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Superfluidity in Bose-Fermi Mixtures

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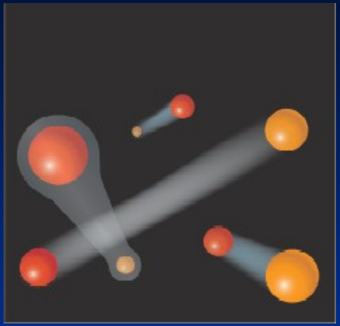
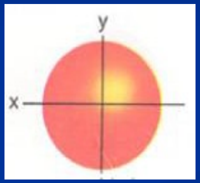
Outline

- Motivation.
- Fermionic superfluidity in Bose-Fermi mixtures.
- Exotic superfluidity in nature.
- Dipolar Bose-Fermi Mixtures.
- Vortex Excitations.
- Future Work.

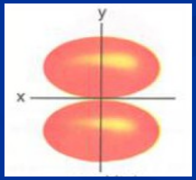
Motivation



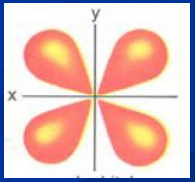
- Non-standard pairing symmetry.
- High enough interaction strength.
- Possible breaking of time-reversal symmetry.
- Excitations obeying non-abelian statistics.



s-wave



p-wave



d-wave

Bose-Fermi Mixture

- Different kind of phenomena:

  Pairing of fermions mediated by bosons.

  Novel pairing between bosons and fermions.

  Different exotic quantum phases.

Fermionic Superfluidity in Bose-Fermi Mixture

- Analogous to phonon-mediated superfluidity in metals.
- Induced interaction depends on momentum.
- Stability of the mixed state : $\dfrac{g_{bf}^2 N_0}{g_{bb}} < 1$
- Interaction strength for p-wave ~ 0.1.

$$\mathrm{T_c} \sim \mathrm{T_f} \exp\left(-10\right) \sim 10^{-5}\,\mathrm{T_f}$$

Hard to observe experimentally with ultracold atoms.

Unconventional pairing in nature

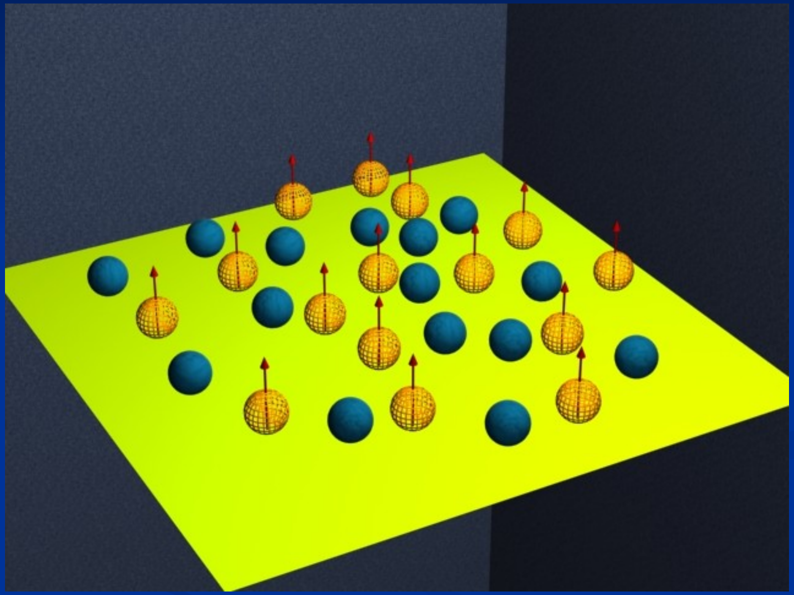
- Various phases in He^3.
- Possible p-wave pairing in Sr_2RuO_4.
- P-wave pairing in ultracold atoms using Feshbach resonances.
- D-wave pairing in Cuprates.

Higher order pairing

- F-wave pairing in a honeycomb optical lattice.
- Superconductivity in $Na_xCoO_2 \cdot yH_2O$.
- Hubbard models with disconnected Fermi surfaces.

