Istituto Nazionale per la Fisica della Materia



Research and Development Center on

Bose-Einstein Condensation

Trento, Italy

Scientific Report

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Overview

The mission of the Center

The aim of the INFM Research and Development Center on Bose-Einstein Condensation (BEC) is to promote theoretical research on the various phenomena related to Bose-Einstein condensation and to the physics of cold atomic gases in traps. Since the first observation of BEC in cold gases in 1995, the study of ultracold gases has become an emerging area of research at the crossing point of several disciplines, including atomic, molecular and optical (AMO) physics, statistical mechanics and condensed matter physics. BEC has been presently achieved with Rb, Na, Li, K, H, He, Cs and Yb. More than twenty groups worldwide have reported experimental production of BEC and there is a vast range of theoretical and computational activity.



Interest in BEC derives from several factors. BEC is the only known phase transition taking place also in the absence of interactions, its origin being purely quantum mechanical. Thus it is one of the cornerstones of quantum statistical mechanics. Before 1995 BEC had only been explored in strongly interacting systems, such as superfluid helium, where the effects of the interaction mask some crucial features of BEC. The dilute gas experiments of the past years have made it possible to compare in a systematic way experimental data with the predictions of first principle theory. The theoretical approaches have been mainly based, in the first years, on the use of Gross-Pitaevskii theory for the order parameter and have proven quite successful, especially to predict the behaviour of these systems at low temperature, both at equilibrium and out of equilibrium.

Despite this great success there remain a number of important problems of conceptual relevance which are presently the object of intense research activity. These include, among others, the dynamics of the condensate at finite temperature, the kinetic phenomena in the presence of BEC, the nucleation of quantized vortices, the dynamics of vortex arrays, the nature of the phase transition for reduced dimensionalities and in the presence of array geometries, the emergence of new quantum phases, like number squeezed and Schroedinger cat states, the occurrence of new topological structures in multicomponent condensates, the behaviour of BEC for large scattering lengths and the role of Feshbach resonances, the occurrence of chaos in the dynamics of BEC, the fluctuations of the condensate for small samples, the theory of the order parameter beyond mean field, the mechanisms of decoherence of the phase, the stability of solitons and vortex rings. The remarkable property exhibited by BEC of generating a macroscopic population of atoms in the same quantum state has also opened up the new field of coherent atomic optics. This has already led to the development of coherent matter waves sources (the so called atom laser) to be employed for interferometry. This is a major step towards the ultimate control of fundamental characteristics of atomic beams with important applications like precision spectroscopy, frequency standards, atomic gyroscope, atom lithography and holography, sensors etcetera.

In the last few years an impressive activity in the field of ultracold gases has also concerned the study of Fermi gases. Despite the initial difficulty in cooling such systems experimentalists have been quite successful in obtaining highly degenerate samples, providing new concrete perspectives in the study of Fermi superfluidity, including the long sought BCS-BEC crossover. At present Bose-Einstein condensation of molecules (pairs of fermions) has been succefully achieved and new challenging experimental and theoretical perspectives are characterizing the international scene. Finally, BEC research boosted several important applications in related branches of physics. A few examples concern the use of BEC to generate gases with very high non-linear optical susceptibility, where light propagates at extremely low speed and the perspective of using ultracold gases to implement logical operations with important links with the field of quantum information.

In the last years, the growth of the BEC field has crucially benefited by the cooperative efforts of experimental and theoretical groups in many laboratories. The aim of the BEC Center is to reinforce the interdisciplinary links of the theoretical research. On the other hand the Center is intended to reinforce the scientific collaborations between theoretical and experimental activities, establishing direct and systematic links with the main laboratories in the world.

The first two years

The BEC Center was established by the Istituto Nazionale per la Fisica della Materia in Trento in June 2002, following a selection made by an international panel. The Center is hosted by the Department of Physics of the University of Trento on the basis of an official agreement with INFM. Scientists belonging to the BEC Center include INFM researchers as well as personnel from the University, together with a large number of PhD students and post-doctoral fellows, who are partly funded by INFM and partly by the University. At present the scientists active in the Center are about 20. The budget of the BEC Center is provided by INFM and by the Provincia Autonoma di Trento (PAT) on the basis of an official agreement with INFM. The research activity of the Center is also supported by the Italian Ministry of Research. the the worldwide development of research activities in the field of ultracold gases. The Trento BEC Center has significantly contributed to the the worldwide development of research activities in the field of ultracold gases through a long series of scientific publications, the reinforcement and the creation of international collaborations, the organization of workshops and conferences, as well as through the training of young scientists.

Detailed information on the activity of the BEC Center can be found at the website: http://bec.science.unitn.it

May, 2004

Organization

Management

Director

* Sandro Stringari

Scientific board

- * Jean Dalibard
- * Chris Pethick
- * William D. Phillips
- * Gora Shlyapnikov
- * Peter Zoller

Secretariat

- * Flavia Evandri (October 2002-September 2003)
- * Daniela Zecca (since October 2003)

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Personnel of University

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- * Meret Kraemer (thesis defended in February 2004)
- * Grigory E. Astrakharchik
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- * Sara Ianeselli
- * Christian Trefzger

Technical Staff

- * Valerio Varriale
- * Giuseppe Froner

Scientific report

Selected research lines

This report provides an overview of the main scientific activities on Bose-Einstein condensation and related topics, with reference to the period June 2002- May 2004. Most of the scientific work carried out at the Center can be naturally classified according to the following research lines:

- Rotating quantum gases
- Quantum gases in low dimensions
- Excitations in Bose-Einstein condensates
- Dynamics of BEC in optical lattices
- Breakdown of coherence in optical lattices
- Ultracold Fermi gases
- Bose-Fermi mixtures
- Numerical simulations in quantum gases

We have also identified two research lines recently developed at the Center where we expect more systematic activity in the near future:

- Quantum information applications
- Interferometry and sensors with ultracold gases

ROTATING QUANTUM GASES

One of the most important implications of superfluidity concerns the rotational behaviour of quantum fluids at low temperature. This problem, of historical importance in the physics of superfluid helium, has recently become popular also in the context of atomic gases [1]. The implications of superfluidity are directly connected with the constraint of irrotationality imposed by the existence of an order parameter. The superfluid velocity field is in fact directly related to the gradient of the phase Φ of the order parameter

$$\boldsymbol{v} = \frac{\hbar}{m} \, \boldsymbol{\nabla} \Phi \tag{1}$$

with non trivial implications also at a topological level. Some important consequences imposed by the irrotationality constraint are:

- Superfluids cannot rotate in a rigid way and their moment of inertia consequently differs from the rigid value.
- Angular momentum can be carried through the creation of quantized vortices.

Notice that the existence of quantized vortices means that the viscosity of a superfluid is exactly zero. In fact, in the presence of an even small viscosity the vortices would diffuse. In cold atomic gases the effects predicted by superfluidity have been extensively explored both experimentally and theoretically and in its first two years of activity the BEC Center has provided a series of significant contributions.

