D.-H. Kim, P. Törmä, and J.-P. Martikainen, arXiv:0901.4769, PRL, accepted.

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Induced Interactions for Ultra-Cold Fermi Gases in Optical Lattices

Dong-Hee Kim[1], Päivi Törmä[1], Jani-Petri Martikainen[2]

[1]Department of Applied Physics, Helsinki University of Technology, Finland

[2]NORDITA, Sweden

What we want to study:

Superfluid transition of Fermi gases (Two component) (Attractive interaction)

in Optical Lattices (3D, 2D, …)

within the Mean-Field Theory (weak coupling regime)

beyond BCS predictions (many-body effect)

Medium-Induced Interaction

: Gorkov—Melik-Barkhudarov (GMB) correction

Medium effect in FREE gases

BCS result in vacuum :

$$k_B T_c^{(0)} = \frac{8\gamma}{\pi e^2} \epsilon_F \exp\left(-\frac{\pi}{2k_F|a|}\right)$$

$$U_0 = \frac{4\pi\hbar}{m} a$$

Sá de Melo et al., PRL **71**, 3202 (1993).

Stoof et al., PRL **76**, 10 (1996).

Correction due to a Medium effect:

Gorkov—Melik-Barkhudarov (GMB) correction

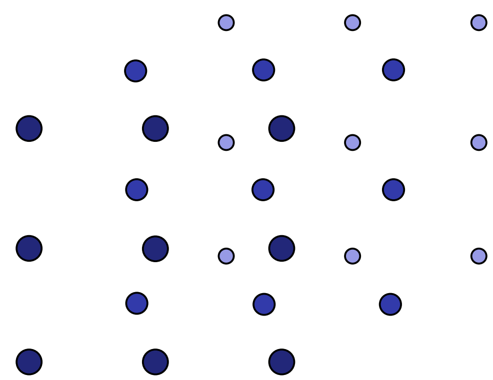
$$\frac{T_c^{(0)}}{T_c} = (4e)^{1/3} \approx 2.22$$

: 2nd order correction to the interaction.

Gorgov and Melik-Barkhudarov, Sov. Phys. JETP **13**, 1018 (1961).

Heiselberg et al., PRL **85**, 2418 (2000).

Superfluidity in optical lattices?



Indirect experimental evidence:

J. K. Chin, D. E. Miller, Y. Liu, C. Stan, W. Setiawan,
C. Sanner, K. Xu, and W. Ketterle, Nature **443**, 961 (2006):

~ long range order?

Q. Medium effects in optical lattices?

The difference could be even more dramatic.

$$T_c \propto \exp\left(-\frac{\pi}{2k_F|a|}\right)$$: correction changes the prefactor.

$$k_F|a| \to k_F|a|(1+\delta k_F|a|)$$: 2nd order correction to interaction

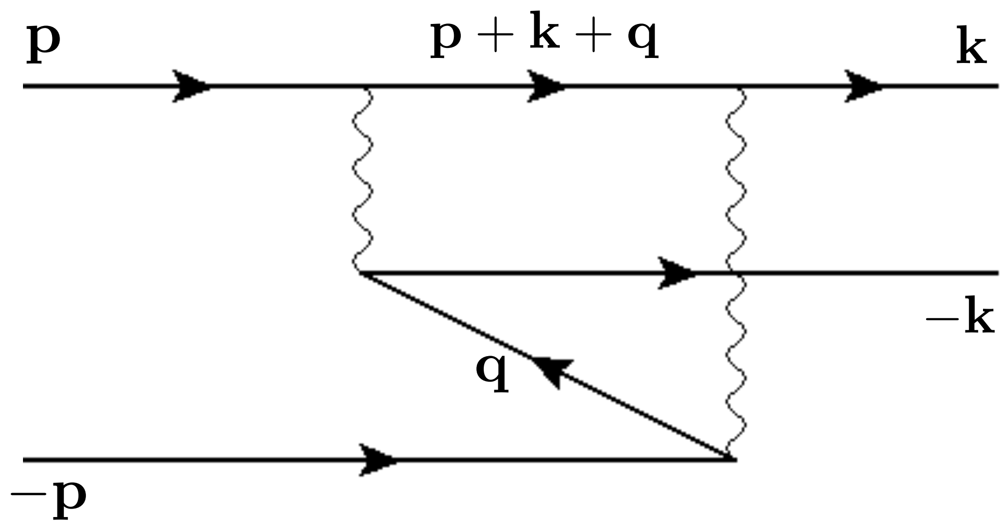
$$T_c \to e^{\delta} T_c$$: varies *sensitively* with the correction

