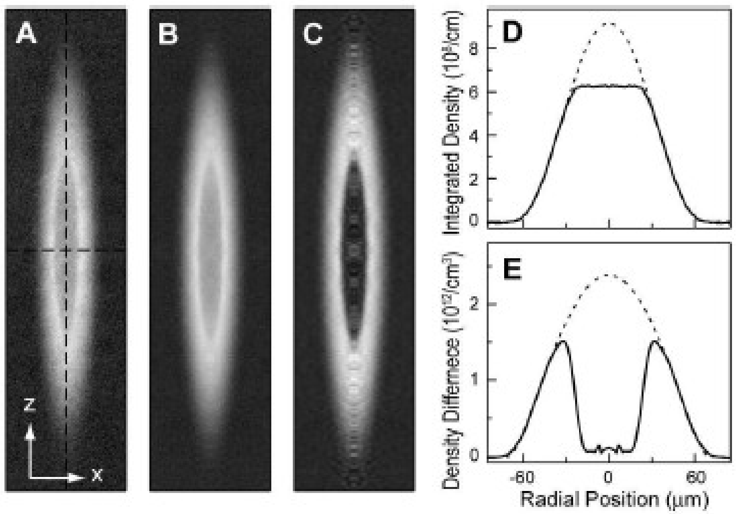
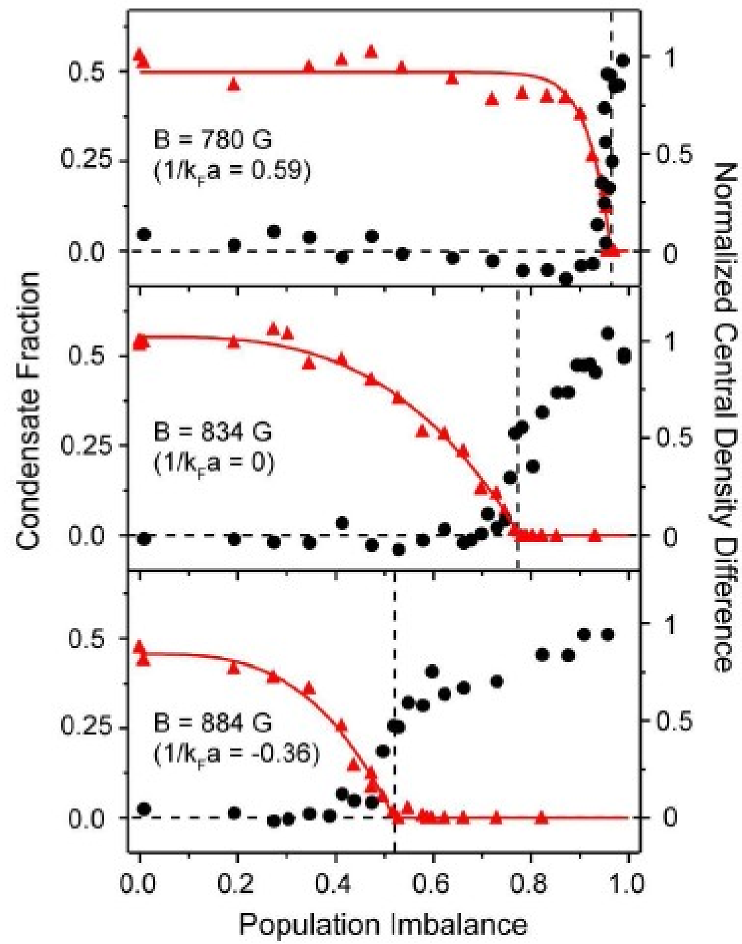




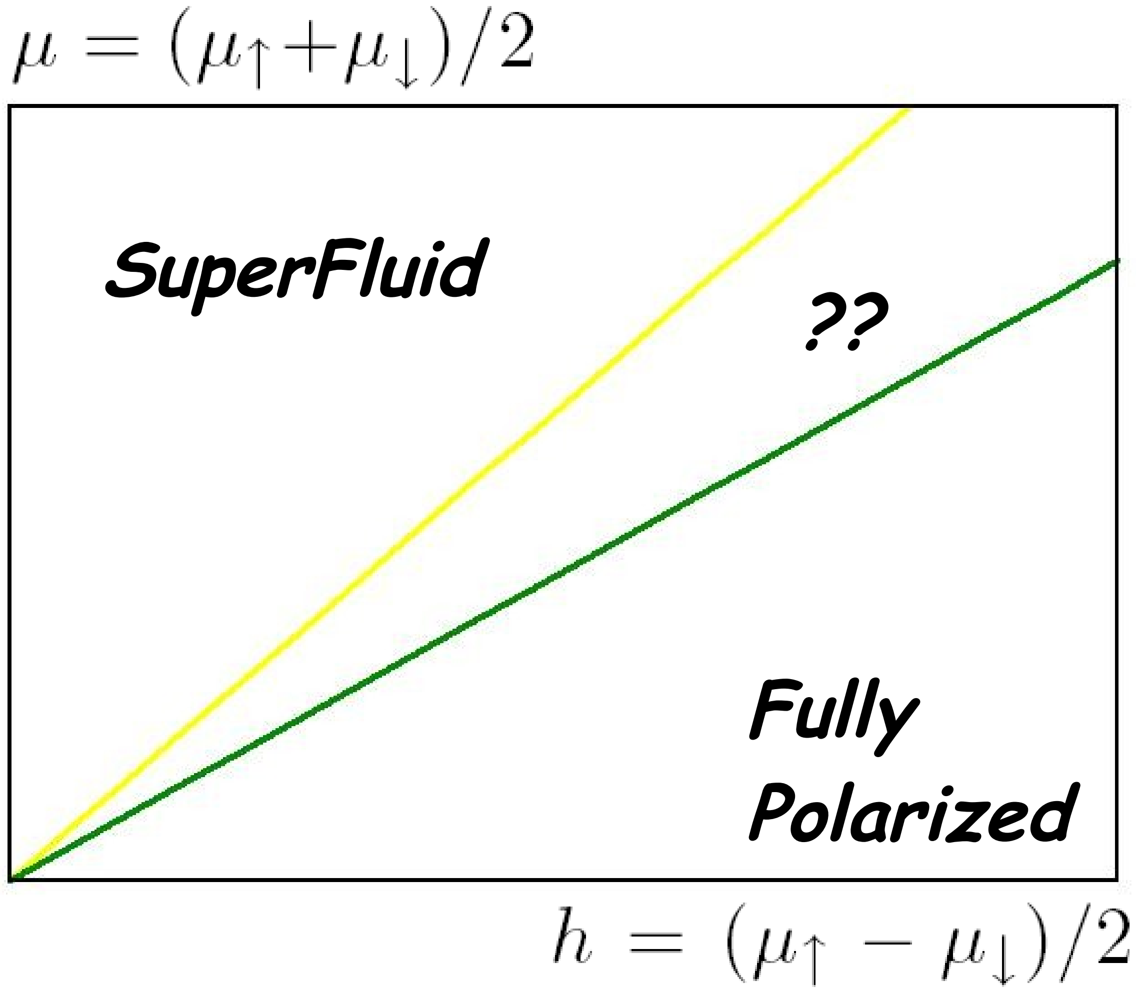
BEC

Unitarity

BCS

[MIT, Phys. Rev. Lett. **97**, 030401 (2006)]

Phase diagram



E.g., only
SF and P phase

$$P_p = P_s \Rightarrow \mu = \frac{(2\xi_S)^{3/5}}{2-(2\xi_S)^{3/5}} h$$

Normal phase of polarized Fermi gas at unitarity

Assumption:
at high polarization homogeneous phase,
NORMAL FERMI LIQUID: consider a very dilute mixture of spin-↓ atoms immersed in non-interacting gas of spin-↑ atoms

Energy expansion for small concentration $x = \frac{n_\downarrow}{n_\uparrow}$

$$\frac{E(x)}{N_\uparrow} = \frac{3}{5}\epsilon_{F\uparrow}\left(1 - Ax - \frac{m}{m^*}x^{5/3} + \cdots\right) = \frac{3}{5}\epsilon_{F\uparrow}\epsilon(x)$$

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Non interacting gas

single-particle energy

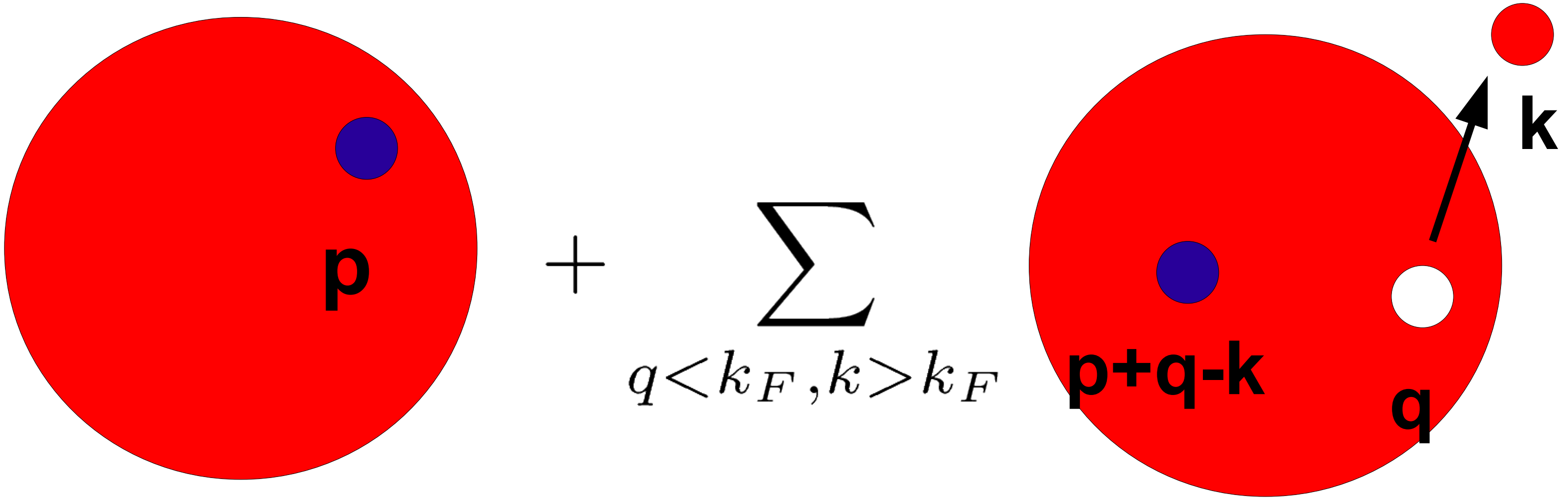
quantum pressure
of a Fermi gas of quasi-particles
with an effective mass

Normal phase of polarized Fermi gas at unitarity

Consider a SINGLE down atom interacting with an ideal Fermi gas (up-atoms).