Scissors mode in a rotating BEC

At high angular velocity Bose-Einstein condensates are known to exhibit spontaneous breaking of rotational symmetry [2, 3]. This phenomenon, which is the consequence of two-body repulsive interactions, shows up in the occurrence of considerable deformations of the atomic cloud even in the presence of a tiny trap deformation. This peculiar regime can be reached in the absence of vortices by increasing adiabatically the angular velocity and is characterized by novel dynamic features. In a joint experimentaltheoretical collaboration [4] with the team of J. Dalibard at the Ecole Normale Superieure (ENS) we have investigated the scissors mode, corresponding to the oscillation of the gas around the principal axis of the trap in the rotating plane. The frequency of the scissors mode, evaluated in the rotating frame, has been predicted to decrease with the deformation of the trap. This surprising behaviour originates from the fact that the restoring force of the oscillation is proportional to the trap deformation, while the



Figure 1: Left (a): typical scissors oscillation measured at ENS [4]. Right (b): measured scissors frequency [4].

moment of inertia remains finite due the considerable deformation of the gas caused by the spontaneous symmetry breaking. In Fig. 1(a) we show a typical scissors oscillation measured at ENS. In this experiment the deformation of the trap is determined by the laser intensity producing the rotation. In Fig. 1(b) the measured scissors frequency is shown to decrease linearly with the laser intensity, as predicted by theory.

Rotating Fermi superfluids

In Ref. [5] we have investigated the dynamic properties of a Fermi superfluid rotating at low angular velocity. The main motivation is to compare such a behaviour with the corresponding effect exhibited by a non superfluid gas in the collisional regime. Actually the macroscopic dynamics of both a superfluid and of a normal gas in the collisional regime are governed by the *same* equations of hydrodynamics, so that at first sight one would conclude that it is impossible to infer superfluidity by looking at the dynamic behaviour of the gas. However this is true only if non rotational configurations are considered. In the presence of a rotating trap the dynamics of a non superfluid system are not subject to the constraint of irrotationality and different dynamical effects can take place. For example in Fig. 2 we show the time dependence of the density profile of a Fermi gas initially confined in a trap rotating with angular velocity Ω . At t = 0 the deformation of the trap is suddenly removed and the gas is no longer in equilibrium, giving rise to quadrupole shape oscillations. The behaviour of the oscillation is however different depending on whether the gas is superfluid or is in a collisional regime. In the first case (superfluid) the oscillation does not exhibit precession, while in the second



Figure 2: Stroboscopic imaging of the quadrupole oscillation described in the text: (a) superfluid case, no precession; (b) collisional case, precession.

one (collisional non superfluid) the oscillation precesses with angular velocity $\Omega/2$. The angular velocity of the precession is actually determined by the moment of inertia carried by the system according to

$$\Omega_{\rm prec} = \frac{\langle \ell_z \rangle}{2m \langle r^2 \rangle} \tag{2}$$

The possibility of distinguishing between superfluid and collisional hydrodynamics might provide an important signature of superfluidity in ultra-cold Fermi gases.

Shape oscillations of a BEC containing a vortex lattice

When a Bose-Einstein condensate rotates at high angular velocity the most stable configuration contains one or more vortex lines. It is now possible to produce samples containing a large number of vortex lines. These form a regular lattice, similar to the Abrikosov lattice of superconductors. If the number of vortex lines is high, the system can carry a large amount of angular momentum and can be described within a classical hydrodynamic framework, introducing the concept of diffused vorticity. The resulting equations of rotational hydrodynamics have proved to be quite efficient in describing the collective oscillations of a Bose-Einstein condensate containing a large number of vortex lines (see for example the experimental findings of the E.A. Cornell team at Jila [6]). In Ref. [7] we have investigated the shape deformations of the condensate resulting from a sudden switch on of a tiny deformation of the confining trap. If the angular velocity is close to the critical value fixed by the radial trapping frequency, the gas exhibits a soft quadrupole mode which is strongly excited by suddenly adding a static quadrupole perturbation. As a consequence, a large deformation is produced in



Figure 3: Vortex lattice dynamics for a rotating Bose-Einstein condensate: (a) experimental findings [8]; (b) theoretical calculation based on rotational hydrodynamics [7].

the condensate and the vortex lines are forced to rearrange themselves, giving rise to stripe-like configurations. In Fig. 3 we show a typical prediction compared with the experimental findings of Ref. [8]. The equations of rotational hydrodynamics have been also used in Ref. [9] to investigate the Kelvin oscillations of a vortex lattice. The Kelvin modes are macroscopic oscillations of a vortex line and in this work their study has been generalized to the case of a vortex lattice in a 3D harmonic trap.

Tkachenko oscillations of a vortex lattice

In Ref. [10] we have investigated the elastic oscillations of a vortex lattice (Tkachenko oscillations). The frequency of these oscillations is governed by the quantum nature of vortex lines, being determined by the corresponding quantum of circulation. In a pure hydrodynamic description these oscillations occur at zero frequency [9], their restoring force being fixed by a macroscopic elastic effect. In an incompressible fluid Tkachenko predicted that the dispersion law of these oscillations depends linearly on the wavevector q [11]. In a dilute gas these modes are strongly affected by compressional effects and behave like q^2 at small wavevectors [12]. The Tkachenko modes have been first observed experimentally in Ref. [13] (see Fig. 4). In our paper we have developed a theoretical study based on sum rules taking into account the effects of the finite size, inhomogeneity and compressibility of the gas. We have calculated (see Fig. 5)

the frequency of the Tkachenko modes in the so called Thomas-Fermi limit where the size of vortex cores is much smaller than the distance between vortices. The agreement with experiments is remarkable, except close to the critical angular velocity where the Thomas-Fermi approximation is no longer adequate. Another important prediction of our paper [10] is that, despite their elastic nature, Tkachenko oscillations are easily excited by density perturbtions. This effect, which is the consequence of the high compressibility of the gas, might provide new efficient methods for the experimental excitation of these exhotic states. The sum rule approach has also permitted to derive the behaviour of the static structure factor. In a uniform system we predict the law

$$S(q) \to \frac{\hbar q^2}{4m\Omega}$$
 (3)

differently from what happens in a non rotating interacting Bose fluid where the static structure factor vanishes linearly with q.



Figure 4: Tkachenko oscillation of a vortex lattice [13].

Vortices in a harmonic plus quartic potential

In a purely harmonic trap it is not possible to induce rotations in stationary conditions with angular velocity larger than the harmonic trapping frequency. Inclusion of a quartic term in the confinement avoids this limitation and permits access to new configurations of major physical interest. The first experiments in this direction have been carried out at ENS [15]. Several theoretical works have already addressed this problem (see for example Refs. [16]). In Ref. [17] we have carried out a numerical simulation based on the solution of the Gross-Pitaevskii equation. A typical prediction is reported in Fig 6. At small angular velocities one sees a vortex lattice similar to that seen in harmonic traps. However, as one increases the angular velocity a density depression



Figure 5: Lowest Tkachenko frequency of a trapped Bose gas in units of $\omega_0 = \sqrt{\hbar\Omega/4mR_{\perp}^2}$ as a function of the angular velocity Ω . The full line is the sum rule result, while the dashed line is the prediction of Ref. [12]. Experimental points are taken from Refs. [13, 14].

appears in the center. The critical value of the angular velocity corresponding to the transition from a regular vortex lattice to a vortex lattice with a macroscopic hole agrees very well with the analytic prediction based on rotational hydrodynamics. Such configurations have not yet been realized experimentally.