Gorkov–Melik-Barkhudarov correction

Physical interpretation

H. Heiselberg, C. H. Pethick, H. Smith, and L. Viverit, PRL **85**, 2418 (2000).

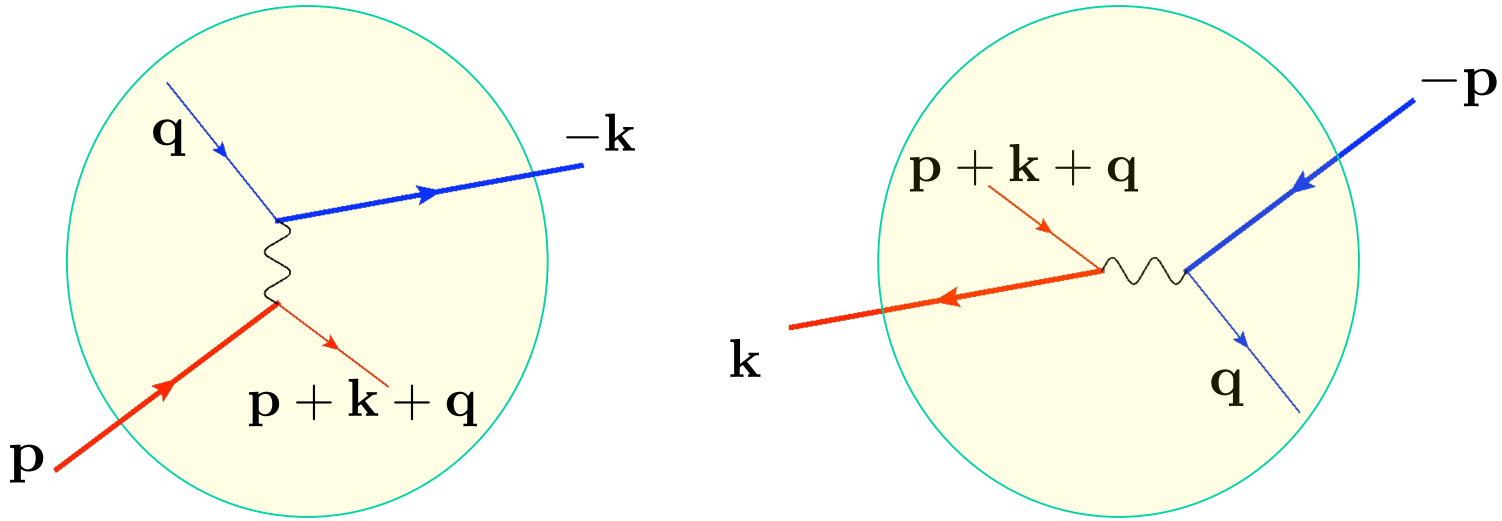
C. J. Pethick and H. Smith 2008.



Two fermions in the medium
≠ two fermions *in vacuo*.

"medium-induced interaction"

Scattering in the medium:



Induced Interaction calculation

Induced interaction:

$$U_{\rm ind}(\mathbf{p},\mathbf{k}) = -U_0^2 \int \frac{d\mathbf{q}}{(2\pi)^D} \frac{f_{\uparrow,\mathbf{p}+\mathbf{k}+\mathbf{q}} - f_{\downarrow,\mathbf{q}}}{\xi_\uparrow(\mathbf{p}+\mathbf{k}+\mathbf{q}) - \xi_\downarrow(\mathbf{q})}$$

$$\xi_\sigma(\mathbf{k}) = \epsilon_\sigma(\mathbf{k}) - \mu_\sigma$$

$$f_{\sigma,\mathbf{k}} = \frac{1}{1+\exp\left[\beta\xi_\sigma(\mathbf{k})\right]}$$

Effective interaction:

Assume only the momenta at the Fermi surfaces contribute:

$$\langle U_{\rm ind}\rangle = \frac{1}{|S_\uparrow||S_\downarrow|}\int_{S_\uparrow} dS_{\mathbf{p}} \int_{S_\downarrow} dS_{\mathbf{k}} U_{\rm ind}(\mathbf{p},\mathbf{k})$$

$$U_{\rm eff} = U_0 + \langle U_{\rm ind}\rangle$$

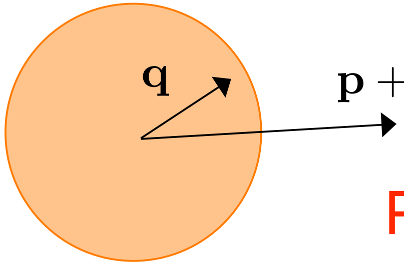
: effective interaction replacing U_0

TWO INTERESTING FEATURES

Induced Interaction: continued

1. Lindhard function (static) – susceptibility for non-interacting case

$$U_{\text{ind}}(\mathbf{p},\mathbf{k}) = -U_0^2 \int \frac{d\mathbf{q}}{(2\pi)^D} \frac{f_{\uparrow,\mathbf{p+k+q}} - f_{\downarrow,\mathbf{q}}}{\xi_\uparrow(\mathbf{p}+\mathbf{k}+\mathbf{q}) - \xi_\downarrow(\mathbf{q})}$$



POSITIVE : screens the negative interatomic interaction U_0

suppresses T_C or the order parameter.

2. Perfect screening occurs at large interactions.

$$U_{\text{eff}} = U_c + \langle U_{\text{ind}} \rangle = 0$$

- parameter space of validity

At this large induced interaction, a RPA charge susceptibility can diverge.

$$\chi_{RPA} = \frac{\chi_0}{1 + U_0\chi_0} \rightarrow \infty$$

- connection to other physics.

(Charge density waves)

Mean-Field calculations

Hubbard Hamiltonian:

$$\mathcal{H} = -\sum_{\sigma\alpha}\sum_{i_\alpha} t_{\sigma\alpha}(\hat{c}^\dagger_{\sigma,i_\alpha+1}\hat{c}_{\sigma,i_\alpha} + h.c.) + U\sum_{\mathbf{i}}\hat{n}_{\uparrow\mathbf{i}}\hat{n}_{\downarrow\mathbf{i}} - \mu\sum_{\sigma,\mathbf{i}}\hat{n}_{\sigma\mathbf{i}}$$

$\alpha \in \{x, y, z\}$

Mean-Field approximation: $\Delta = U\langle\hat{c}^\dagger_{\uparrow\mathbf{i}}\hat{c}^\dagger_{\downarrow\mathbf{i}}\rangle$

$$\mathcal{H}_{MF} = \sum_k \left(\xi_{\uparrow k}\hat{c}^\dagger_{\uparrow k}\hat{c}_{\uparrow k} + \xi_{\downarrow k}\hat{c}^\dagger_{\downarrow k}\hat{c}_{\downarrow k} + \Delta\hat{c}^\dagger_{\uparrow k}\hat{c}^\dagger_{\downarrow -k} + \Delta\hat{c}_{\downarrow -k}\hat{c}_{\uparrow k}\right) - \frac{\Delta^2}{U}$$

$$\epsilon_\sigma(\mathbf{k}) = 2\sum_\alpha t_{\sigma\alpha}\left[1 - \cos(k_\alpha)\right]$$

Energy dispersion of non-interacting system

$U \rightarrow U_0$: the usual BCS calculations

$U \rightarrow U_{\mathrm{eff}}$: the effective interaction with the GMB correction.

The zero temperature order parameter Δ is calculated.