Our system

- Dipolar bosons.
- Single component fermions.
- Trapped along the z direction and homogeneous along the x-y direction.

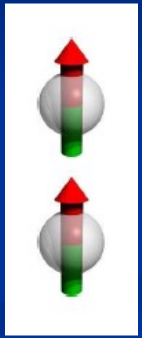


Experimental possibility

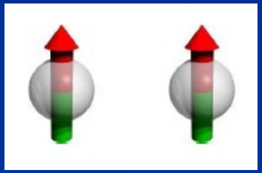
- Ultracold samples of Cr^{52} with s-wave scattering length tuned to zero-Magnetic dipole moment.
- Quantum degenerate samples of Heteronuclear bosonic molecules-Electric dipole moment.
- Fermionic atoms like, K^{40}, Li^{6}, Cr^{53} etc.

Dipolar Particles

- Anisotropic interaction.
- Long-ranged : $g_{dd} \dfrac{\hat{\mu}_1.\hat{\mu}_2 - 3(\hat{\mu}_1.\hat{r})(\hat{\mu}_2.\hat{r})}{r^3}$
- g_{dd}= Dipolar interaction strength.
- Trapping stabilizes the condensate against collapse.



Attractive

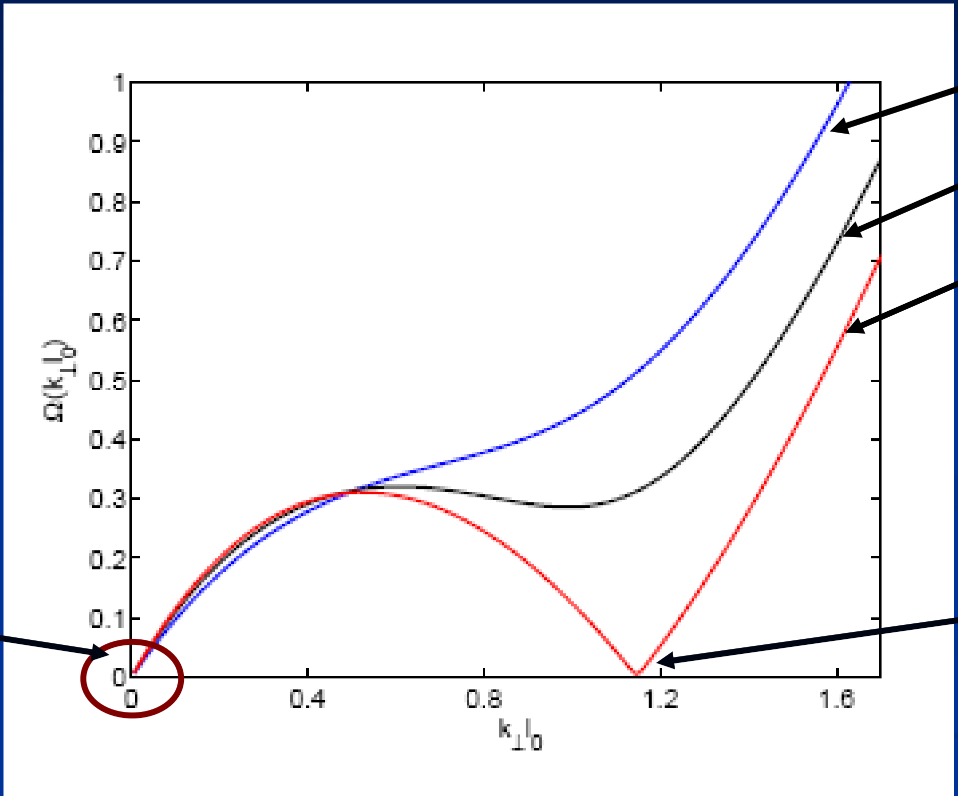


Repulsive

Dipolar condensate

- Bosonic density : $\mathrm{n_b}(x, y, z) = \dfrac{3\mathrm{n_b}}{4\mathrm{R_z}}\left(1 - \dfrac{z^2}{\mathrm{R_z^2}}\right).$
- Thomas-Fermi approximation.
- Minimize the mean field energy.
- Hamiltonian : $\mathrm{H_b} = \sum_{k_\perp} \Omega\,(k_\perp) b_{k_\perp}^\dagger b_{k_\perp}$
- Excitation Spectrum – $\Omega_{k_\perp}$.
- Bogoliubov operators: $\mathrm{b}_{k_\perp}$, $\mathrm{b}_{k_\perp}^\dagger$