Variational Ansatz (single particle hole excitations):

$$|\psi\rangle = \phi_0 |\mathbf{p}\rangle_\downarrow \; |0\rangle_\uparrow + \sum_{q<k_F}^{k>k_F} \phi_{\mathbf{qk}} |\mathbf{p}+\mathbf{q}-\mathbf{k}\rangle_\downarrow \; c^\dagger_{\mathbf{k}_\uparrow} \; c_{\mathbf{q}_\uparrow} \; |0\rangle_\uparrow$$



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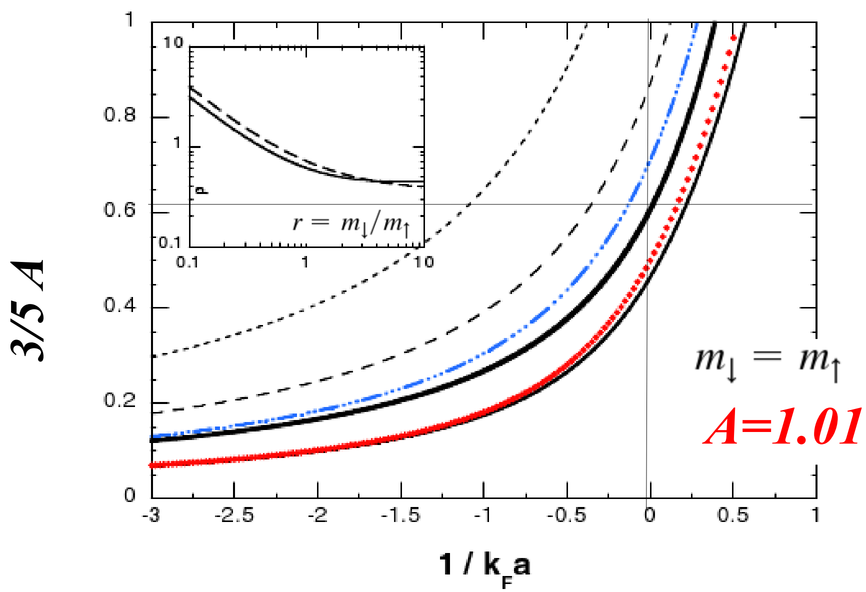

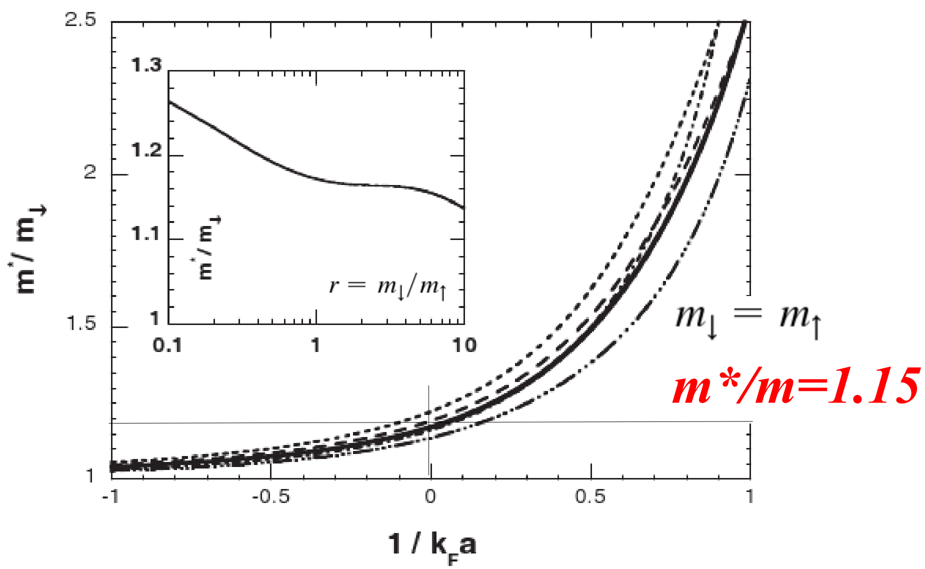

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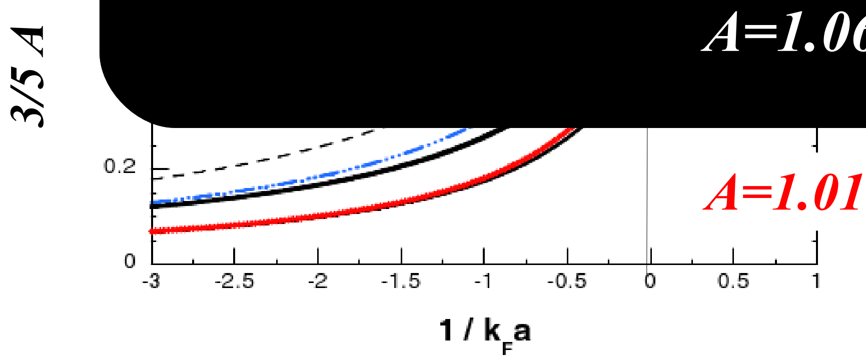
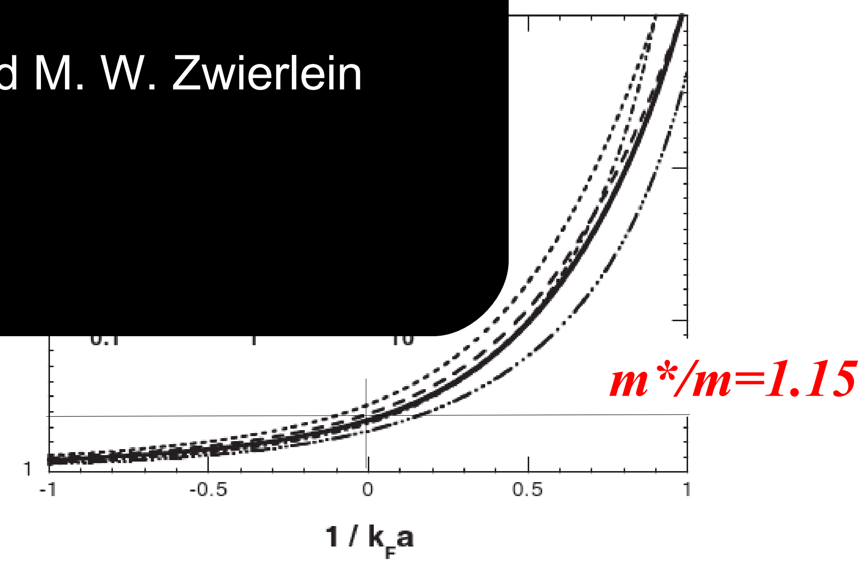
First measurements of the coefficient *A* reported by

A. Schirotzek, C. Wu, A. Sommer, and M. W. Zwierlein
arXiv:0902.3021

A=1.06(7)



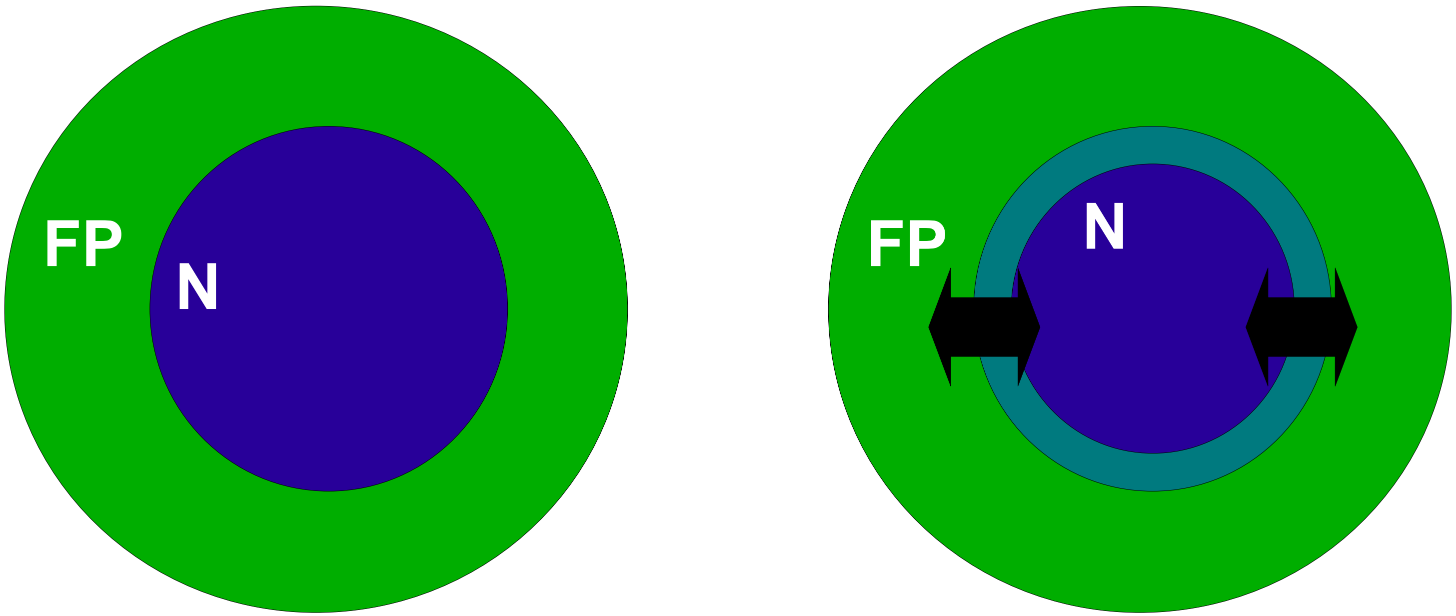


Note: it is equivalent to a T-matrix approach

$$\omega - \epsilon_{\downarrow,k} + \mu_\downarrow - \Sigma(k,\omega) = 0 \quad \Rightarrow \quad \mu_\downarrow = \Sigma(0,0) \;\; \& \;\; \frac{m^*}{m_\downarrow} = \frac{1 - \frac{\partial \Sigma}{\partial \omega}}{1 - 2m_\downarrow \frac{\partial \Sigma}{\partial k^2}}$$

Normal phase of polarized Fermi gas at unitarity

An idea how to measure *A* and *m**:
Sudden change of the scattering lenght in the highly imbalanced case (x~0) the minority compenent would start oscillating



Not interacting
to
Unitarity

Amplitude: $$\frac{m^*}{m}\left(1+\frac{3}{5}A\right)$$

Frequency: $$\sqrt{\frac{m}{m^*}\left(1+\frac{3}{5}A\right)}$$

Superfluid-Normal phase coexistence at unitarity

$$\frac{E(x)}{N_\uparrow} = \frac{3}{5}\epsilon_{F\uparrow}\left(1 - Ax - \frac{m}{m^*}x^{5/3} + Bx^2\right) = \frac{3}{5}\epsilon_{F\uparrow}\epsilon(x)$$

interaction between quasi-particles

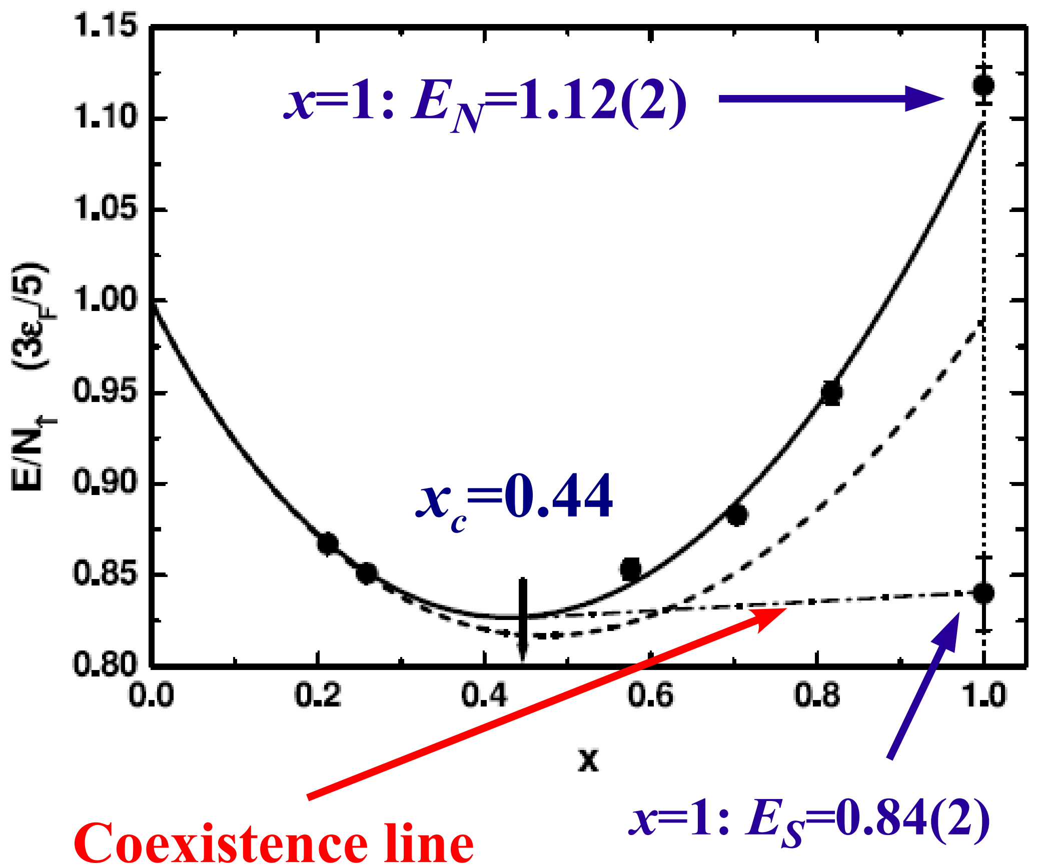
Most recent values using FN-QMC

A = 0.99(2)

m^*/m = 1.09(3)