Mean-Field calculations

Minimization of Free Energy:

T.K. Koponen, T. Paananen, J.-P. Martikainen, and P. Törmä, PRL 99, 120403 (2007).
T.K. Koponen, T. Paananen, J.-P. Martikainen, M.R. Bakhtiari, and P. Törmä, NJP 10, 045104 (2008).

$$\Omega = -\frac{\Delta^2}{U} + \int \frac{d\mathbf{k}}{(2\pi)^D} \left(\xi_\downarrow(-\mathbf{k}) + E_-(\mathbf{k}) - \frac{1}{\beta} \ln(1 + e^{-\beta E_+(\mathbf{k})})(1 + e^{\beta E_-(\mathbf{k})}) \right)$$

Effective interaction $U_{\text{eff}} = U_0 + \langle U_{\text{ind}} \rangle$

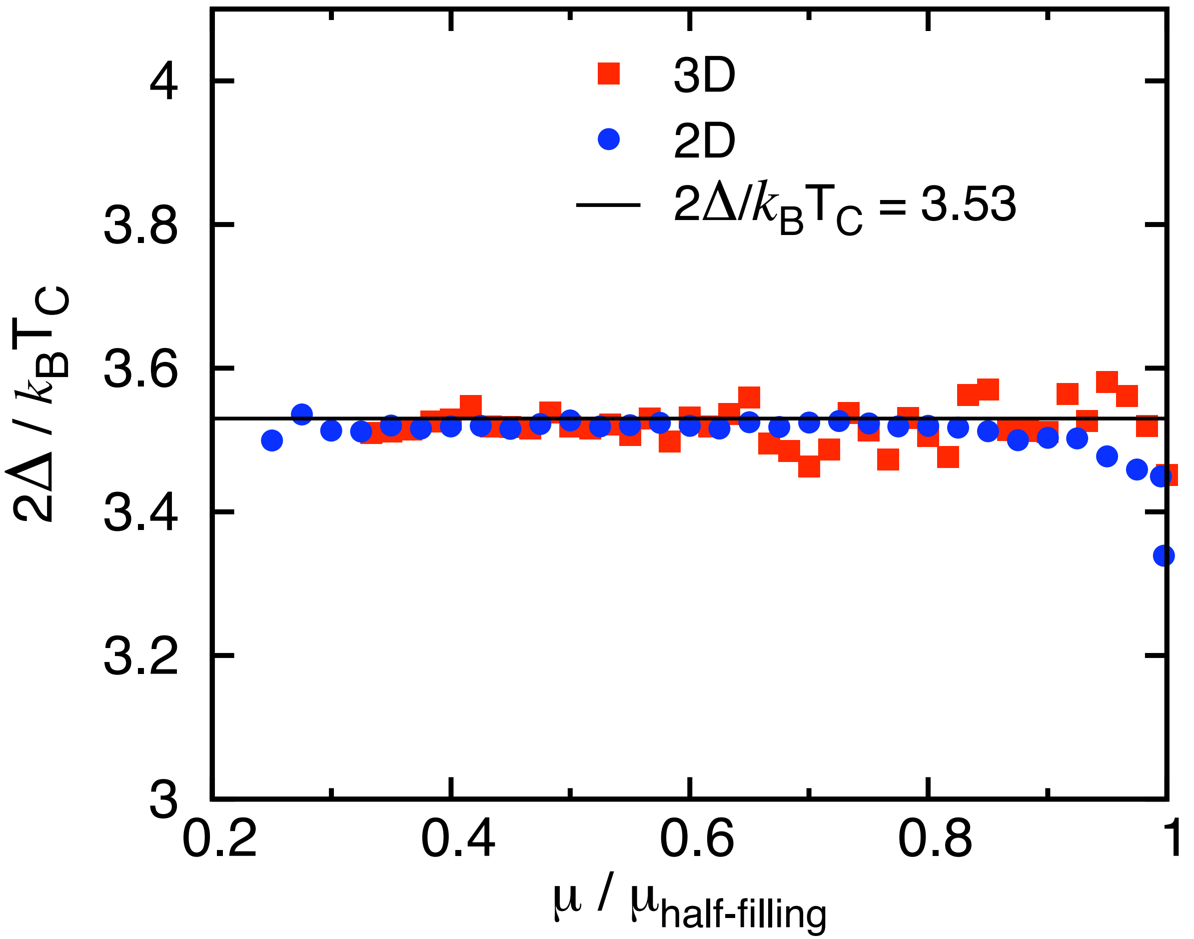
$$E_\pm(\mathbf{k}) = \frac{\xi_\uparrow(\mathbf{k}) - \xi_\downarrow(-\mathbf{k})}{2} \pm \sqrt{\left(\frac{\xi_\uparrow(\mathbf{k}) + \xi_\downarrow(-\mathbf{k})}{2}\right)^2 + \Delta^2}$$

$\mu_\uparrow = \mu_\downarrow$: Perfectly balanced gases are considered.

Numerical Integrations: Monte Carlo algorithm is used.

Δ versus T_C

The zero temperature order parameter Δ is calculated instead of T_C.