The physics of rotating systems is extremely rich in the presence of spin degrees of freedom: thanks to the interplay of the spin and translational degrees of freedom, simple vortices are replaced by more complicated spin textures, e.g. π -disclinations and coreless vortices. These textures have been studied in many different contexts, from ³He-A to quantum Hall systems and, more recently, trapped gaseous BECs [18].

In most experiments so far, imaging is performed in a destructive way by first separating the different spin components using a magnetic field gradient and then by separately imaging them. In the work [19], a different imaging scheme is proposed which provides non-destructive, *in-situ*, images of the spin textures and which directly addresses the geometrical features of the BEC order parameter (density, spin, and nematicity profiles). The method relies on the dependence of the dielectric properties of a spinor atomic gas on the internal state of the atoms. We study the propagation of polarized light across an atomic cloud and we show how information about the spatial profile of the spin texture can be retrieved by analyzing the intensity, phase, and polarization profile of the transmitted light. Our technique has analogies in other areas



Figure 6: Equilibrium configurations for a condensate in a quadratic plus quartic potential at different angular velocities.

of condensed matter physics: the polarization of the electrons in a solid-state sample as well as the order parameter of nematic liquid crystals can be measured by looking at polarization changes in a transmitted, diffracted, or reflected light.



Figure 7: Nematic order of a π -disclination in a rotating spinor BEC with weak antiferromagnetic interactions. Panel (a): Two dimensional projections of the nematicity ellipsoids. Panel (b): Simulated phase-contrast image of the disclination using σ_+ circularly polarized incident probe light.

Other work relative to the vortex structure in ultracold gases concerns:

- Quantum Hall State of Vortices in Rotating Bose Gases [20]
- Zero-temperature damping of Bose-Einstein condensate oscillations by vortex-

antivortex pair creation [21]

- [1] A.L. Fetter and A.A. Svidzinsky, J. Phys.: Condens. Matter 13, R135(2001).
- [2] A. Recati, F. Zambelli, and S. Stringari, Phys. Rev. Lett. 86, 377 (2001).
- [3] K. W. Madison, F. Chevy, V. Bretin, and J. Dalibard, Phys. Rev. Lett. 86, 4443 (2001).
- [4] M. Cozzini, S. Stringari, V. Bretin, P. Rosenbusch, and J. Dalibard, Phys. Rev. A 67, 021602(R) (2003).
- [5] M. Cozzini and S. Stringari, Phys. Rev. Lett. **91**, 070401 (2003).
- [6] P.C. Haljan, I. Coddington, P. Engles, and E.A. Cornell, Phys. Rev. Lett. 87, 210403 (2001).
- [7] M. Cozzini and S. Stringari, Phys. Rev. A 67, 041602(R) (2003).
- [8] P. Engels, I. Coddington, P.C. Haljan, and E.A. Cornell, Phys. Rev. Lett. 89, 100403 (2002).
- [9] F. Chevy and S. Stringari, Phys. Rev. A 68, 053601 (2003).
- [10] M. Cozzini, L. Pitaevskii and S. Stringari, Phys. Rev. Lett. 92, 220401 (2004)
- [11] V.K. Tkachenko, Zh. Eksp. Teor. Fiz. 50, 1573 (1966) [Sov. Phys. JETP 23, 1049 (1966)].
- [12] G. Baym, Phys. Rev. Lett. **91**, 110402 (2003).
- [13] I. Coddington, P. Engels, V. Schweikhard, and E.A. Cornell, Phys. Rev. Lett. 91, 100402 (2003).
- [14] V. Schweikhard, I. Coddington, P. Engels, and E.A. Cornell, Phys. Rev. Lett. 92, 040404 (2004).
- [15] V. Bretin, S. Stock, Y. Seurin, and J. Dalibard, Phys. Rev. Lett. **92**, 050403 (2004).
- [16] A.L. Fetter, Phys. Rev. A 64, 063608 (2001); U.R. Fischer and G. Baym, Phys. Rev. Lett. 90, 140402 (2003); G.M. Kavoulakis and G. Baym, New J. Phys. 5, 51 (2003).

- [17] B. Jackson, A.L. Fetter, S. Stringari, to be published.
- M.R. Matthews, B.P. Anderson, P.C. Haljan, D.S. Hall, C.E. Wieman, and E.A. Cornell, Phys. Rev. Lett. 83, 2498 (1999); A.E. Leanhardt, Y. Shin, D. Kielpinski, D.E. Pritchard, and W. Ketterle, Phys. Rev. Lett. 90, 140403 (2003).
- [19] I. Carusotto and E.J. Mueller, J. Phys. B (At. Mol. Opt. Phys.) 37, S115 (2004).
- [20] U.R. Fischer, P.O. Fedichev, A. Recati, P. Zoller, cond-mat/0212419.
- [21] P.O. Fedichev, U.R. Fischer, and A. Recati, Phys. Rev. A 68, 011602 (2003).

QUANTUM GASES IN LOW DIMENSIONS

The study of quantum degenerate gases in low dimensions has become a very active area of research. The role of correlations and of thermal and quantum fluctuations is greatly enhanced by the reduced dimensionality. Two- (2D) and one-dimensional (1D) quantum gases are well suited systems to study beyond mean-field effects.

Bose gases in 2D and 1D have been realized in highly anisotropic harmonic traps. Complete freezing of the transverse degrees of freedom is obtained if both the thermal energy and the chemical potential are much smaller than the separation between levels in the transverse direction. This condition imposes severe constraints on the temperature and the number of particles in the trap. Fully 2D and 1D kinematics has been achieved for systems prepared in a optical trap (2D) [1] and in a deep optical lattice (1D) [2].

For a homogeneous 2D Bose system at finite temperature it is well known that thermally induced fluctuations of the order parameter destroy the long-range order associated with the broken gauge symmetry. Bose-Einstein condensation (BEC) is absent in this system, but, nevertheless, a phase transition from a normal to a superfluid state, which belongs to the Kosterlitz-Thouless (KT) universality class and is different from its three-dimensional (3D) counterpart, is predicted. The KT transition has been observed in thin films of liquid ⁴He [11]. The study of the transition in 2D in the presence of harmonic confinement is still an open problem. The finite size of the trapped cloud acts as a cutoff for long-range fluctuations which are less efficient in destroying the coherence of the system. At low enough temperature a quasicondensate phase is possible which is completely indistinguishable from the truly BEC phase. However, at higher temperatures thermal fluctuations should dominate the physics and whether the transition is of BEC or KT type is a challenging question both for theorists and experimentalists. In 1D the physical scenario is richer. For homogeneous systems it is predicted that BEC is absent even at zero temperature due to large quantum fluctuations. The presence of the harmonic confinement makes it possible to have a quasicondensate phase at low enough temperatures such that the coherence length exceeds the size of the system. By increasing temperature, the crossover to a condensate with fluctuating phase, corresponding to a partial loss of coherence over the size of the cloud, and finally to a complete incoherent high-temperature regime has been observed experimentally [4].