B = 0.14

*[S. Pilati and S. Giorgini, Phys. Rev. Lett. **100**, 030401 (2008)]*



Critical concentration x_c:

$P_{SF} = P_N$

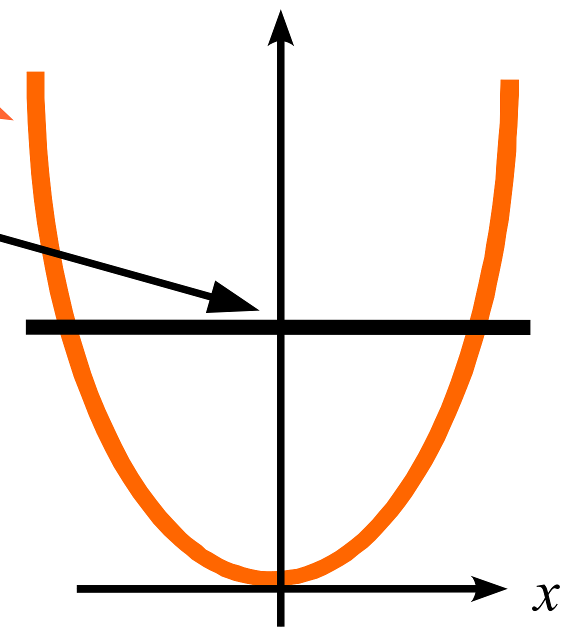
$$\frac{\epsilon'(x_c)}{\epsilon(x_c)} = \frac{5}{3}\frac{\epsilon(x_c)^{3/5} - (2\xi_S)^{3/5}}{x_c - 1}$$

SF	N with x_c=0.44

Phase Separation

Exploring Phase diagram in the Trap: LDA

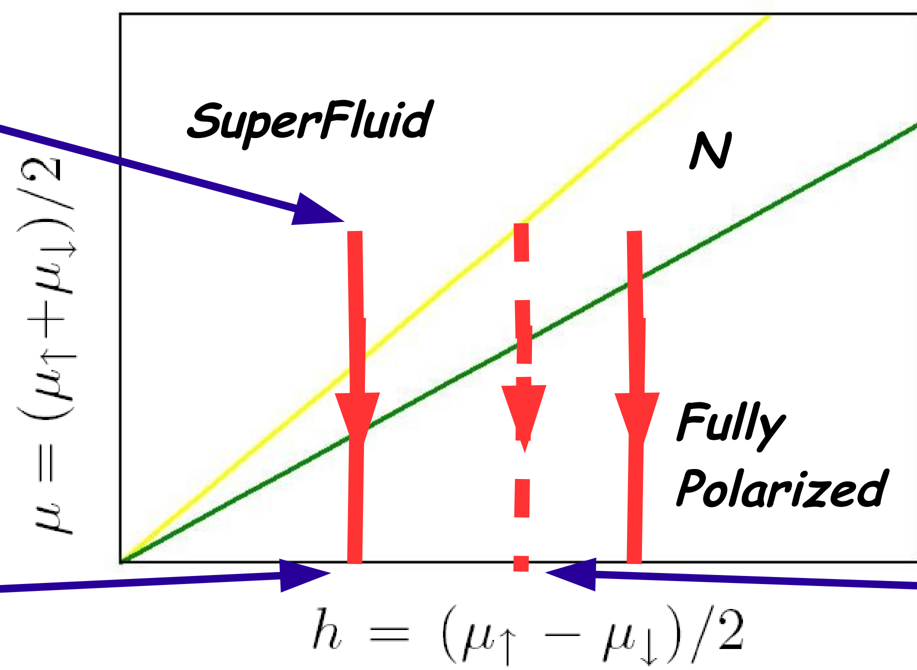
LDA : $$\mu_\sigma(\mathbf{x}) = \mu_\sigma^0 - V(\mathbf{x}) = \mu_\sigma^0 - \frac{1}{2} m\omega x^2$$



$$\mu(\mathbf{x}) = \mu^0 - \frac{1}{2} m\omega x^2$$ Decreasing outward

$$h(\mathbf{x}) = h^0$$ Constant also inside the trap

By the total number of atoms



By the imbalance

Critical imbalance

Exploring Phase diagram in the Trap: LDA

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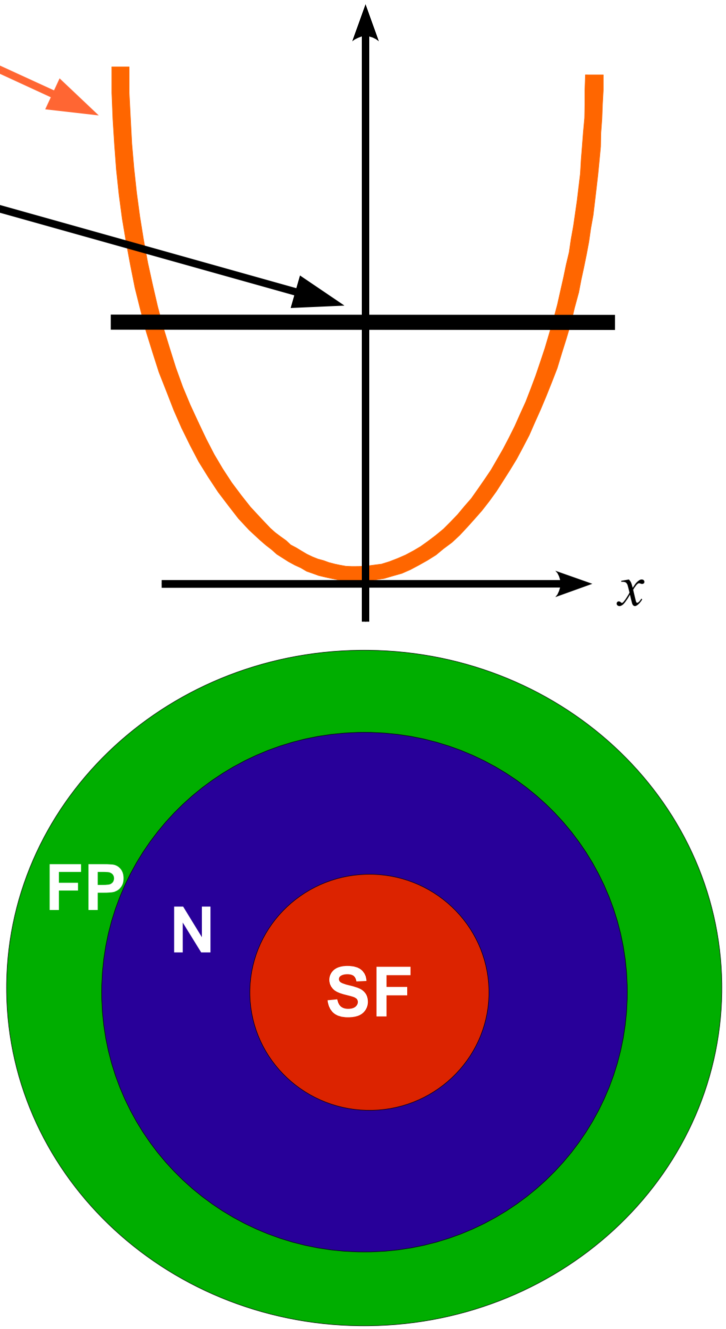
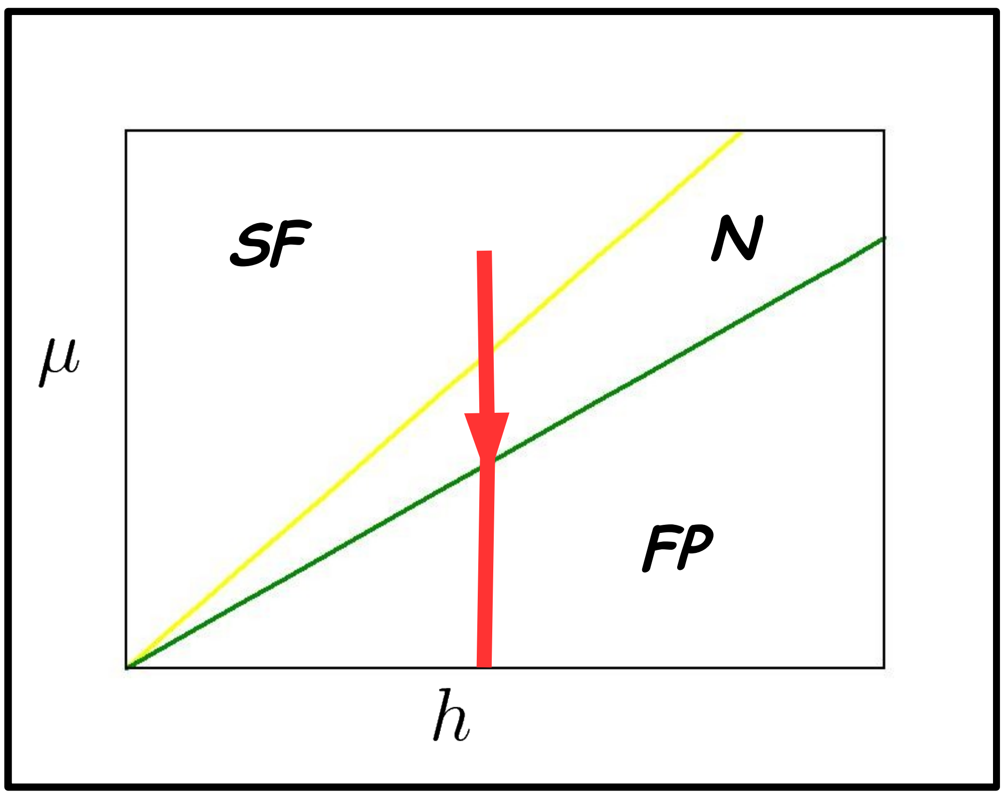
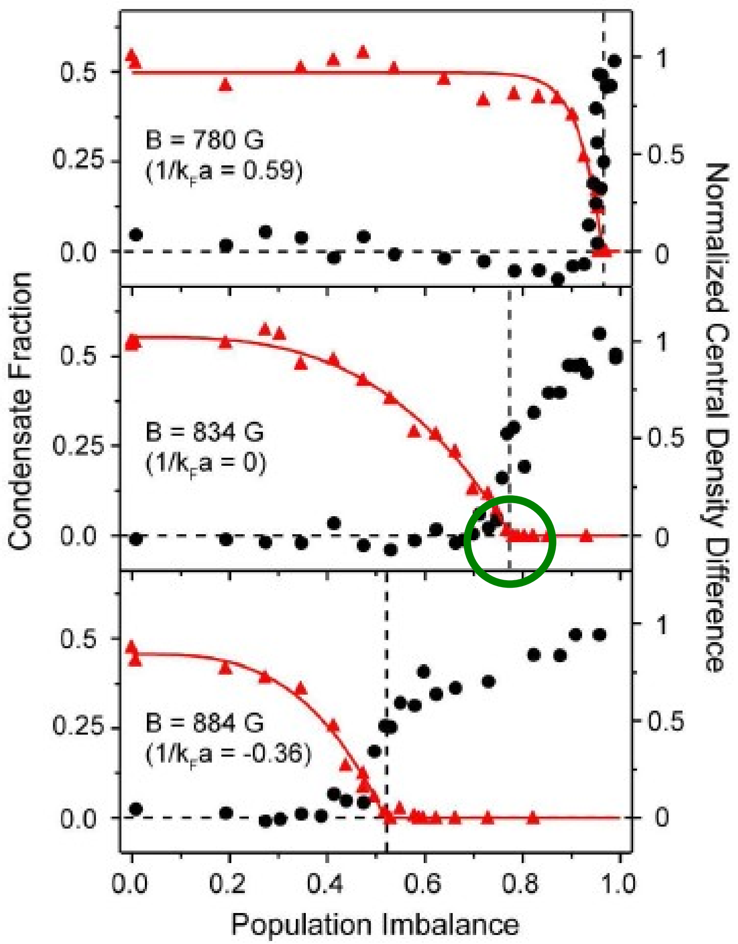