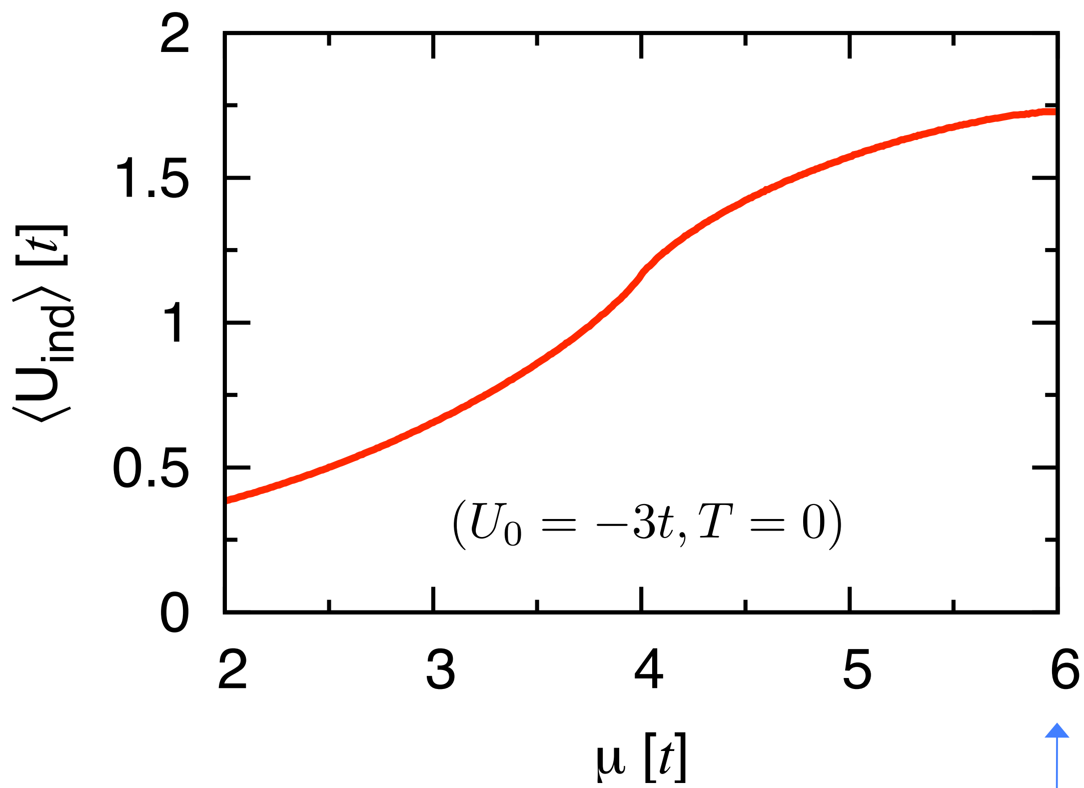


Both are equivalent in the weak coupling regime.

3D Lattices: Induced Interaction

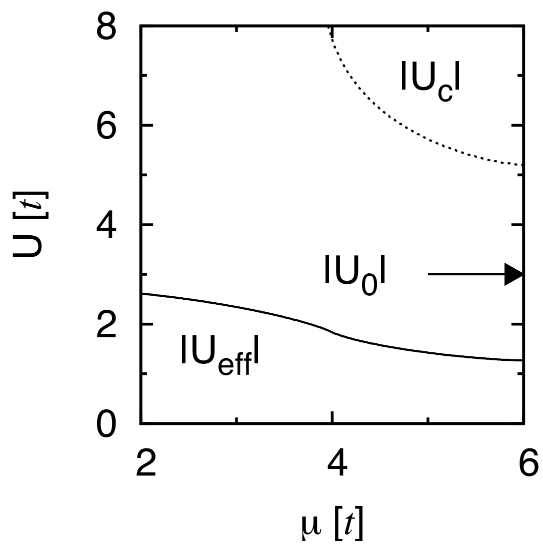
Induced interaction strength increases as chemical potential increases.

$$U_{\text{ind}}(\mathbf{p},\mathbf{k}) = -U_0^2 \int \frac{d\mathbf{q}}{(2\pi)^D} \frac{f_{\uparrow,\mathbf{p}+\mathbf{k}+\mathbf{q}} - f_{\downarrow,\mathbf{q}}}{\xi_\uparrow(\mathbf{p}+\mathbf{k}+\mathbf{q}) - \xi_\downarrow(\mathbf{q})}$$



Half-filling

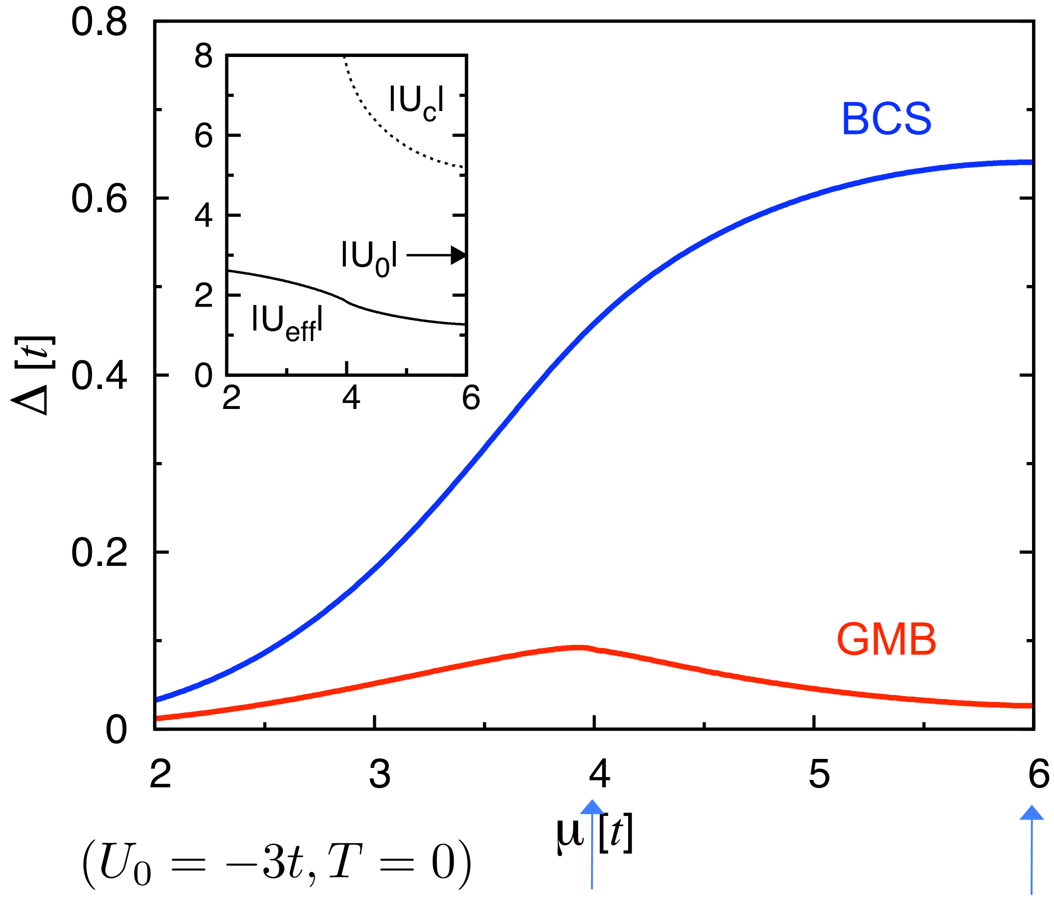
U_{eff} decreases as μ increases.