At very low temperatures 1D Bose gases show quite remarkable features. In a usual 3D quantum many-body system the role of interactions is suppressed by decreasing the particle density n and the system becomes more ideal. In a 1D Bose gas the opposite



Figure 1: Transition between the TG and the 1D mean-field regime: ω^2/ω_z^2 as a function of the parameter Na_{1D}^2/a_z^2 .

is true and the effects of interactions become more important. The reason is that at $T \to 0$ the kinetic energy of a particle is $K \propto n^2$ and it decreases with decreasing density faster than the interaction energy per particle $V \propto n$. The ratio of interaction to kinetic energy $\gamma = V/K$ characterizes the different regimes of the gas. If $\gamma \ll 1$ the properties of the system are well described within a mean-field scheme and the quasicondensate picture applies. On the contrary, if $\gamma \gg 1$, the system enters the Tonks-Girardeau (TG) regime, where the repulsion between particles strongly reduces the wavefunction at short interparticle distances and the system behaves as if it consisted of noninteracting spinless fermions. Achieving such a TG regime and observing fermionization of the 1D Bose gas is a great challenge [2, 5, 6].

The research activity of the Trento BEC-INFM group in this field has been devoted mainly to the study of 1D systems using both standard techniques of quantum manybody theory and numerical simulations with quantum Monte Carlo methods. Some of the most relevant contributions are discussed in more details below.

Collective oscillations of a 1D trapped Bose gas

The study of collective modes in 1D systems can provide a useful experimental technique to investigate the role of interactions and beyond mean-field effects. In this work [7] the frequency of the lowest compressional mode in 1D Bose gases in harmonic traps is investigated as a function of the interaction strength using the hydrodynamic equations of superfluids. For trapped systems the parameter that controls the strength of interactions is given by Na_{1D}^2/a_z^2 , where N is the number of atoms in the trap, a_{1D}



Figure 2: Energy per particle as a function of the 3D scattering length for a system of N=5 particles in a trap with $\omega_z/\omega_\rho = 0.01$. The asterisks refer to the DMC results with the 3D Hamiltonian and the squares to the DMC results with the 1D Hamiltonian with contact interactions and renormalized coupling constant. The horizontal dashed line indicates the TG energy.

is the effective 1D scattering length and $a_z = \sqrt{\hbar/m\omega_z}$ is the oscillator length fixed by the frequency ω_z of the longitudinal harmonic confinement. This parameter plays the role of the ratio of kinetic to interaction energy $1/\gamma = K/V$ introduced above. For $Na_{1D}^2/a_z^2 \gg 1$ the gas is in the weakly-interacting mean-field regime. In this case the frequency of the breathing mode is predicted to have the value $\omega = \sqrt{3}\omega_z$. In the opposite regime $Na_{1D}^2/a_z^2 \ll 1$, the system is a strongly-interacting TG gas and for the frequency of the mode one finds the result $\omega = 2\omega_z$ corresponding to an ideal Fermi gas. By using the exact Lieb-Liniger equation of state of a homogeneous Bose gas with contact interactions [8] and the local density approximation, one can calculate the frequency of the breathing mode in the crossover from mean field to the TG regime. The result is shown in Fig. 1. In Ref. [2] the frequency of the breathing mode in the 1D mean-field regime was measured and the prediction $\omega = \sqrt{3}\omega_z$ has been confirmed.

Quasi-1D Bose gases with large scattering length

In this work [12] we investigate the energetics of Bose gases confined in highly elongated harmonic traps over a wide range of interaction strengths. In normal experimental conditions the 3D s-wave scattering length a_{3D} is much smaller than the transverse harmonic oscillator length $a_{\rho} = \sqrt{\hbar/m\omega_{\rho}}$, fixed by the tight harmonic confinement with frequency ω_{ρ} . Under these conditions the effective 1D coupling constant is simply proportional to a_{3D} [10] and, although the system exhibits 1D kinematics, the scattering properties are still 3D. However, utilizing a magnetic atom-atom Feshbach resonance a_{3D} can be tuned to very large values both positive and negative. For large $|a_{3D}|$ the scattering of two particles is strongly affected by the tight transverse confinement and the effective 1D coupling constant is renormalized due to virtual transverse excitations [11]. We consider a system of N particles described by a 3D Hamiltonian, which takes into account explicitly the transverse confinement and where interparticle interactions are modeled by a realistic short-range potential. By using exact diffusion Monte-Carlo (DMC) methods to calculate the energy of the gas at zero temperature we find that the system (i) is well described by a 1D model Hamiltonian with contact interactions and renormalized coupling constant for any value of a_{3D} ; (ii) behaves like a TG gas for a critical positive value of a_{3D} ; (iii) reaches a unitary regime for large values of $|a_{3D}|$, where the properties of the 1D Bose gas become independent of the actual value of a_{3D} and are similar to those of a hard-rod gas; and (iv) becomes unstable against cluster formation for a critical negative value of a_{3D} . In Fig. 2 we report the results for the energy per particle as a function of the 3D scattering length.

Interacting Fermi gases in highly elongated harmonic traps

Two-component atomic Fermi gases in 1D have not been realized experimentally yet; however, their realization in highly-elongated, needle-shaped traps is within reach of present-day techniques. The behavior of quasi-1D two-component Fermi gases can, if the confinement is chosen properly, be characterized to a very good approximation by an effective 1D coupling constant which encapsulates the interspecies atom-atom interaction strength. This coupling constant can be tuned to essentially any value, including zero and $\pm \infty$, by varying the 3D *s*-wave scattering length a_{3D} through application of an external magnetic field in the proximity of a Feshbach resonance. The role of interactions in 1D atomic Fermi gases has been studied mainly in connection with Luttinger liquid theory. Recati *et al.* [12] investigate the properties of a two-component Fermi gas with *repulsive* interspecies interactions confined in highly-elongated harmonic



Figure 3: Square of the lowest breathing mode frequency, ω^2 , as a function of the coupling strength Na_{1d}^2/a_z^2 for an inhomogeneous two-component 1D Fermi gas with repulsive $(g_{1d} > 0)$ and attractive $(g_{1d} < 0)$ interactions (solid lines).

traps. In the limit of weak and strong coupling these authors relate the parameters of the Luttinger Hamiltonian, which describe the low-energy properties of the gas, to the microscopic parameters of the system. The prospect of realizing Luttinger liquids with cold fermionic atoms is fascinating since it would allow detailed investigations of strongly correlated many-body systems, which play a central role in condensed matter physics [13], to be conducted. This work [17] investigates the properties of inhomogeneous 1D two-component Fermi gases under harmonic confinement with attractive and repulsive interspecies interactions. The study is based on the exact equation of state of a homogeneous 1D system of fermions with zero-range attractive [15] and repulsive [16] interactions treated within the local density approximation. The energy per particle, the size of the cloud, and the frequency of the lowest compressional mode are calculated as a function of the effective 1D coupling constant, including infinitely strong attractive and repulsive interactions. The predictions for the size of the cloud and for the breathing mode frequency have immediate implications for experimental studies. For attractive interactions we discuss the cross-over from the weak- to the strong-coupling regime and point out the possibility of forming a mechanically stable molecular Tonks-Girardeau gas. In Fig. 3 we show the results for the frequency of the lowest compressional mode as a function of the coupling strength.