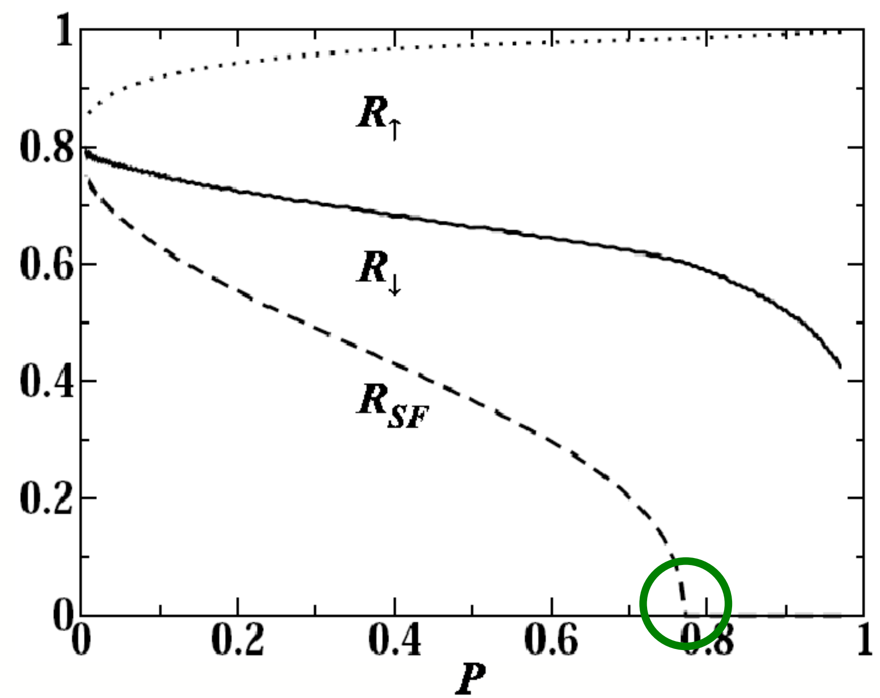


Normal phase of polarized Fermi gas at unitarity: TRAP

1) Critical Polarization (IN TRAP): $P_C = 0.77$

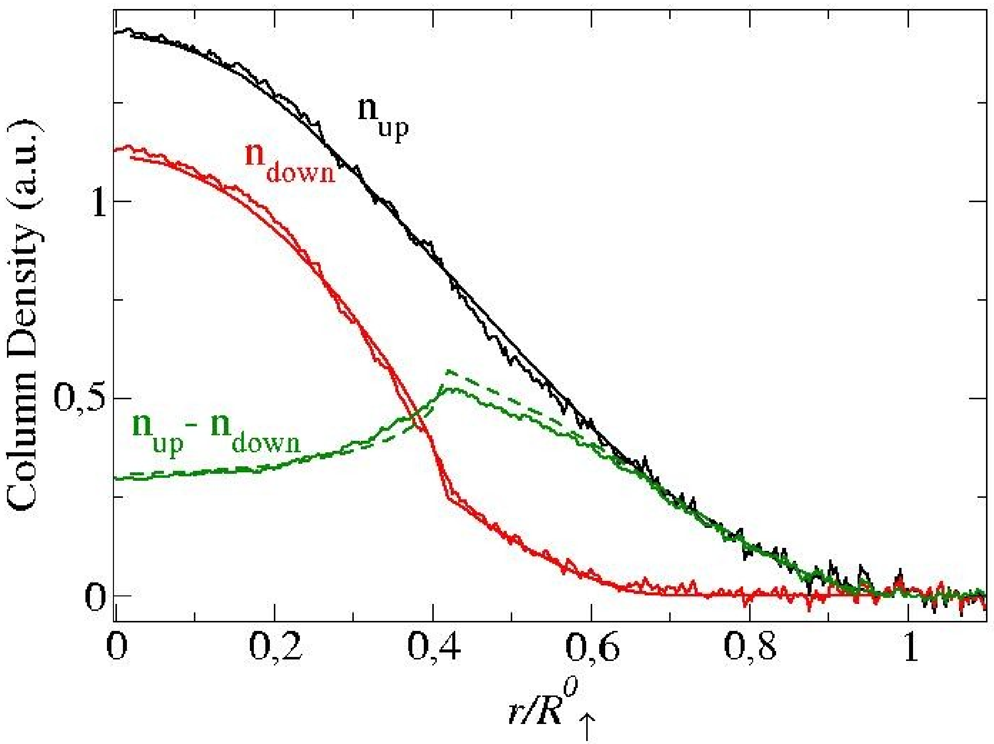
(very good agreement with MIT exps)

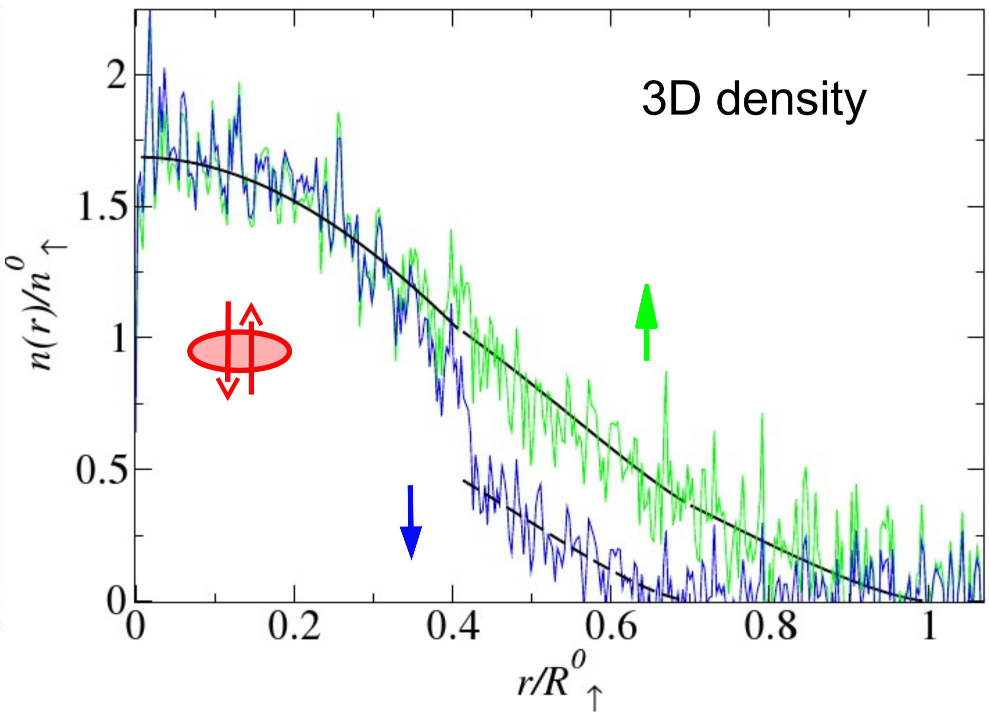




Normal phase of polarized Fermi gas at unitarity: TRAP

2) Density profiles





[Exp. Data from Yong Shin (MIT) compared with theory in A.R., C.Lobo and S. Stringari PRA (2008)]

Normal phase of polarized Fermi gas at unitarity: TRAP

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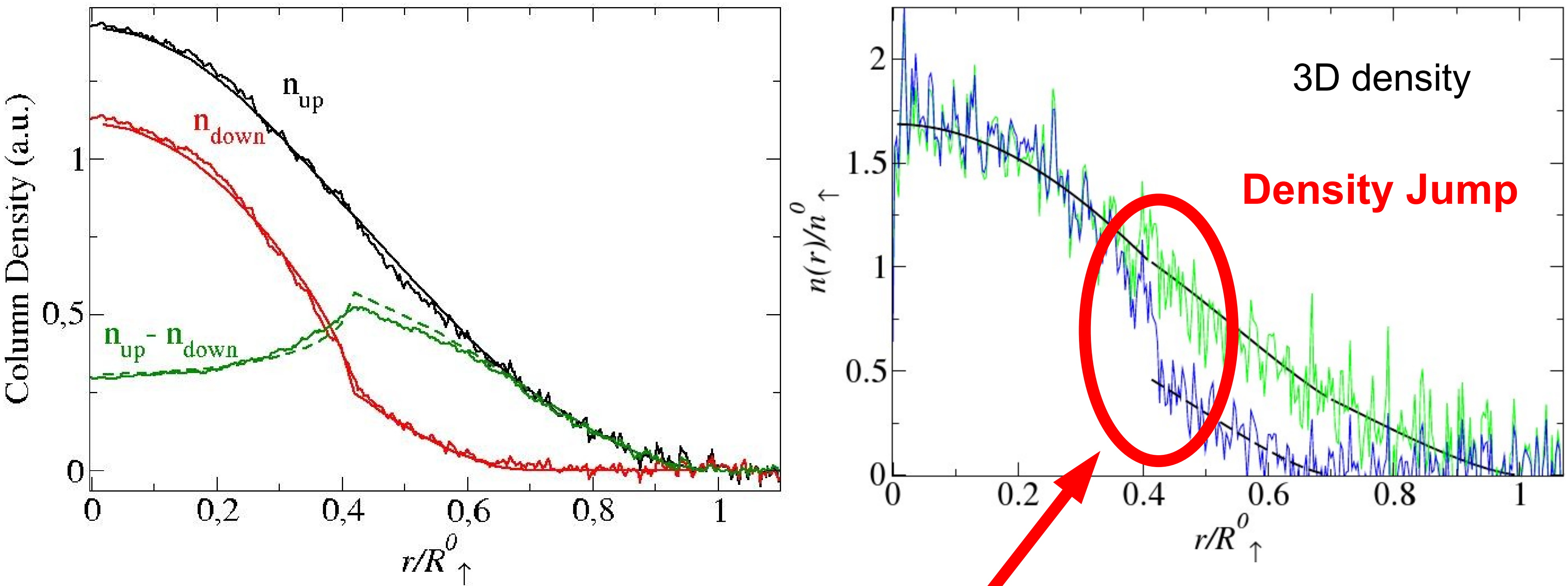
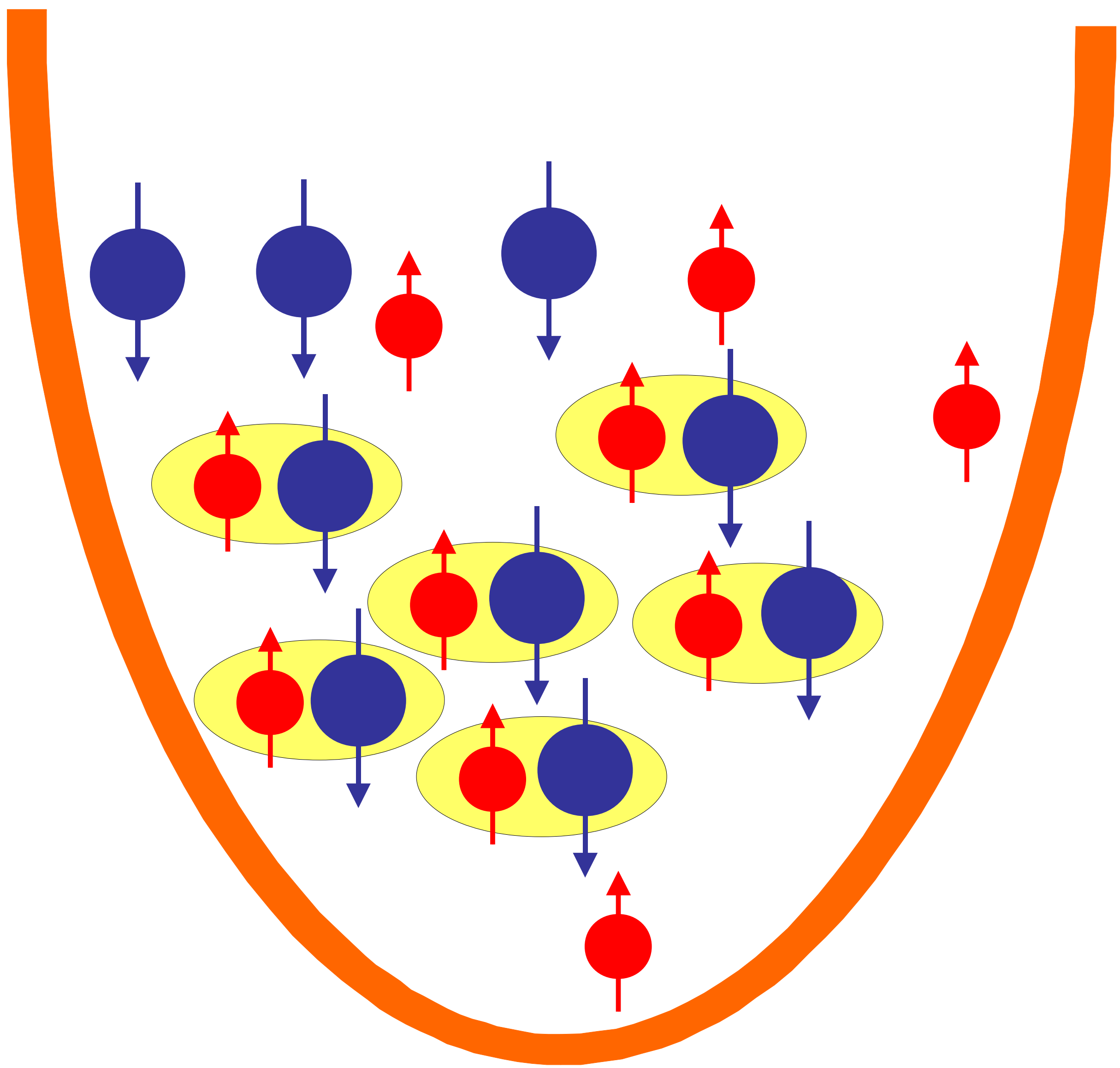