$|U_0| < |U_c|$ for all μ.

3D Lattices: order parameter (T=0)

The order parameter is suppressed much beyond the factor 2.22.



$(U_0 = -3t, T = 0)$

3D van hove singularity

half filling

As the filling factor increases, correction effect becomes larger.

$$\frac{\Delta_{BCS}}{\Delta_{GMB}} \sim 2.22$$ at low density limit.

$$\frac{\Delta_{BCS}}{\Delta_{GMB}} \sim 5$$ at μ = 4t.

$$\frac{\Delta_{BCS}}{\Delta_{GMB}} \sim 25$$ at half filling.

cf.) 1/D correction at high dimension

[van Dongen, PRL 67, 757 (1991)]

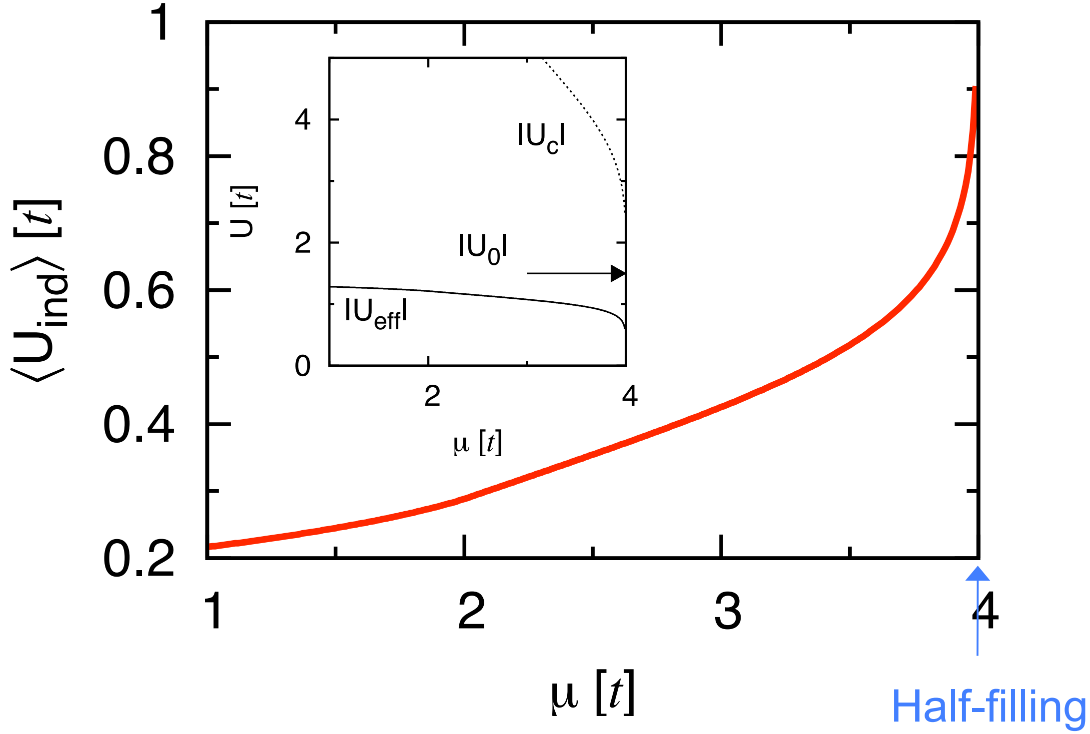
$$T_c^0/T_c \sim \mathcal{O}(1)$$

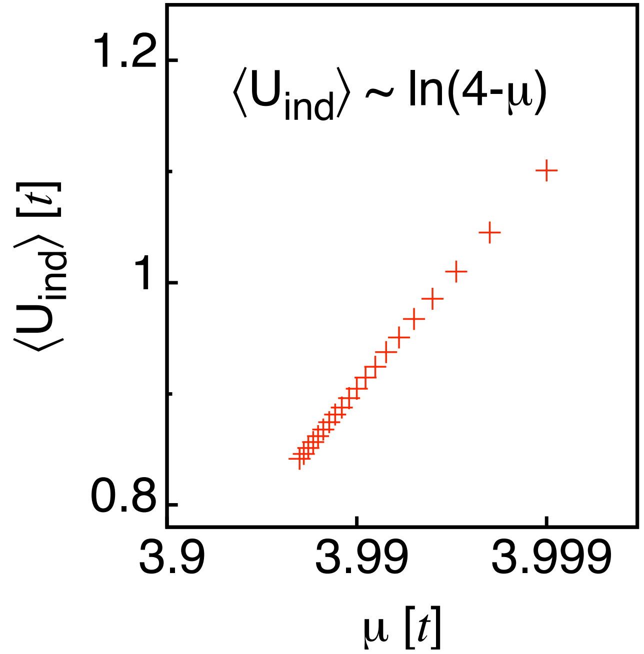
2D Lattices: induced interaction

Induced interaction strength increases as chemical potential increases, but …

… induced interaction diverges at half filling.

$$U_{\rm ind}(\mathbf{p},\mathbf{k}) = -U_0^2 \int \frac{d\mathbf{q}}{(2\pi)^D} \frac{f_{\uparrow,\mathbf{p}+\mathbf{k}+\mathbf{q}} - f_{\downarrow,\mathbf{q}}}{\xi_\uparrow(\mathbf{p}+\mathbf{k}+\mathbf{q}) - \xi_\downarrow(\mathbf{q})}$$