Superfluidity of the weakly interacting 1D Bose gas

Is the weakly interacting 1D Bose gas superfluid? Diverging answers can be found in the literature. As shown in [11], one of the subtleties of the issue is that there are actually different definitions of superfluidity, one of them based on a stationary (that is thermal equilibrium) property of the system, the other one involving a dynamical response of the system. In 1D, these two definitions are found to dramatically differ in the thermodynamic limit. This work [18] is focussed on the thermal equilibrium regime, in a case where the gas can exchange momentum with a rotating vessel with walls that are smooth, at least at the macroscopic scale. Investigations are performed by considering not only the mean momentum of the rotating gas, but also the whole probability distribution of the total momentum. Using the conventional definition of the superfluid fraction, which relies on the variance of the total momentum of the gas in the limit $v_{\rm rot} \rightarrow 0$, we find that the gas has a significant superfluid fraction only in the regime where the coherence length exceeds the length of the ring (BEC regime). In the opposite regime where the length of the ring exceeds the coherence length, we have identified a regime where several peaks appear in the full probability distribution of the total momentum, each peak corresponding to a quasicondensate in a plane wave state with a given winding number, the analog of supercurrents in superconductors.



Figure 4: Probability distribution p(P) of the total momentum P in units of $Nmv_1 = 2\pi N\hbar/L$. Temperature: $T/T_v = 0.25, 0.5, 1, 2, 3$ (from top to bottom), $T_v = 2\pi^2 N\hbar^2/L^2$, N = 1000.

Other contributions to the field of quantum gases in low dimensions involving members of the Trento BEC-INFM group are listed below:

- Spin-charge separation in ultra-cold quantum gases [12]
- Three-dimensional quasi-Tonks gas in a harmonic trap [19]
- Violation of self-similarity in the expansion of a 1D Bose gas [20]
- Irregular dynamics in a one-dimensional Bose system [21]
- D. Rychtarik, B. Engeser, H.-C. Nägerl and R. Grimm, Phys. Rev. Lett. 92, 173003 (2004).
- [2] H. Moritz, T. Stöferle, M. Köhl and T. Esslinger, Phys. Rev. Lett. 91, 250402 (2003).
- [3] D.J. Bishop and J.D. Reppy, Phys. Rev. Lett. 40, 1727 (1978).
- [4] S. Dettmer *et al.*, Phys. Rev. Lett. **87**, 160406 (2001).
- [5] B. Laburthe Tolra *et al.*, preprint cond-mat/0312003.
- [6] B. Paredes *et al.*, Nature **429**, 277 (2004).
- [7] C. Menotti and S. Stringari, Phys. Rev. A 66, 043610 (2002).
- [8] E.H. Lieb and W. Liniger, Phys. Rev. 130, 1605 (1963).
- [9] G.E. Astrakharchik, D. Blume, S. Giorgini and B.E. Granger, Phys. Rev. Lett. 92, 030402 (2004).
- [10] D.S. Petrov, G.V. Shlyapnikov and J.T.M. Walraven, Phys. Rev. Lett. 85, 3745 (2000).
- [11] M. Olshanii, Phys. Rev. Lett. 81, 938 (1998).
- [12] A. Recati, P.O. Fedichev, W. Zwerger and P. Zoller, Phys. Rev. Lett. 90, 020401 (2003).
- [13] see for example: J. Voit, Rep. Prog. Phys. 57, 977 (1994).

- [14] G.E. Astrakharchik, D. Blume, S. Giorgini and L.P. Pitaevskii, preprint condmat/0312538.
- [15] M. Gaudin, Phys. Letters **24A**, 55 (1967).
- [16] C.N. Yang, Phys. Rev. Lett. **19**, 1312 (1967).
- [17] A.J. Leggett, Rev. Mod. Phys. 71, S318 (1999).
- [18] I. Carusotto and Y. Castin, Comptes Rendus Physique 5, 107-127 (2004).
- [19] P. Pedri and L. Santos, Phys. Rev. Lett. **91**, 110401 (2003).
- [20] P. Pedri, L. Santos, P. Ohberg and S. Stringari, Phys. Rev. A 68, 043601 (2003).
- [21] G.P. Berman, F. Borgonovi, F.M. Izrailev and A. Smerzi, Phys. Rev. Lett. 92, 030404 (2004).

EXCITATIONS IN BOSE-EINSTEIN CONDENSATES

The study of elementary excitations is a task of primary importance of quantum many-body theories. In the case of Bose fluids, in particular, it plays a crucial role in the understanding of the properties of superfluid liquid helium and was the subject of pioneering work by Landau, Bogoliubov and Feynman. After the experimental realization of BEC in trapped Bose gases, there has been an extensive study of the excitations in these systems. Measurements of the frequency of the lowest modes have soon become available and the direct observation of the propagation of wave packets has been also obtained. In the meanwhile, on the theoretical side, a variety of papers has been written to explore several interesting features exhibited by the dynamic behavior of trapped Bose gases.

The group of Trento has been working on the excitations of trapped BEC since the very beginning [1] using different techniques: superfluid hydrodynamic equations, linearized Gross-Pitaevskii (GP) theory, time dependent GP equation, sum rules. More recently, the problem of the response of BEC to light (Bragg) scattering has been investigate in detail [2] in connection with experiments performed at MIT [3]. Among the main results, one can mention the direct measurement of the Bogoliubov quasiparticle amplitudes [4] based on the suggestion and the theoretical analysis of Ref. [5]. The research activity of the Trento BEC-INFM group in this field has been devoted mainly to the detailed study of the Bogoliubov spectrum of elongated condensates and the behavior of the excitations during a free expansion of the gas.

Bogoliubov spectrum and Bragg spectroscopy of elongated BEC

The crossover between phonon and single-particle excitations in the Bogoliubov spectrum of a weakly interacting Bose gas [6] is one of the main "textbook" concepts that can be directly tested in the case of trapped Bose-Einstein condensed gases. In the analysis of the first experiments done at MIT [3, 4] and at the Weizmann Institute [7], a local density approximation (LDA) has been used to adapt the Bogoliubov theory of uniform gases to the actual inhomogeneous condensates. This approach is expected to be accurate for large condensates, where the density profile varies smoothly on the scale of the excitation wavelength and the system behaves locally as a piece of uniform gas with a local Bogoliubov spectrum. However, the finite size of the condensate produces a discreteness of the spectrum, which is ignored in LDA. In the work [8] we showed that the response of the condensate to a Bragg pulse is indeed significantly affected by the radial degrees of freedom. In particular, if the duration of the pulse is longer than the radial trapping period, the condensate responds resonantly at the frequencies $\omega_{n_r}(k)$ of axial quasiparticles with n_r nodes in the radial direction. By using Bragg pulses longer than in previous measurements, the multi-branch spectrum was indeed resolved (see Fig. 1), finding good agreement with the predictions of Gross-Pitaevskii (GP) theory.