[Exp. Data from Yong Shin (MIT) compared with theory in A.R., C.Lobo and S. Stringari PRA (2008)]

Directly related to the Chandrasekhar-Clogston limit

Fermi mixtures



$$\left\{ \begin{array}{l} x = \dfrac{n_\downarrow}{n_\uparrow} \\[2ex] \kappa = \dfrac{m_\downarrow}{m_\uparrow} \end{array} \right.$$

Fermi mixtures

Equation of state of unpolarized SF:

$$\frac{E_S(\kappa)}{N_S} = \xi_S(\kappa)\frac{3}{5}\frac{\hbar^2}{4m_\kappa}(6\pi^2 n_S)^{2/3} \quad ; m_\kappa : \text{reduced mass}$$

Equation of state of polarized N:

$$\frac{E(x,\kappa)}{N_\uparrow} = \frac{3}{5}\epsilon_{F\uparrow}\left(1 - A(\kappa)x - \frac{1}{\kappa}\frac{m_\downarrow}{m^*(\kappa)}x^{5/3} + B(\kappa)x^2\right)$$

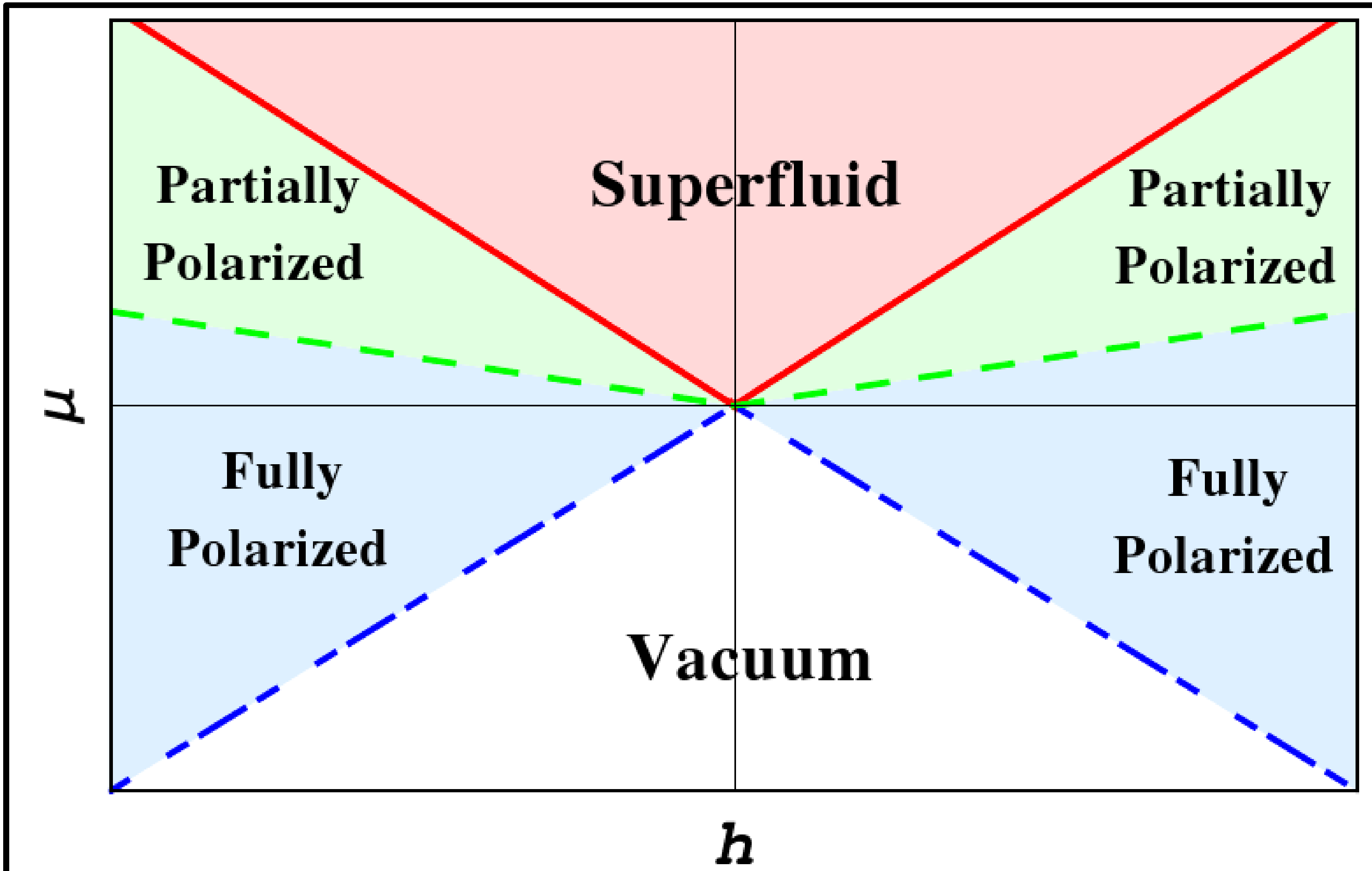
$B(\kappa)$ such that: $$\frac{E(1,\kappa)}{N} = \frac{E_N(\kappa)}{N} = \xi_N(\kappa)\frac{3}{5}\frac{\hbar^2}{4m_\kappa}(6\pi^2 n_N)^{2/3}$$

Note: Recently first QMC results for $\kappa = 6.5$ and $1/6.5$

[A. Gezerlis, S. Gandolfi, K. E. Schmidt, J. Carlson, arXiv:0901:3148]