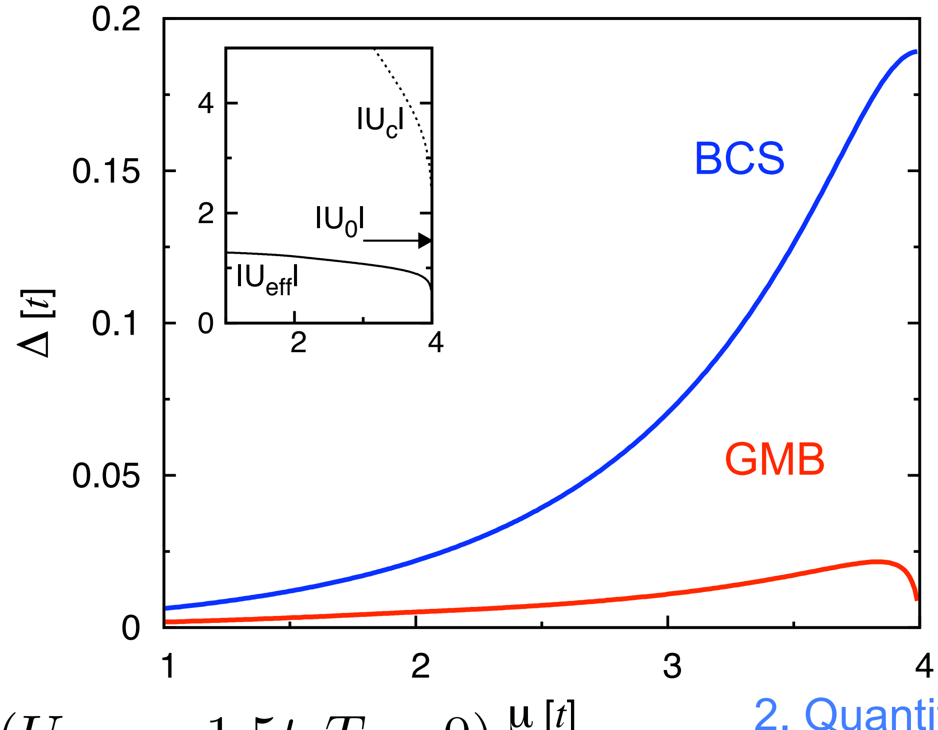




$(U_0 = -1.5t, T = 0)$

2D Lattices: order parameter (T=0)

As μ increases,
correction effect becomes larger.



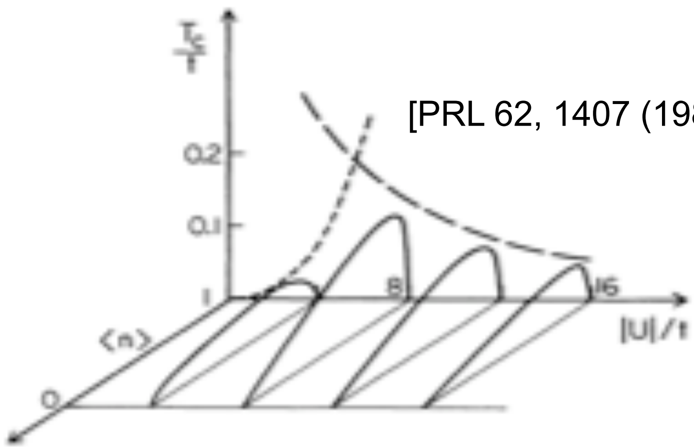
$(U_0 = -1.5t, T = 0)$

- Effect of GMB correction

$$\frac{\Delta_{BCS}}{\Delta_{GMB}} \sim 7 \qquad \text{at } \mu = 3t.$$

- Comparisons with QMC

1. Decrease near half filling



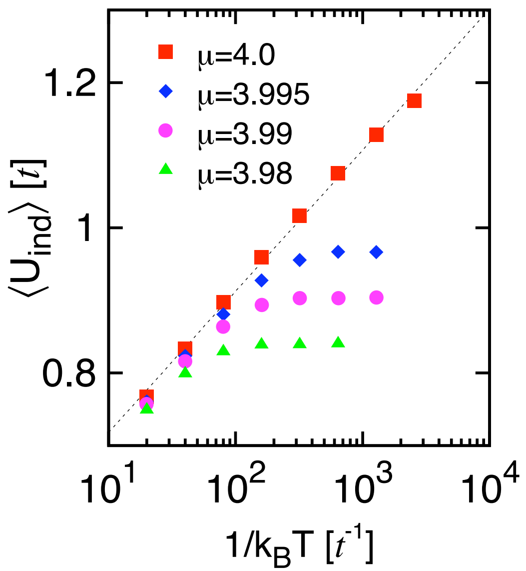
2. Quantitative comparison

At quarter filling and U_0=-4t, $k_B T_c \sim 0.05t$

[PRL 62, 1407 (1989); PRL 66, 946 (1991)]

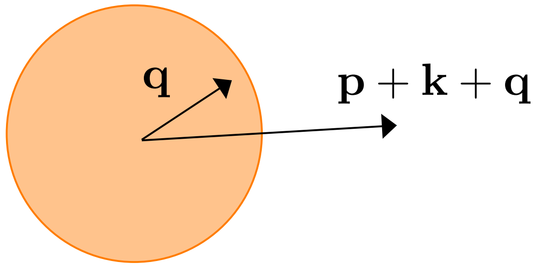
GMB correction: $k_B T_c \sim 0.02t$

2D Lattices : divergence at half filling



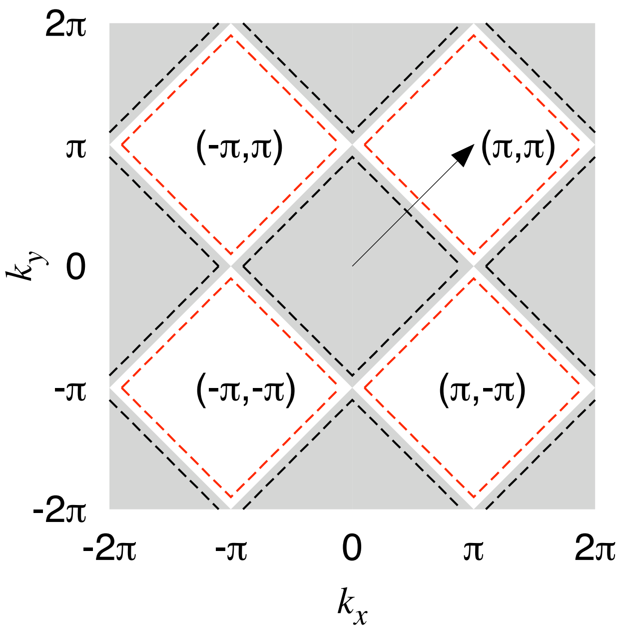
The induced interaction diverges at half filling at T=0.