Figure 1: A Bose-Einstein condensate is excited by light (Bragg) scattering. The Bragg pulse has frequency ω , wavevector k, strength V_B and duration t_B . The measured quantity is the total momentum transferred to the condensate. In this figure, the momentum P_z , in units of $N\hbar k$, is plotted as a function of ω , for a fixed wavevector $k = 3.1 \mu \text{m}^{-1}$. The duration of the pulse is 6 msec. Points with error bars are measured values. The upper curve is the result of GP simulations with $V_B = 0.2\hbar\omega_{\rho}$, where $\omega_{\rho} = 2\pi 220$ Hz is the radial trapping frequency. The lower curve is the same but with V_B ten times smaller. For the lower curve, P_z is multiplied by 20. The arrows indicate the frequency of axial Bogoliubov excitations with 0, 1 and 2 radial nodes.

In a second paper [9], we presented a detailed theoretical analysis of the multibranch spectrum, supporting our previous interpretation of the observed spectra and giving further information about the role played by Bogoliubov excitations in Bragg spectroscopy. A useful insight into this problem is obtained by studying the case of an infinite condensate, unbound along z and harmonically trapped in the radial direction ρ . We showed that the response of such a cylindrical condensate retains all the relevant properties needed to interpret the observed behavior of a finite elongated condensate. In addition to the numerical solution of the full time dependent GP equation, we explicitly determined the time evolution of the order parameter in the linear (small amplitude) limit, using the quasiparticle projection method of Refs. [10]. The cylindrical geometry allows one to simplify the calculations and, more important, to make the connection between the momentum transferred and the dynamic structure factor more direct than for a finite condensate. By comparing the predictions of this approach to the results of the numerical integration of the time dependent GP equation, one can also distinguish the different effects of linear and nonlinear dynamics.

The fate of phonons in freely expanding BEC

In most of the experiments with excited condensates, the observations are performed after switching-off the confining potential and letting the condensate expand. The comparison between theory and experimental data often ignores the dynamics of the expansion. This is justified if one looks for quantities that are conserved during the expansion like, for instance, the total momentum of the condensate. However, the expansion of a condensate initially dressed with phonon-like modes is interesting from both the conceptual and experimental viewpoints. Conceptually, the behavior of an excited state that starts as a quasiparticle and evolves into a particle is a remarkable and nontrivial example of quantum process, which shows some similarities with the quantum evaporation process at the surface of superfluid helium [11] and with the evolution of two-level systems with nonhermitian Hamiltonian [12]. From the experimental viewpoint, on the other hand, the characterization of the observable density and velocity distributions of the expanded gas in terms of specific initial configurations is important in order to use those distributions as a probe of in-trap quasiparticles.

In the paper [14] we investigated the basic mechanisms of this phonon evaporation process by using the Gross-Pitaevskii theory and dynamically rescaled Bogoliubov equations. We first performed three-dimensional GP simulations, by assuming the initial configuration to be a stationary trapped condensate at zero temperature; quasiparticle states were excited by acting with an external Bragg potential and, then, the condensate was let expand. We showed that the expansion exhibits quite different behaviors depending on the wavelength of the initial phonons: i) short wavelength phonons are converted into a separate cloud of excited atoms moving out of the condensate (Fig. 2b); ii) long wavelength phonons remain inside the expanding condensate in the form of density modulations (Fig. 2d). In both cases, the final density distribution shows nontrivial features, such as a radial distortion of the density modulations in the condensate and a "shell" structure of the released-phonon cloud.

A deeper understanding was obtained by studying the evaporation process in the expansion of a cylindrical condensate, in which the different roles of radial and axial degrees of freedom can be easily pointed out. In this case we numerically solved the Bogoliubov equations for the amplitudes of in-trap quasiparticles. Then we followed



Figure 2: Density $n(\rho, z)$ of the expanding condensate, initially excited with phonons of momentum q. In (a) and (b): $qa_{\rho} =$ 2.03. The density in (a) is obtained by computerized tomography of TOF column density absorption images, $n_{\rm col}(y, z)$, while (b) corresponds to a full GP simulation. In both cases, note the clearly separated excitation cloud on the right of the condensate. The same quantities are plotted in (c) and (d) for $qa_{\rho} = 0.31$. Note the strong density modulation, with no significant outcoupled fraction. The quantity a_{ρ} is the harmonic oscillator length $a_{\rho} = [\hbar/(m\omega_{\rho})]^{1/2}$, where ω_{ρ} is the radial trapping frequency. Distances are given in mm.

the expansion of a condensate initially dressed with one of these quasiparticles by using the scaling properties of the GP equation in two-dimensions and solving rescaled Bogoliubov-like equations. This approach allowed us to characterize the behavior of the excitations during the expansion, at different levels of approximation, and to find the relevant timescales for the evaporation process. Interesting results were also obtained by averaging out the slow radial motion of the excitations in the rescaled coordinates. The problem was thus mapped into the evolution of a quasiparticle in a uniform gas with a decreasing time-dependent density. We showed that this process can be either adiabatic (conversion of a quasiparticle into a single particle with the same momentum) or non-adiabatic depending on the phonon wavelength and on the chemical potential of the gas.

A similar analysis of the expansion of condensates with long wavelength phonons was



Figure 3: Fringe visibility as a function of the Bragg excitation frequency. Points are experimental results. The error bars represent the uncertainty due to four measurements. We observe a clear double-peaked spectrum, which is finite-time broadened. The peaks are found at $\omega = \pm 137 \pm 10$ Hz, close to the expected Bogoliubov frequency (138 ± 5 Hz). The solid line is a full GP simulation of the experiment. The dashed line is the analytic result for an uniform gas having density which decreases in time as the average density of the inhomogeneous expanding condensate.

previously presented in Ref. [13], where random phase fluctuations were included in the configuration of a very elongated condensate at the beginning of the expansion, in order to simulate the behavior of a quasi-condensate with thermal excitations. In our analysis, conversely, one considers the situation where the initial excitations are imprinted in a controllable way and characterized by looking at the shape of the expanded gas, thus allowing for a new type of spectroscopic studies of Bogoliubov quasiparticles.

High sensitivity phonon spectroscopy of BEC using matter-wave interference

An interesting application of the problem of expanding condensates with phonons is the work [15], which was done again in a theory-experiment collaboration with the group of Nir Davidson. In the experiments the condensates were excited by Bragg pulses and imaged after a free expansion. As in previous experiments, at high momenta the excitations were clearly separated from the expanding condensate, and could be counted and quantified. Even in the phonon regime (wavelength larger than but comparable to the healing length) a well defined excitation cloud could still be distinguished from the condensate (see Fig. 2a) and was found to be amenable to direct atom counting

methods. However, at sufficiently low momentum (wavelength much larger than the healing length), we observed a new regime, in which the excitations and the condensate no longer separate, regardless of the duration of the expansion. The excitations were manifested in a clear density modulation of the cloud in the absorption images (see Fig. 2c). Using both the Gross-Pitaevskii equation and a dynamically rescaled Bogoliubov theory [14] we found that, in our highly-elongated condensate, axial low-momentum phonons are adiabatically converted by the (mainly radial) expansion into free atoms with the same axial momentum. The overlap of these moving free particles with the expanding ground state of the condensate results in the axial periodic density modulations observed in the TOF images. Whereas the sensitivity of atom or momentum counting methods scales quadratically with the quasiparticle excitation amplitude, we showed that the fringe visibility scales linearly with these amplitudes. We used the fringe visibility of these density modulations after TOF, as an extremely sensitive spectroscopic probe of the excitation strength (see Fig. 3).