Excitation Spectrum



$g_{3d} = 2.0$

$g_{3d} = 3.0$

$g_{3d} = 3.61$

Phonons

Roton instability.

$$g_{3d} = \frac{8\pi\, m_b\, g_{dd}\, n_b l_0}{5\hbar^2}$$

l_0 = Oscillator length.

n_b = Bosonic density

Degenrate fermions

- Single component fermions.
- Fermionic Density : $\mathrm{n_f}(x, y, z) = \frac{\mathrm{n_f}}{\pi\, \ell_f^2} \exp\left[- \frac{x^2 + y^2}{\ell_f^2} \right]$
- Width of the Gaussian = Oscillator length.
- Assume a two-dimensional Fermi surface.

Boson-fermion interaction

- Condensate-fermion Hamiltonian :

$$H_{bf} = \frac{3g_{bf}}{4\sqrt{\pi}\,R_z} \sum_{k_\perp, q_\perp} \gamma(k_\perp) c^\dagger_{\vec{k}_\perp} c_{\vec{q}_\perp - \vec{k}_\perp} \left[b^\dagger_{\vec{k}_\perp} + b^\dagger_{-\vec{k}_\perp} \right].$$

g_{bf} = Boson-fermion contact interaction.

$$\gamma(k_\perp) = \sqrt{\frac{2n_b \varepsilon_b(k_\perp)}{\Omega(k_\perp)}}$$

Kinetic energy for bosons.

Effective Interaction

- Integrate out the bosonic degrees of freedom.
- Momentum exchanges occur around the Fermi surface.
- Expand the interaction in angular momentum basis :

$$V_{\mathrm{eff}}(\varphi) = \frac{3g_{\mathrm{bf}}^2 N_0}{8\pi g_{\mathrm{dd}} l_0} \sum_{m=\ldots-1,0,1,\ldots} \lambda_m e^{im\varphi}$$

interaction at channel *m*

Effective Interaction

$$\lambda_m = \alpha^2 \int_0^{2\pi} \frac{\exp[im\phi]d\phi/2\pi}{\frac{\eta R_z^2}{g_{3\mathrm{d}}\ell_f^2}(1-\cos\phi) + \underbrace{\frac{R_z^2}{\ell_0^2}\mathcal{V}\left(\sqrt{\frac{R_z}{\ell_f}\eta(1-\cos\phi)}\right)}_{\text{Effective dipolar interaction}}}$$

Interaction at channel *m*.

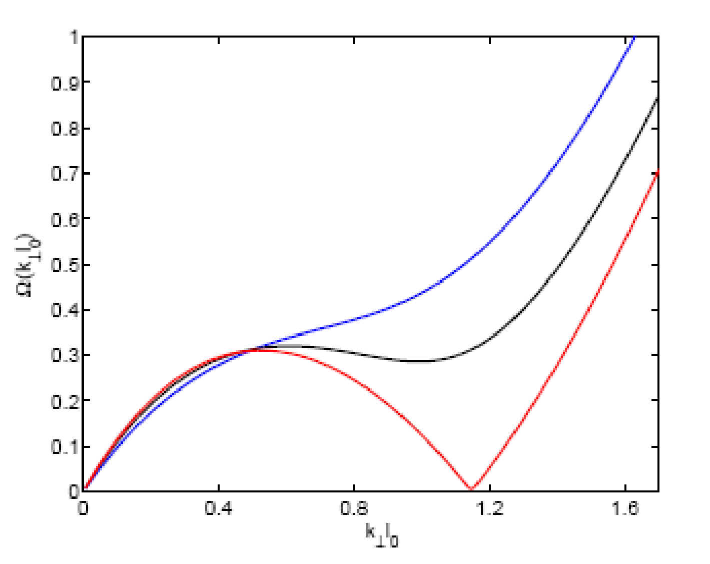
- Dimensionality parameter : $\eta = \dfrac{\varepsilon_F \text{(Fermi energy)}}{\hbar\omega_0 \text{(Trapping frequency)}}$
- Dimensionless dipolar interaction : $g_{3d} = \dfrac{8\pi\, m_b\, g_{dd}\, n_b l_0}{5\hbar^2}$