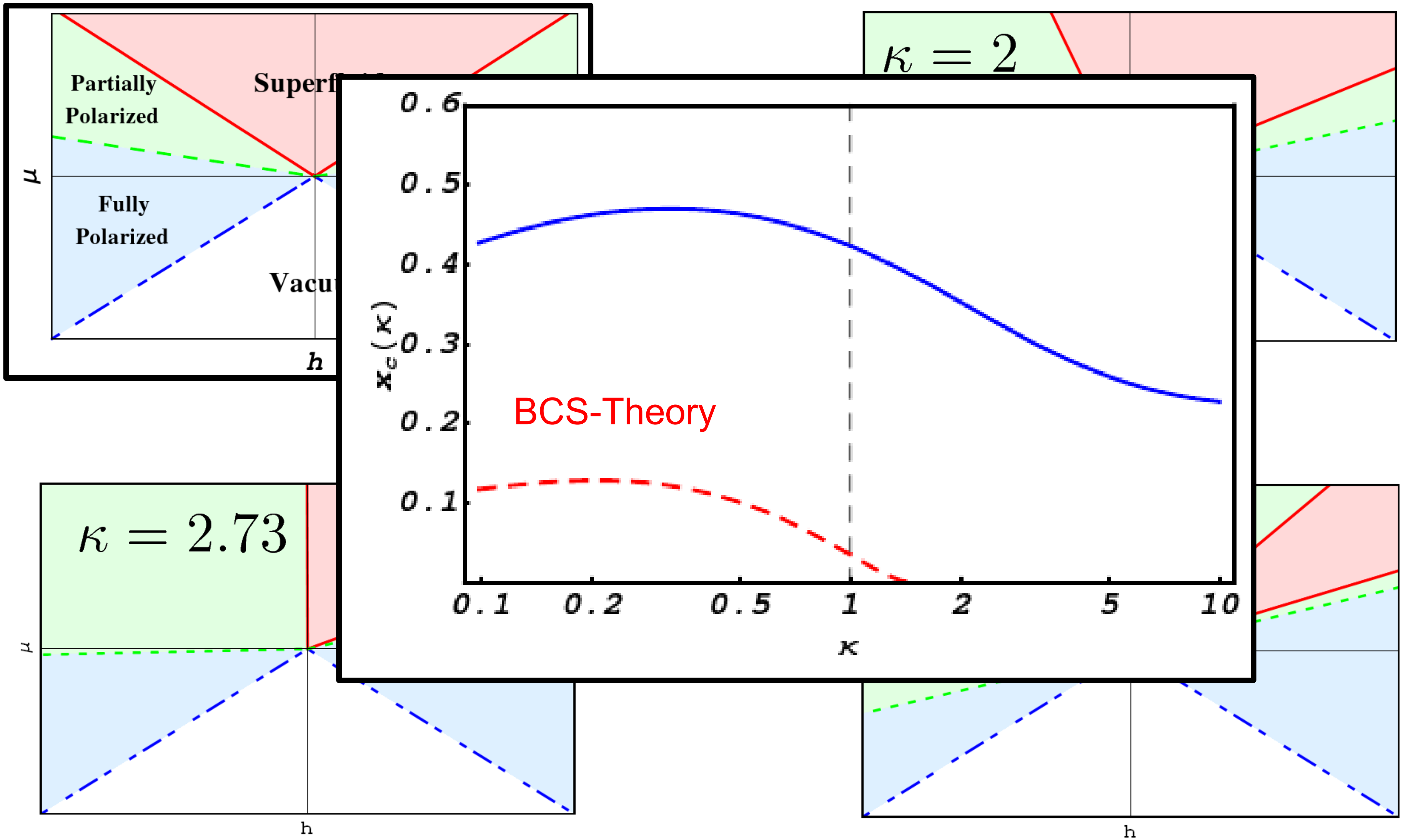
Fermi mixtures

Equal Mass Case

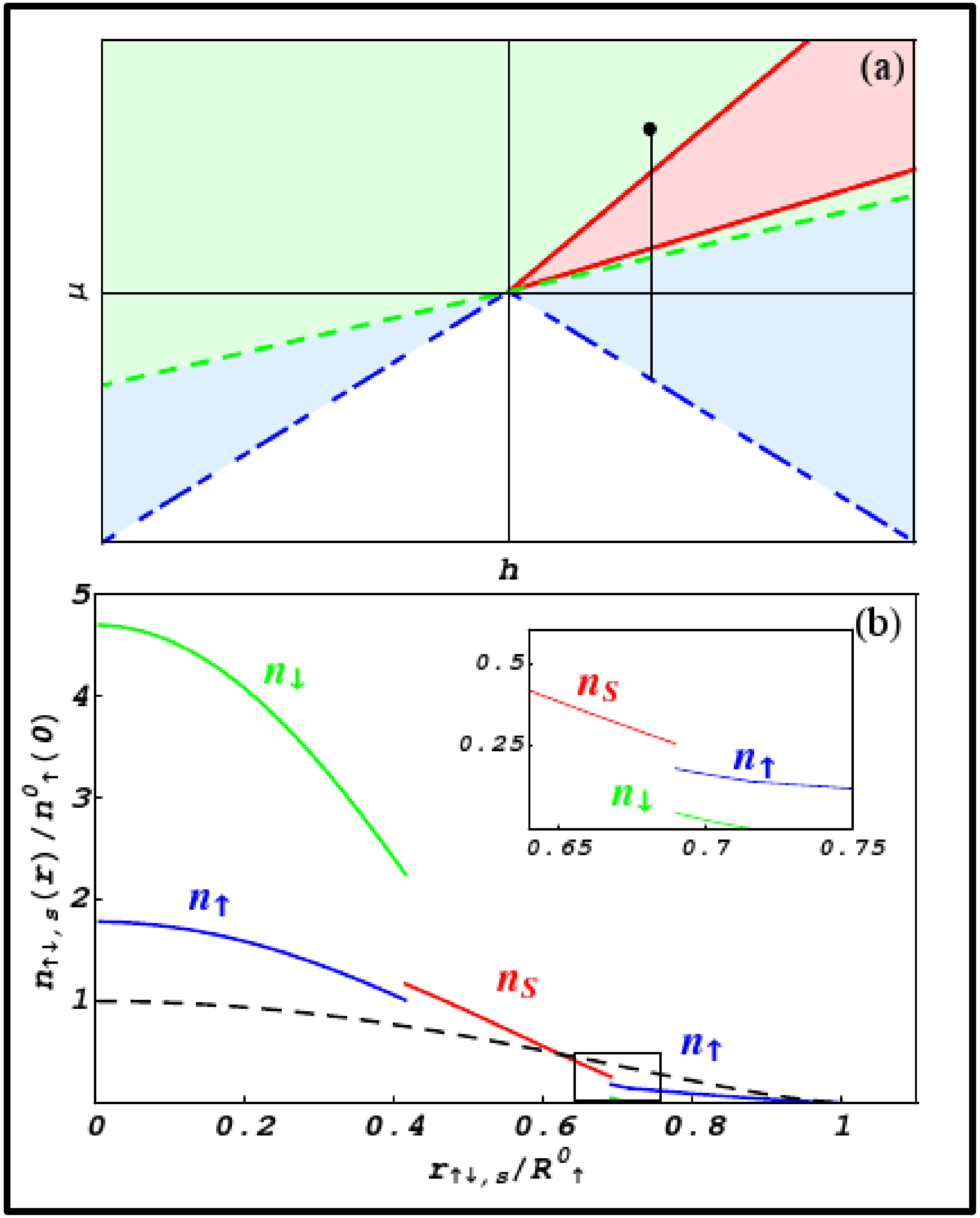


Fermi mixtures: Phase Diagram

Unequal Mass Case



Fermi mixtures: LDA & Configurations



1) 3-Shell configuration:

A SF between two normal phases

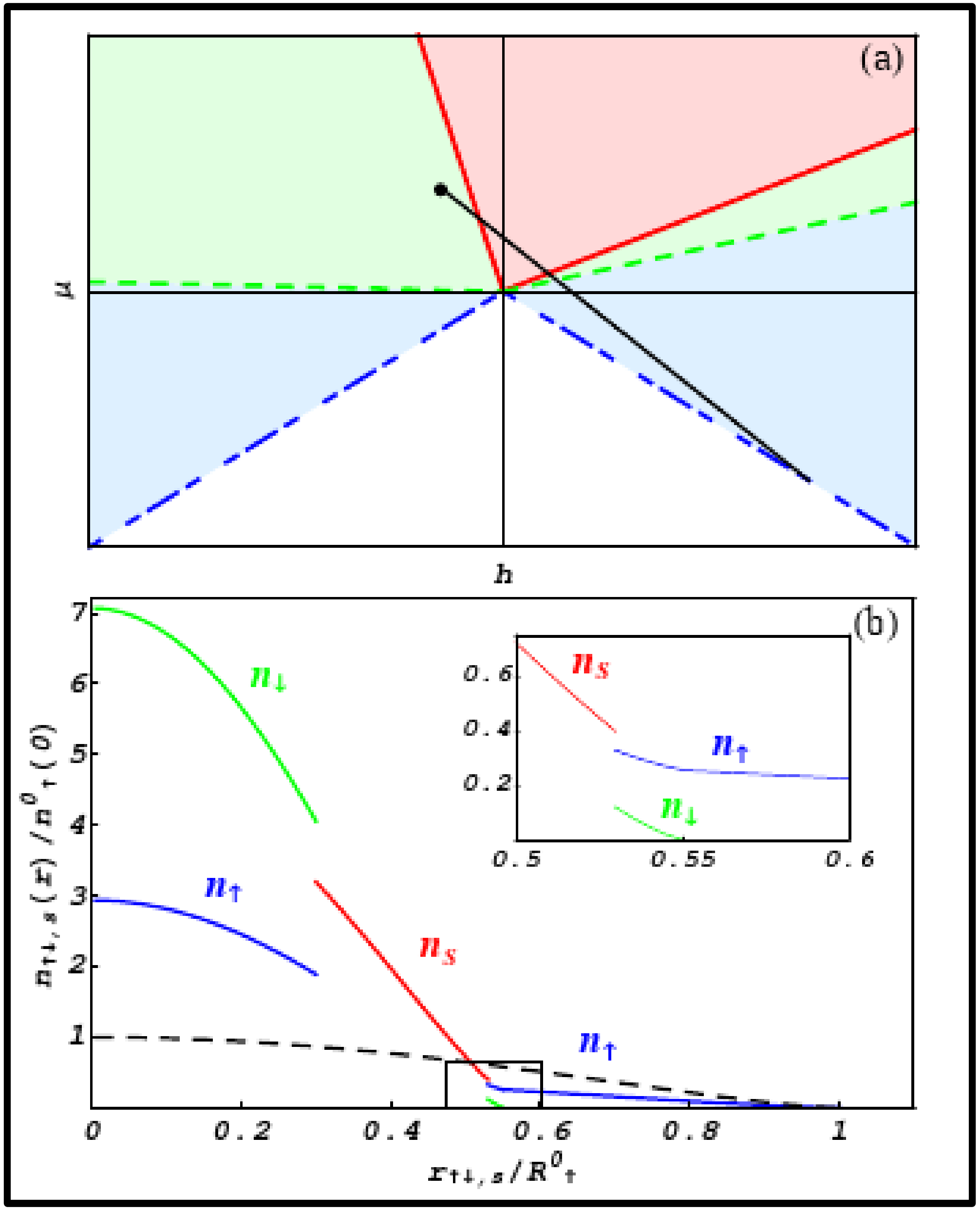
For equal trapping potential possible only if κ or 1/κ > 2.73

In Fig: ^{6}Li-^{40}K with P= -0.13

$$n_\downarrow(H) = 1.92 n_S$$
$$n_\uparrow(H) = 0.86 n_S$$
$$n_\downarrow(L) = 0.17 n_S$$
$$n_\downarrow(L) = 0.71 n_S$$

[Found also within BCS-meanfield approaches: critical mass ratio = 3.8]

Fermi mixtures: LDA & Phase Separation

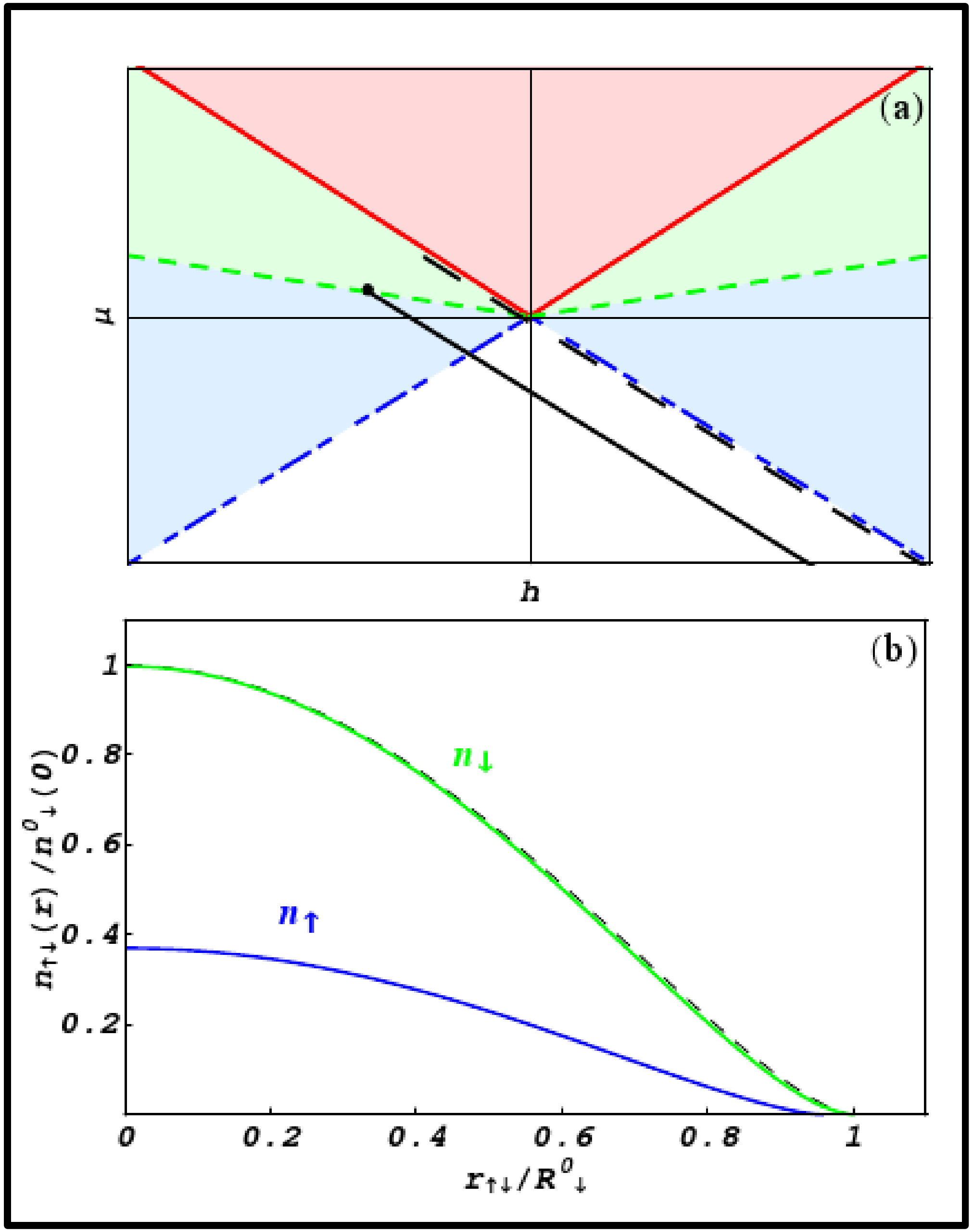


2) Trapping Anisotropy

$$\mu_\sigma = \mu_\sigma^0 - V_\sigma(x) = \mu_\sigma^0 - \frac{1}{2}\alpha_\sigma r^2$$

In Fig: κ=2.2, P=0, $\alpha_\downarrow/\alpha_\uparrow = 8$

Fermi mixtures: LDA & Phase Separation



2) No-Trapping for minority component

In Fig: κ=1, P= -0.5, $\alpha_\uparrow = 0$

For P close to 1:

$$\mu_\downarrow^0 = \frac{\hbar^2}{2m}[6\pi^2 n_\downarrow(\mathbf{r})]^{2/3} + V_\downarrow(\mathbf{r})$$

$$\mu_\uparrow^{0\prime} = \frac{\hbar^2}{2m^*}[6\pi^2 n_\uparrow(\mathbf{r})]^{2/3} + V_\uparrow'(\mathbf{r})$$

$$V_\uparrow'(\mathbf{r}) = V_\uparrow(\mathbf{r}) + \tfrac{3}{5} A V_\downarrow(\mathbf{r})$$

induced trapping

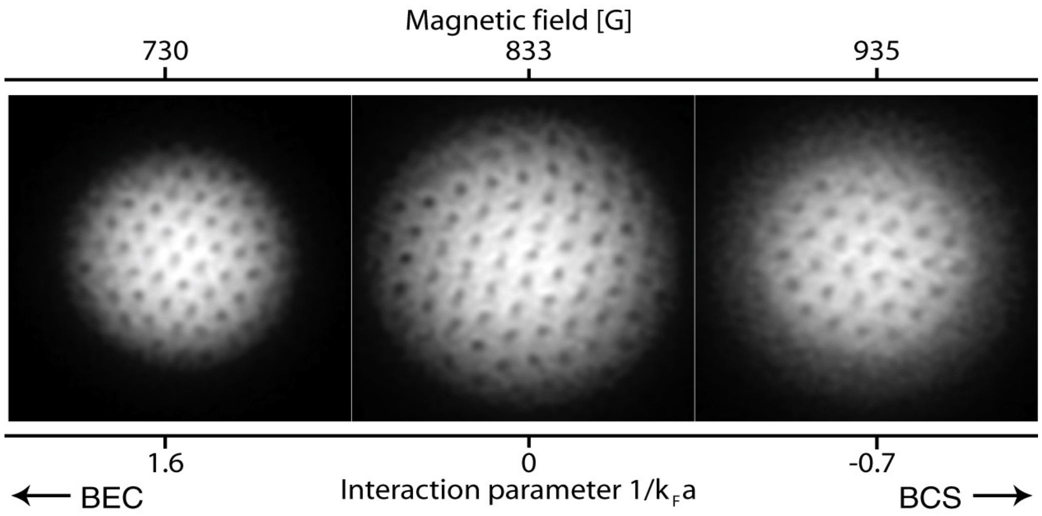
[Not possible within BCS-meanfield approaches]