$$U_{\rm ind}(\mathbf{p},\mathbf{k}) = -U_0^2 \int \frac{d\mathbf{q}}{(2\pi)^D} \frac{f_{\uparrow,\mathbf{p}+\mathbf{k}+\mathbf{q}} - f_{\downarrow,\mathbf{q}}}{\xi_\uparrow(\mathbf{p}+\mathbf{k}+\mathbf{q}) - \xi_\downarrow(\mathbf{q})}$$

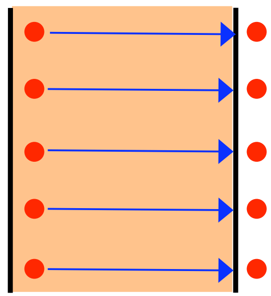


$\mathbf{p}+\mathbf{k}\sim 0$:constant

ex. 1D: $\mathbf{p}+\mathbf{k}\sim 2\mathbf{k}_F$



Fermi surface nesting:

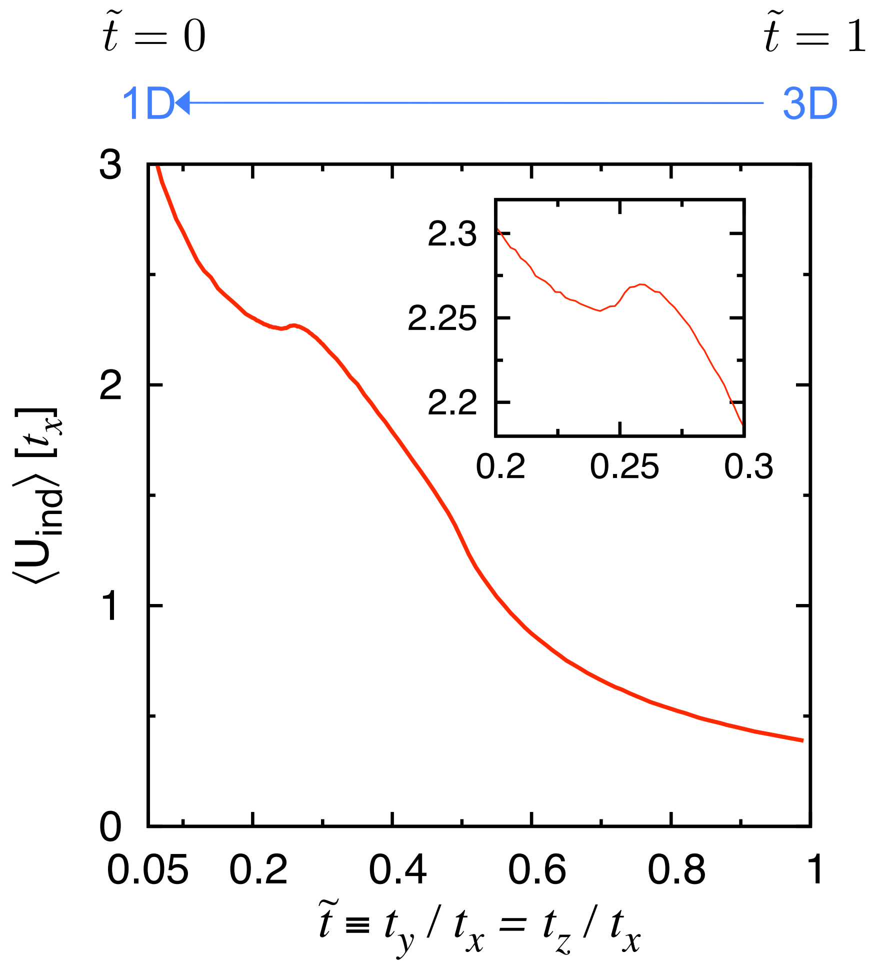


Divergence of the Lindhard function

⟶ signature of CDW

Superfluidity and CDW coexist in the ground state at half filling in 2D lattices.

Lattice Anisotropy: crossover from 3D to 1D



$$U_0 = -3t$$
$$\mu = 2t$$

Lattice anisotropy: $\tilde{t} = t_y / t_x = t_z / t_x$

1D limit: $t_{y,z} \to 0$

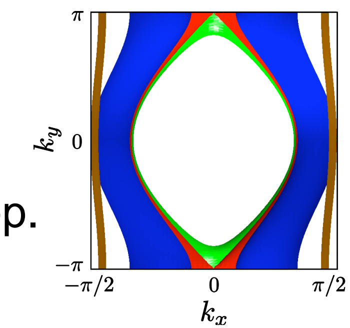
Induced Interaction increases as the lattice goes toward quasi 1D.

Kinks are found at two points of changes in Fermi surface shapes.