$$\frac{R_z}{l_0} \approx \left(2.5 g_{3d}\right)^{\frac{1}{3}} \qquad \frac{R_z}{\ell_f} = \frac{R_z}{l_0} \times \frac{m_f}{m_b}$$

Effective interaction

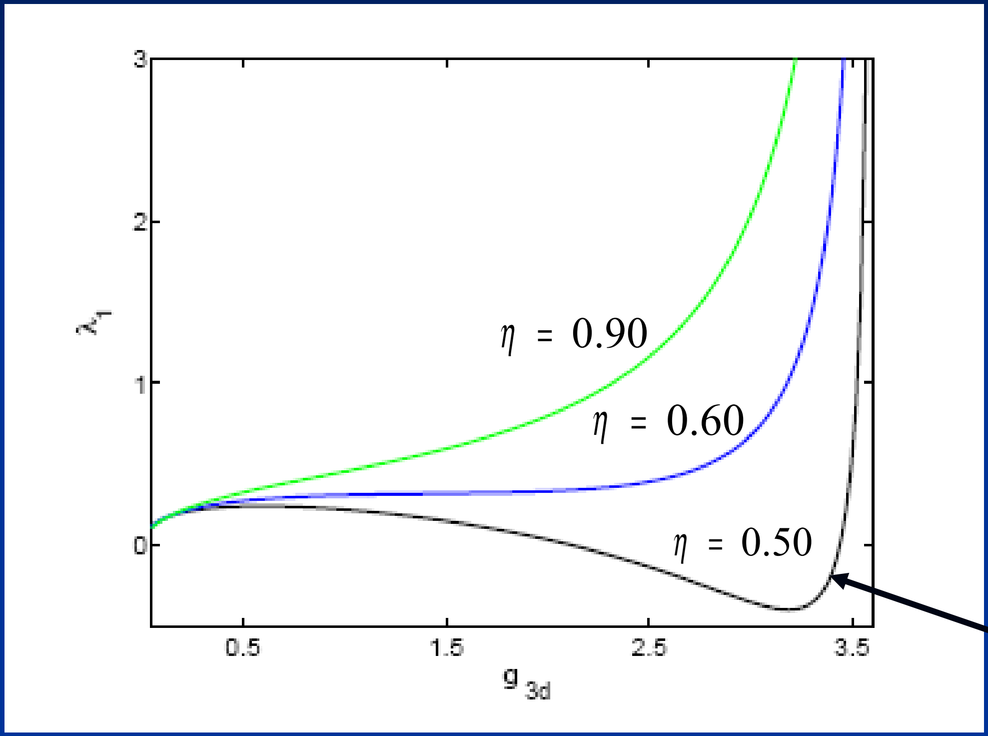
$$\lambda_m = \alpha^2 \int_0^{2\pi} \frac{\exp[im\phi]d\phi/2\pi}{\frac{\eta R_z^2}{g_{3d}\ell_f^2}(1-\cos\phi) + \underbrace{\frac{R_z^2}{\ell_0^2}\mathcal{V}(\sqrt{\frac{R_z}{\ell_f}\eta(1-\cos\phi)})}}$$

$$\mathcal{V}(\tilde{k}_\perp) = \frac{1}{\tilde{k}_\perp^5}\left[4\tilde{k}_\perp^3 - 6\tilde{k}_\perp^2 - 6(1+\tilde{k}_\perp^2)\exp(-2\tilde{k}_\perp) + 6\right] - \frac{8}{15}$$



Non-trivial angular dependence!!