Destroying superfluidity by rotation

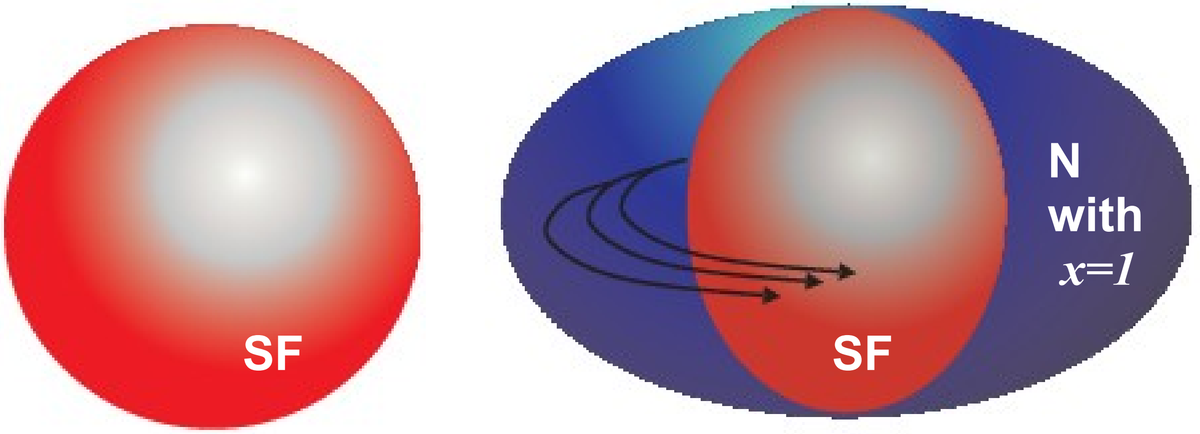
What does it happen if we "apply a rotation / rotate" to the system?

Already seen: **Vortices**

the superfluid lower its energy by allowing some rotation in the form of vorticity – BUT topological defects, energy barrier



The normal part can rotate...
why not ***phase separating in order to minimize the energy***?
A normal phase with concentration
x=1

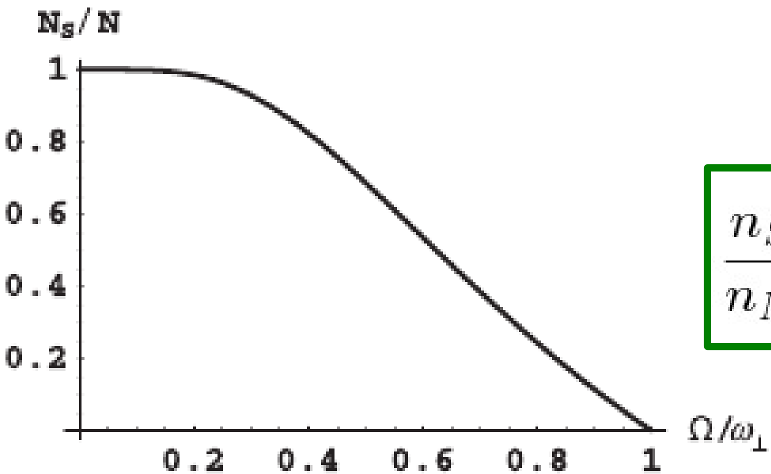


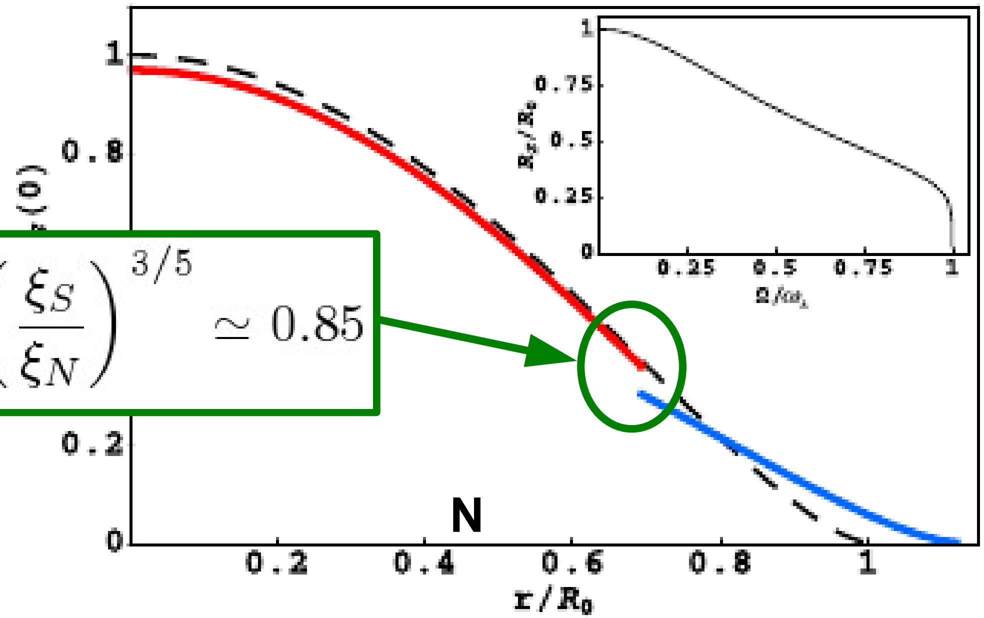
Destroying superfluidity by rotation

Normal phase with concentration ***x=1***: **Strongly interacting Landau-Fermi Liquid**

$$\epsilon_S = \xi_S \frac{3}{5}\frac{\hbar^2}{2m}(6\pi^2 n)^{5/3} \quad < \quad \epsilon_N = \xi_N \frac{3}{5}\frac{\hbar^2}{2m}(6\pi^2 n)^{5/3}$$

BUT, **normal phase gains energy in the rotating frame** $-m\Omega(\mathbf{r}\times\mathbf{v})_z$





$$\frac{n_S}{n_N} = \left(\frac{\xi_S}{\xi_N}\right)^{3/5} \simeq 0.85$$

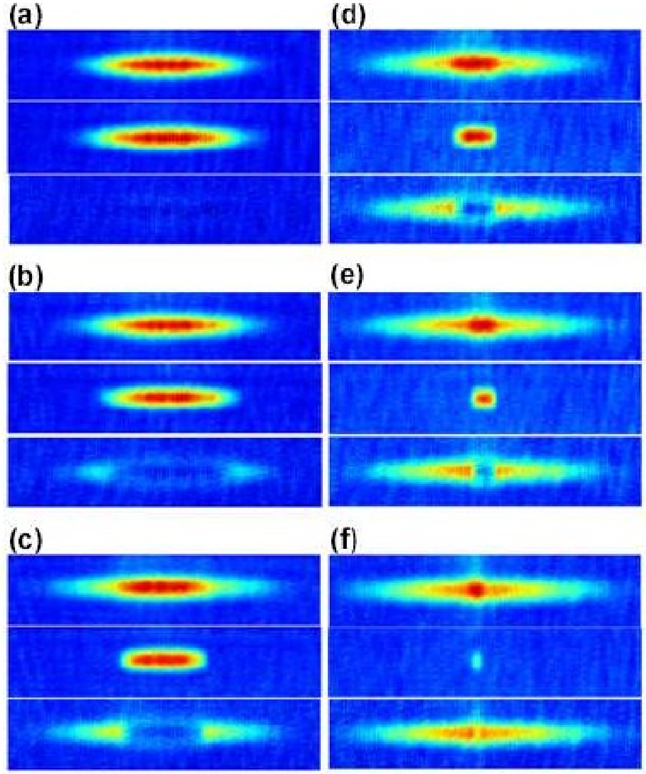
A Bogoliubov-De Gennes approach (quantitavely wrong at unitarity) shows the presence of a third phase at the interface: a superfluid with broken pairs.

[M. Urban, P. Schuck, PRA (2008)]

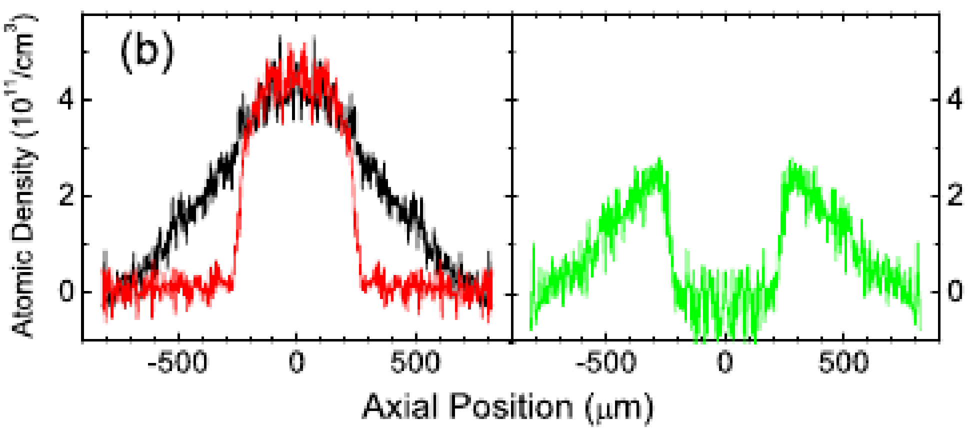

Grazie!
Thanks to
Lev Pitaevskii,
Martin Zwierlein,
Randy Hulet,
Wolfgang Ketterle
for very useful discussions
and
Yong Shin for the MIT data

Everything's clear?

- Randy Hulet's experiments with fewer atoms and large trap frequency ratio



$N=10^5$ and aspect ratio=30
vs.
$N=10^7$ and aspect ratio=7



- How a polaron become a molecule?
- Finite temperature polarons
- More exotic phases (polarized Superfluid, FFLO, Sarma...)
- Casimir-like (“vacuum” fluctuation) interaction
- More than 2 species (analogies with color superfluidity?)
- .....