Other contributions to the field of excitations in Bose-Einstein condensates involving members of the Trento BEC-INFM group are:

- Finite size effects on the collective oscillations of a trapped Bose gas [16].
- Landau damping of transverse quadrupole oscillations of an elongated Bose-Einstein condensate [17].
- Landau damping in trapped Bose-condensed gases [18].
- S. Stringari, Phys. Rev. Lett. **77**, 2360 (1996); L.P. Pitaevskii and A. Rosch, Phys. Rev. A **55**, R853 (1997); F. Dalfovo, C. Minniti, S. Stringari, and L. Pitaevskii, Phys. Lett. A **227**, 259 (1997); F. Dalfovo, S. Giorgini, M. Guilleumas, L.Pitaevskii and S. Stringari, Phys. Rev. A **56**, 3840 (1997); S. Stringari, Phys. Rev. A **58**, 2385 (1998).
- [2] F. Zambelli, L.P. Pitaevskii, D.M. Stamper-Kurn, and S. Stringari, Phys. Rev. A 61, 063608 (2000); A. Brunello, F. Dalfovo, L. Pitaevskii, S. Stringari, and F. Zambelli, Phys. Rev. A 64, 063614 (2001).
- [3] D. M. Stamper-Kurn, A. P. Chikkatur, A. Görlitz, S. Inouye, S. Gupta, D. E. Pritchard, and W. Ketterle, Phys. Rev. Lett. 83, 2876 (1999).

- [4] J. M. Vogels, J. M. Vogels, K. Xu, C. Raman, J. R. Abo-Shaeer, and W. Ketterle, Phys. Rev. Lett. 88, 060402 (2002).
- [5] A. Brunello, F. Dalfovo, L. Pitaevskii, and S. Stringari, Phys. Rev. Lett. 85, 4422 (2000).
- [6] N. N. Bogoliubov, J. Phys. (USSR) **11**, 23 (1947).
- [7] J. Steinhauer, R. Ozeri, N. Katz, and N. Davidson Phys. Rev. Lett. 88, 120407 (2002).
- [8] J. Steinhauer, N.Katz, R. Ozeri, N. Davidson, C. Tozzo, and F. Dalfovo, Phys. Rev. Lett. 90, 060404 (2003).
- [9] C. Tozzo and F. Dalfovo, New J. Phys. 5, 54 (2003).
- [10] S.A. Morgan, S. Choi, K. Burnett, and M. Edwards, Phys. Rev. A 57, 3818 (1998);
 P.B. Blakie, R.J. Ballagh, and C.W. Gardiner, Phys. Rev. A 65, 033602 (2002).
- [11] M. Brown and A.F.G. Wyatt, J. Phys.: Condens. Matter 2, 5025 (1990).
- [12] J.C. Garrison and E.M. Wright, Phys. Lett. A 128, 177 (1988); M. Gorlicki, Phys. Rev. A 49, 4339 (1994).
- [13] S. Dettmer, D. Hellweg, P. Ryytty, J.J. Arlt, W. Ertmer, K. Sengstock, D.S. Petrov, G.V. Shlyapnikov, H. Kreutzmann, L. Santos, and M. Lewenstein, Phys. Rev. Lett. 87, 160406 (2001).
- [14] C. Tozzo and F. Dalfovo, e-print cond-mat/0401359, Phys. Rev. A, in press.
- [15] N. Katz, R. Ozeri, N. Davidson, C. Tozzo and F. Dalfovo, e-print condmat/0405222.
- [16] F. Zambelli and S. Stringari, Laser Physics 12, 240 (2002)
- [17] M. Guilleumas and L.P. Pitaevskii, Phys. Rev. A 67, 053607 (2003).
- [18] B. Jackson, E. Zaremba, New J. Phys. 5, 88 (2003).

DYNAMICS OF A BEC IN OPTICAL LATTICES

Lasers, electric and magnetic fields allow for a large variety of trapping potentials. Among them optical lattices, i.e. non dissipative potentials created by standing waves of light, have provided in the recent years challenging perspectives for the realization of new quantum phases, including the study of the superfluid-Mott insulator transition [1]. The transition point depends strongly on the geometry. In particular in 1D optical lattices, where the occupancy of the single well is high, the transition takes place for rather large intensities of the optical lattice, so that there is a very extended range of parameters where the gas can be described as a fully coherent superfluid.

Some of the concepts familiar from solid state physics apply also for condensates in optical lattices: this is the case for quasi-momentum, energy bands, Bloch states, effective mass. However interactions can modify the situation in a non trivial way. First of all, one finds that there are three energies associated to the system (energy per particle, chemical potential, energy of elementary excitations) leading to different bands spectra. Second, due to the non linear term in the Gross-Pitaevskii equation, condensate wavefunctions obeying the Bloch theory are not the only stationary states, but period-doubled and other exotic solutions can be found [2, 3]. Finally, the dynamics of such systems show many new features which are absent in the non interacting case: the system substains elementary excitations, and dynamical instabilities can arise due to interactions [4, 5].

In the last two years, our work has focused mainly on one dimensional optical lattices $V(x) = sE_R \sin^2(\pi x/d)$. The characteristic quantities are the optical potential depth s, the lattice spacing d, the Bragg momentum $q_B = \hbar \pi/d$ and the recoil energy $E_R = q_B^2/2m$. A detailed study of the properties of the condensate in the regime of full coherence has been carried out, with special attention on their dependence on the optical lattice depth. We investigated in particular the effective mass, Bloch (macroscopic) excitations and Bogoliubov (elementary) excitations [6], the structure factor [7], the propagation of sound [8] and the onset of energetical and dynamical instabilities for condensates in motion [5]. As theoretical tools we have used Gross-Pitaevskii and Bogoliubov theories. Recently the investigation into the insulating regime has also started [9, 10] and some studies of the dynamics in two and three dimensional optical lattices have been carried out [18, 20].

Bloch states, effective mass and superfluid density

For a given quasi-momentum k, stationary solutions of the Gross-Pitaevskii equation
can be found in the form of a Bloch state. They have well defined quasi-momentum k and give rise to an energy per particle band spectrum $\varepsilon(k)$. The behaviour of the lowest band is quadratic at small quasi-momenta and its curvature defines the effective mass m^* . The relation between the effective mass m^* and the tunneling parameter δ (describing the half width of the lowest band in the tight binding regime) is given by $m/m^* = \pi^2 \delta/2E_R$. The calculation of the lowest band by solving the Gross-Pitaevskii equation is actually the most accurate method to estimate the tunneling parameter.

Due to the non linear term in the Gross-Pitaesvkii equation, the effective mass is density dependent. One can introduce a second effective mass m_{μ}^{*} linked to the curvature of the chemical potential band and related to m^{*} by $m_{\mu}^{*} = m^{*} + n\partial m^{*}/\partial n$. Both effective masses correspond to the bare mass m in absence of the periodic potential and become very large for increasing optical lattice depth, indicating the strong decrease of tunneling through the potential barriers.