P-wave interaction

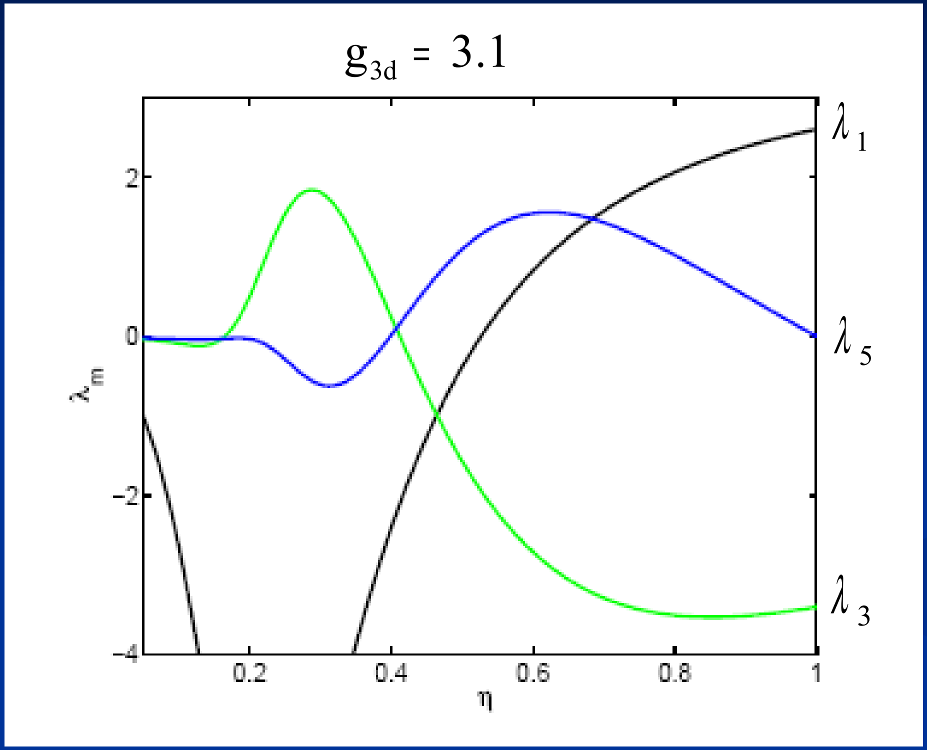


$$\eta = \frac{\varepsilon_F(\text{Fermi energy})}{\hbar\omega_0(\text{Trapping frequency})}$$

$$g_{3d} = \frac{8\pi\, m_b\, g_{dd}\, n_b l_0}{5\hbar^2}$$

Repulsive interaction

p-, f-, h- wave interactions



$$\eta = \frac{\varepsilon_F}{\hbar\omega_0}$$

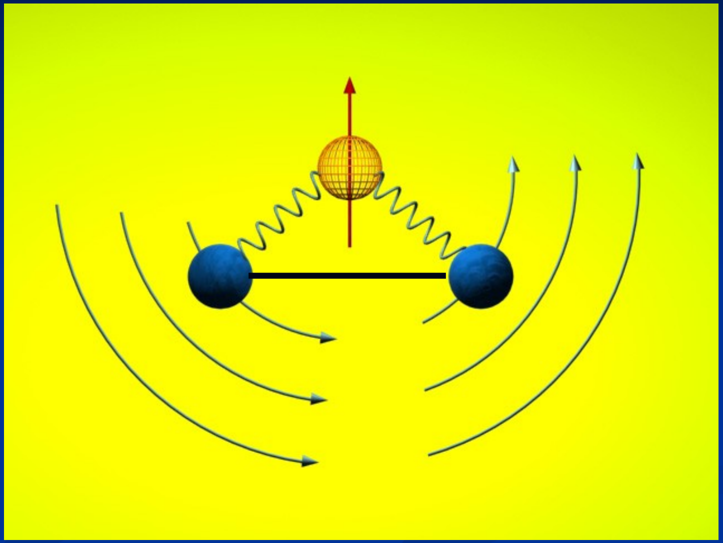
Transition from p-wave to h-wave !!