$\tilde{t} = 0.5$ The surface opens: van Hove singularity

$\tilde{t} = 0.25$

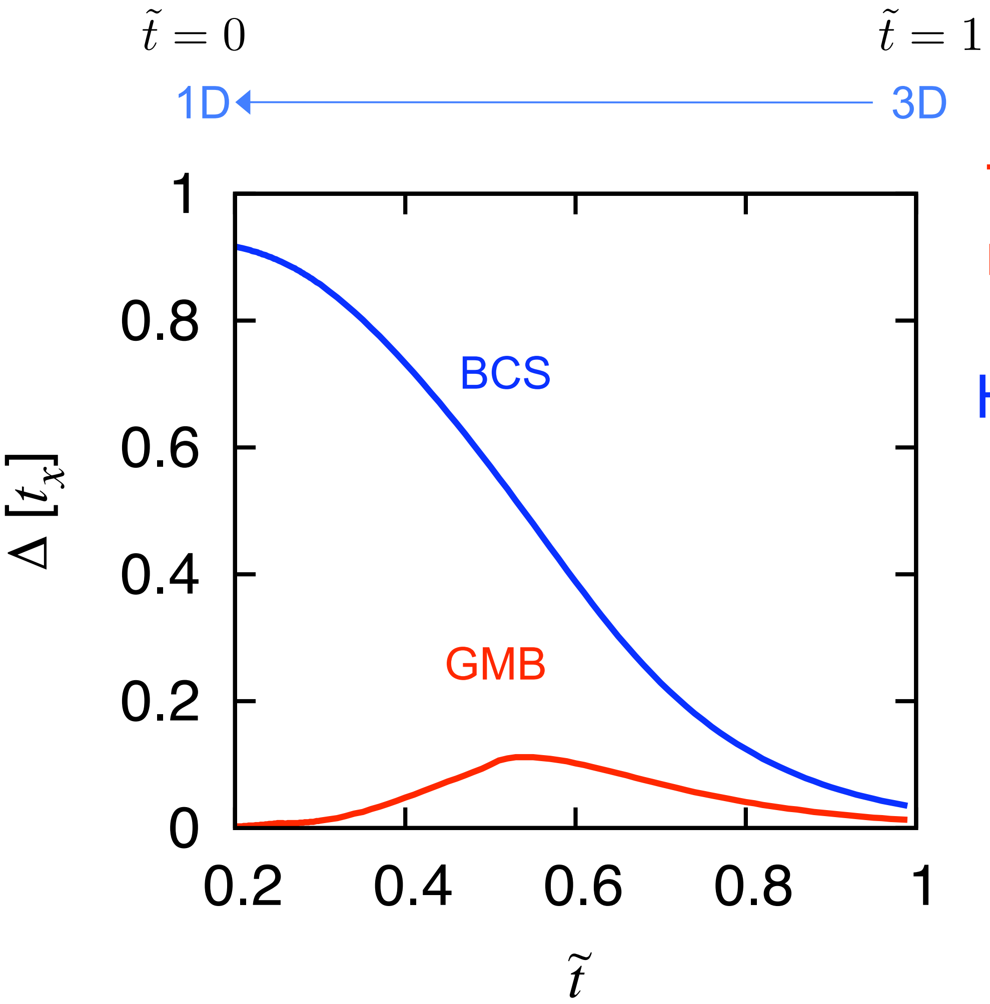
Quasi 1D shapes start to develop.



$\tilde{t} \to 0$: Fermi surface nesting occurs.

Correction diverges.

Lattice Anisotropy: crossover from 3D to 1D



$U_0 = -3t$

$\mu = 2t$

The order parameter decreases much beyond the BCS prediction.

However, the theory fails at 1D limit.

U_{ind} diverges in 1D lattices.

$U_{eff} = 0$ at some point: $\Delta \rightarrow 0$

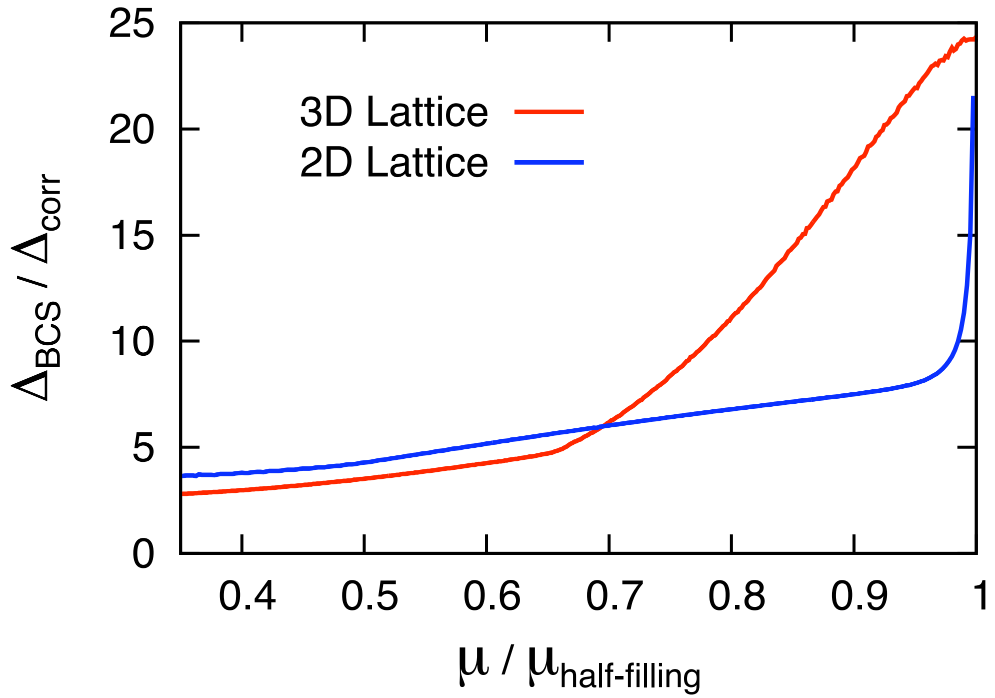
(incorrect!)

In quasi 1D lattices, finite gap exists.

(Larkin and Sak, PRL 1977; PRB 1978.)

Summary

D.-H. Kim, P. Törmä, and J.-P. Martikainen, arXiv:0901.4769, accepted in PRL.



$$\Delta_{corr} \ll \Delta_{BCS}$$

The presence of the optical lattice significantly enhances the effect of induced interactions on the BCS superfluidity.

The induced interaction correction extends the applicability of the mean-field calculations in lower dimensions.

- Predictions closer to QMC values in 2D lattices

- Divergence due to Fermi surface nesting : connection to different physics