Hawking radiation from acoustic black holes in atomic Bose-Einstein condensates

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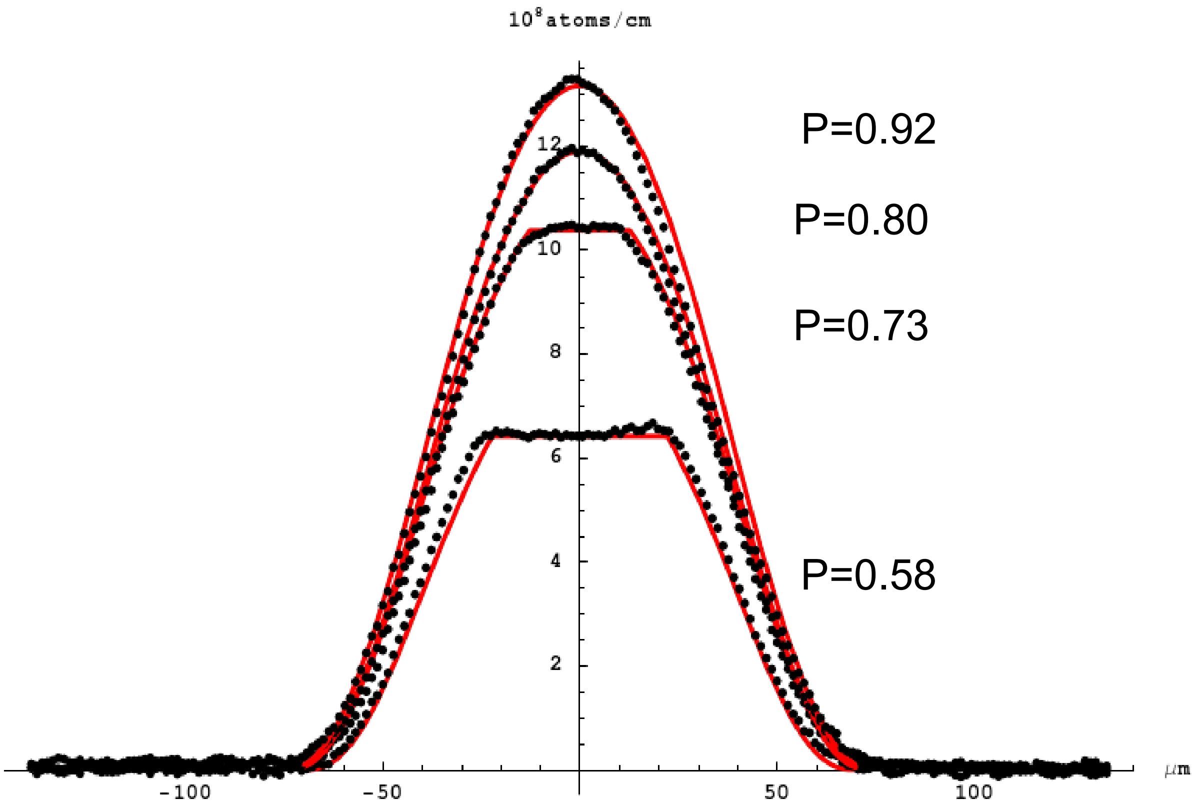
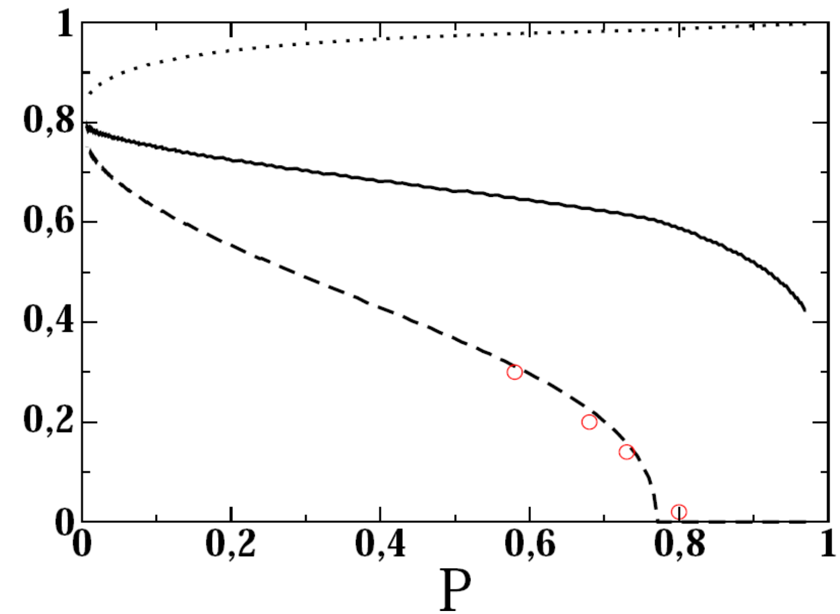
Normal phase of polarized Fermi gas at unitarity: TRAP

2) Density profiles and Radii





$$n_d(x) = 2\pi \int_{-\infty}^{\infty} d\rho \rho (n_\uparrow(x,\rho) - n_\downarrow(x,\rho))$$

Some Insight into the highly polarized Normal phase

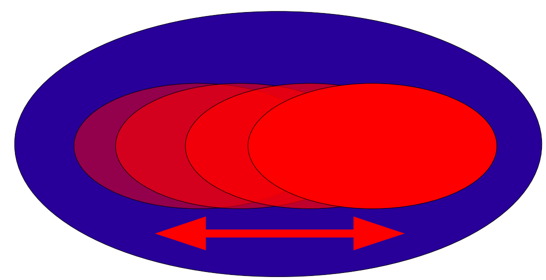
Dipole frequency at high polarization:

the majority component is not affected, the minority can be still think as a non-interacting gas but with *renormalized mass* and *trapping potential*

$$H_{sp} = \frac{p^2}{2m^*} + V(\mathbf{r})\left(1 + \frac{3}{5}A\right)$$

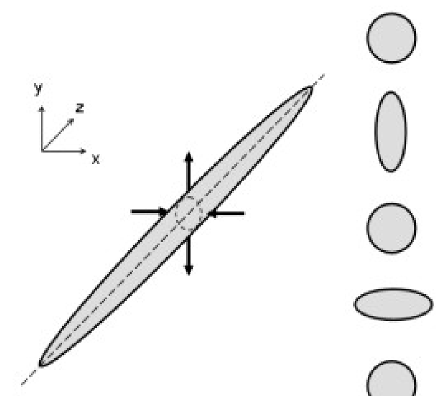
Spin-dipole mode

$$\omega_D^{(s)} = \omega_i \sqrt{\frac{m}{m^*}(1 + (3/5)A)} \simeq 1.26\omega_i$$



Spin-radial-quadrupole mode

$$\omega_Q^{(s)} = 2\omega_\perp \sqrt{\frac{m}{m^*}(1 + (3/5)A)}$$



Altmeyer *(IBK)*

Decaying time of the collective modes

We consider the *momentum relaxation* of an homogeneous highly polarized Fermi gas.

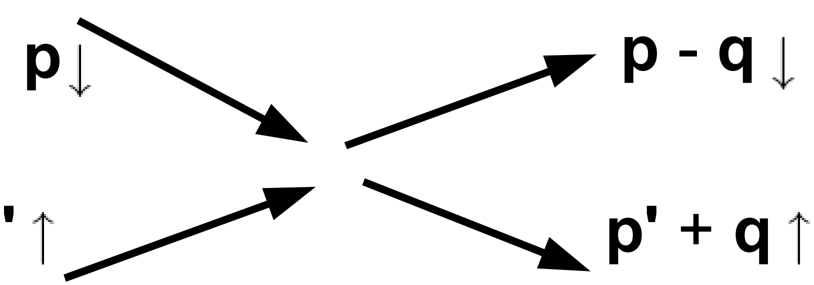
$$\frac{d\mathbf{P}_\downarrow}{dt} = -\frac{\mathbf{P}_\downarrow}{\tau_\mathbf{p}}$$

The minority component have a mean momentum **k** with respect to the majority one: total momentum per unit volume $\mathbf{P}_\downarrow = n_\downarrow \mathbf{k}$

$$\frac{d\mathbf{P}_\downarrow}{dt} = -2\pi \frac{|U|^2}{V^3} \sum_{\mathbf{p},\mathbf{p}',\mathbf{q}} \mathbf{p}[n_p n_{p'}(1 - n_{\mathbf{p}-\mathbf{q}})(1 - n_{\mathbf{p}'+\mathbf{q}}) - n_{\mathbf{p}-\mathbf{q}} n_{\mathbf{p}'+\mathbf{q}}(1 - n_p)(1 - n_{p'})]\delta(\epsilon_p + \epsilon_{p'} - \epsilon_{\mathbf{p}-\mathbf{q}} - \epsilon_{\mathbf{p}'+\mathbf{q}})$$

$$n_{\mathbf{p}\downarrow} = f[\beta(\epsilon_{p\downarrow} - \mathbf{p}\cdot\mathbf{v} - \mu_\downarrow)]$$

$$\epsilon_{p\downarrow} = p^2/2m^*_\downarrow$$



$$\epsilon_{p'\uparrow} = p'^2/2m_\uparrow$$

$$n_{p'\uparrow} = f[\beta(\epsilon_{p'\uparrow} - \mu_\uparrow)]$$

Decaying time of the collective modes

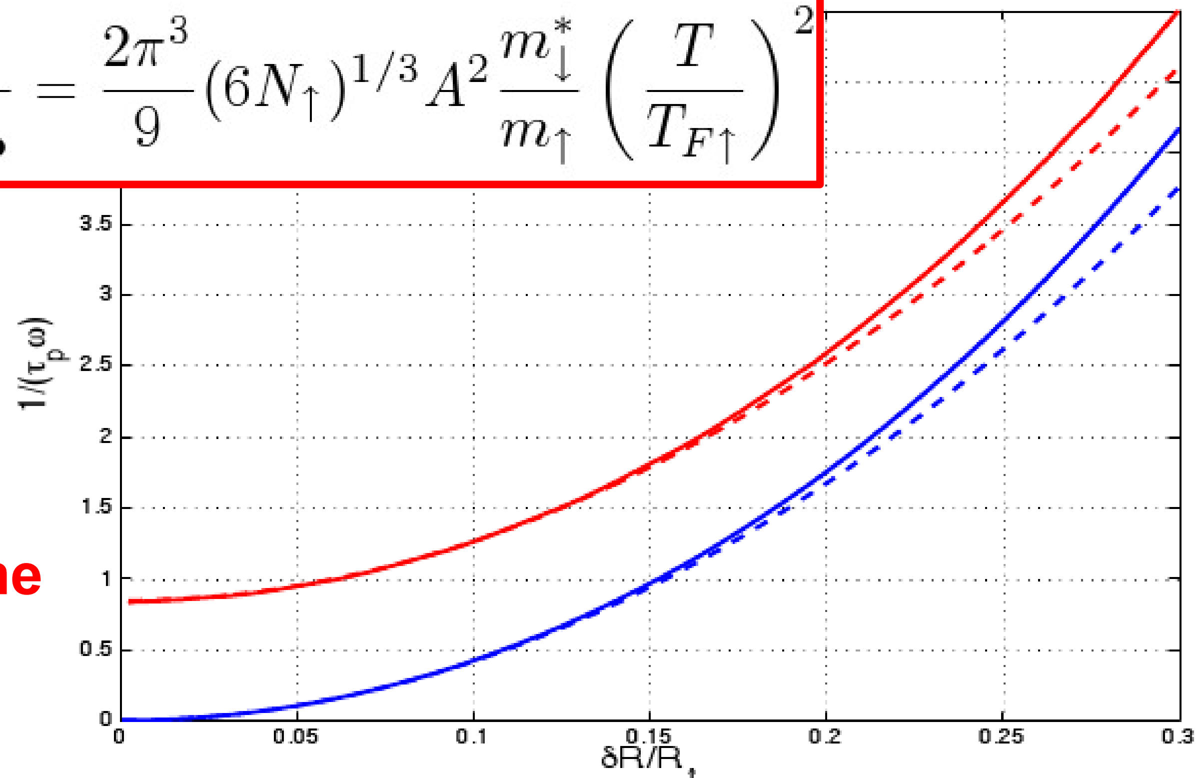
$$\begin{cases} \omega_D \tau_{\mathbf{p}} \gg 1 \\ \omega_D \tau_{\mathbf{p}} \ll 1 \end{cases}$$

Collisionless regime: possible to see the dipole mode

Hydrodynamic regime: the dipole mode overdamped

$$\delta R/R_\downarrow \ll T/T_{F\downarrow} : \frac{1}{\omega \tau_{\mathbf{p}}} = \frac{2\pi^3}{9} (6N_\uparrow)^{1/3} A^2 \frac{m_\downarrow^*}{m_\uparrow} \left(\frac{T}{T_{F\uparrow}} \right)^2$$

MIT regime



$$T = 0 : \frac{1}{\omega \tau_{\mathbf{p}}} = \frac{8\pi}{25} (6N_\uparrow)^{1/3} A^2 \frac{m_\downarrow^*}{m_\uparrow} \left(\frac{T_{F\downarrow}}{T_{F\uparrow}} \right)^2 \left(\frac{\delta R}{R_\uparrow} \right)^2$$