Chirality

p-wave : $k_x + ik_y$

f-wave : $(k_x + ik_y)^3$

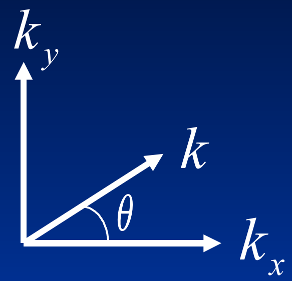
h-wave : $(k_x + ik_y)^5$



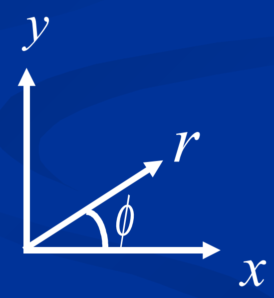
Excitations: $\sqrt{\left(\frac{k^2}{2m_f} - \mu\right)^2 + |\Delta|^2 k^{2m}}, \quad m = 1,3,5$

Vortex Excitations

Order Parameter : $\Delta = \Delta_0(\vec{r})\left(\frac{k}{k_f}\right)^m e^{im\theta}$



Vortex structure : $\begin{cases} \Delta_0(\vec{r}) = 0 & , \mathrm{r} \leq \xi \\ \Delta_0(\vec{r}) = \Delta_0 e^{i\phi} & , \mathrm{r} > \xi \end{cases}$



Single vorticity

Quasi-particle excitations inside a vortex

For $r \gg \xi$

$$
\begin{aligned}
H_0 u_m + (-i)^m \frac{\Delta_0}{k_f^m} e^{i\phi/2} \left[e^{-i\phi} \left(\frac{\partial}{\partial r} - \frac{i}{r} \frac{\partial}{\partial \phi} \right) \right]^m e^{i\phi/2} v_m &= E u_m \\
-H_0 v_m + (i)^m \frac{\Delta_0}{k_f^m} e^{-i\phi/2} \left[e^{i\phi} \left(\frac{\partial}{\partial r} + \frac{i}{r} \frac{\partial}{\partial \phi} \right) \right]^m e^{-i\phi/2} u_m &= E v_m,
\end{aligned}
$$

$\begin{pmatrix} u_m \\ v_m \end{pmatrix}$ = Excitation amplitudes for $m = 1.3, 5.$

Zero-energy solution: $u_m^* = v_m$

Energy : $\mathrm{E}_n = n\omega_0, \quad \omega_0 \sim \dfrac{\Delta_0^2}{\varepsilon_\mathrm{F}}, \quad n =$ integers.

Zero-energy modes

For $r \gg \xi$

$$\begin{bmatrix} u_1 \\ \\ u_3 \\ \\ u_5 \end{bmatrix} \sim \begin{bmatrix} \exp\left(-\frac{\Delta_0}{v_f} r\right) \\ \exp\left(-\frac{m_f^2 v_f^3}{6\Delta_0} r\right) e^{2i\phi} \\ \exp\left(-\left[\frac{m_f^4 v_f^5}{2\Delta_0}\right]^{1/3} r\right) e^{4i\phi} \end{bmatrix}$$

Quasi-particle operator : $\gamma_m = \int d^2 r \left[u_m c^\dagger(r) + v_m c(r) \right]$

Majorana Fermions : $\gamma_m = \gamma_m^\dagger$

Ground state degeneracy

Topological properties

- Two well-separated vortices i and j.
- Each vortex carries a Majorana zero-energy mode.
- Exchange of two such vortices : $|0\rangle \to \gamma_m^i \gamma_m^j |0\rangle$.
- Non-abelian nature : $\gamma_m^i \gamma_m^j \neq \gamma_m^j \gamma_m^i$.
- $2n$ such vortices have degeneracy 2^n.

(Ivanov-2001)

Conclusion

- Superfluidity in Bose-Fermi mixture.
- Effect of roton minimum on the pairing symmetry.
- Feshbach resonance like character.
- Possibility of non-standard f-wave and h-wave superfluidity.
- Non-Abelian statistics.

Future Study

- Identification of different pairing symmetries.
- Ways to create and manipulate vortices.
- Identification of the statistics of the excitations.
- Looking into the different pairing possibilities within each angular momentum channel.
- Stability of Fermi surface.

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