The Bloch state represents a state with constant current $I = nv_g$, where the group velocity $v_g = \partial \varepsilon / \partial k$ is the velocity of the condensate with respect to the lattice. Alternatively for small k, the current can be described as the product of the superfluid velocity $v_s = k/m$ and the superfluid density $n_s = nm/m^*$. This results implies that in presence of the lattice the superfluid density is always smaller than the total density. The persistence of a superfluid flow in a lattice for quasi-momentum different from zero must be analysed carefully by checking the stability of the stationary state.

The effective mass determines also the frequency $\omega_D = \sqrt{m^*/m} \omega_x$ of dipole oscillations and the frequency $\omega_Q = \sqrt{5/2} \times \sqrt{m^*/m} \omega_x$ of quadrupole oscillations in presence of a lattice and an external harmonic confinement along the direction of the lattice. These predictions have been already successfully tested [11, 12], and provide a useful method in the experimental determination of the effective mass (see Fig.1).

Bogoliubov excitations and dynamic structure factor

Numerical solutions and analytic expressions in the tight binding regime for the Bogoliubov excitations can be easily found. The excitation spectrum shows a linear behaviour $\hbar\omega = c|q|$ at small quasi-momenta of the excitation and the system admits sound waves with velocity

$$c = \sqrt{\frac{n(\partial \mu/\partial n)}{m^*}}.$$
(1)

Increasing the optical potential depth, the sound velocity is reduced with respect to the uniform case, due to the strong increase of m^* . Furthermore a phononic regime is



Figure 1: (a) Experimental data for the dipole oscillation frequency as a function of lattice depth s and theoretical predictions based on the calculation of the effective mass (full line) and on a numerical simulation of the time-dependent GPE (triangles) [12, 1]; (b) Quadrupole oscillation frequency vs. dipole oscillation frequency: the slope is in good agreement with the predicted value $\sqrt{5/2}$ (figure from [14]).



Figure 2: Bogoliubov bands for s = 10 and $gn = 0.5E_R$; excitation strengths Z_j towards the states in the first three bands for $p = -1.2q_B$, $p = 0.8q_B$ and $p = 2.8q_B$.



Figure 3: Strength $Z_1(p)$ towards the first band as a function of momentum transfer p and optical lattice depth s for $gn = 0.5E_R$. On the back: the projection of the strength for s = 0, 1, ... 10enphasise that $Z_1(p)$ vanishes at all even multiples of q_B in correspondence of the phononic regime.

found at all even multiples of the Bragg momentum q_B due to the periodicity of the excitations in quasi-momentum space (see Fig.2) [6].

Bogoliubov excitations can be selectively probed transferring momentum p and energy $\hbar\omega$ to the condensate. The dynamic structure factor describes the capability of the system to respond to such a probe [7]. In the presence of the lattice, the excitations at quasi-momentum q have many momentum components at $q + 2\ell q_B$, with ℓ integer. Hence, such excitations can be addressed with different strengths by any momentum trasfer p differing from q a quantity $2\ell q_B$. Moreover, for a given value of the momentum transfer p, it is possible to excite several states, corresponding to the different bands in the spectrum. An example of the non trivial excitation strengths in presence of a lattice is shown in Fig.2.

Particularly interesting is the excitation strength towards the first band $Z_1(p)$. First, $Z_1(p)$ vanishes at all $p = 2\ell q_B$ reflecting directly the phonon behaviour of the excitation spectrum (see Fig.3). Moreover $Z_1(p)$ exhibits characteristic oscillations suppressed at large p. The range of p where the strength is not negligible increases with optical lattice depth, reflecting the better localisation of the condesate and Bogoliubov amplitudes at the bottom of the lattice wells.

The strength Z_1 is quenched both by increasing interaction and optical potential



Figure 4: Sound velocity as a function of lattice depth s. Bogoliubov prediction (solid line) and results "measured" based on the simulation (circles) with respective signal amplitudes Δn for $gn = 0.5E_R$. The signal amplitude Δn is defined as indicated in the inset.

depth. This is due to the fact that the excitations in the lowest Bogoliubov band acquire a strong quasi-particle character and also reflects an increase of the quantum depletion in the condensate. When the quantum depletion becomes important, the assumption of full coherence leading to those results is no longer valid.

Sound propagation

Alternatively to the probing of single excitations by Bragg spectroscopy, one can produce wavepackets, i.e. superposition of different excitations. In particular it is interesting to address only small quasi-momenta in the phononic regime, and create in such a way sound packets, as shown in the inset of Fig.4.

The main question addressed in [8] is the strong interplay between nonlinear effects and the periodicity of the external potential. In spite of the phononic behaviour of the Bogoliubov spectrum at small quasi-momenta, it is not obvious whether a sound signal of observable amplitude can propagate also in deep lattices, where the tunneling rate is very small and nonlinear corrections to the Bogoliubov theory can be important (see also [5]).

We performed simulations in the non linear regime, and extracted the value of the Bogoliubov sound velocity with high accuracy. For increasing lattice depths, the



Figure 5: (a) AHE. Semiclassical trajectory of the wave packet center for a force **f** in the positive y direction: not only the atoms perform Bloch oscillations in the direction of the force, but also they drift along the perpendicular direction because of the geometrical phase accumulated. (b) SHE. Distribution of the spin vertical component as a function of momentum **k** when a finite force $\mathbf{f} = f_x \hat{x}$ is applied. The sign of the z-component of the spin depends on the sign of the y component of the wavevector (red is positive, blue is negative). The blue arrows show effective **k**-dependent magnetic field $\mathbf{B}_{\mathbf{k}}$ close to the central Γ point.

maximal attainable signal amplitude decreases very strongly as $\sim 1/\sqrt{m^*n(\partial \mu/\partial n)}$. However, the strong decrease of the sound velocity as a function of lattice depth s can be measured up to s = 20 with signal amplitudes larger than 5%, as shown in Fig.4.

We have investigated the role of non linearities at fixed interaction and lattice depth, and increasing strength of the external perturbation producing the sound signal. For very small perturbations, the system evolves in the linear regime. Increasing the perturbation, the first non linear effects appear due to mode–coupling among Bogoliubov excitations through the production of shock waves, which in deep lattices propagate at a velocity smaller than the sound velocity. For even larger perturbations in deep lattices, the amplitude of the sound signal saturates. It propagates without deformation at exactly the Bogoliubov sound velocity and leaves behind a wake of noise, due to the formation of dephased currents on the back of the signal.

Spin-orbit coupling and Berry phase with ultracold atoms in 2D optical lattices

In solid state systems, a non-vanishing geometric Berry phase in the crystal momentum space may be responsible for the anomalous Hall effect (AHE) [16] (i.e. the generation of a transverse current by an electric field even in the absence of a magnetic field) and the spin Hall effect (SHE) [17] (i.e. the production of a transverse spin current by an electric field). This requires that time-reversal and/or spatial inversion symmetries are broken in the lattice, or an appreciable spin-orbit coupling.

In the work [18], the expertise on the light-matter interaction in the presence of spin degrees of freedom developed in [19] has been combined to the one of the Austin group on spin-orbit coupling issues in solid-state systems. By choosing the polarization of the optical lattice beams appropriately, the internal degrees of freedom of the atom can be coupled to their momenta as in the (relativistic) spin-orbit effect for electrons in solids. We predict that effects similar to AHE and SHE can be observed by studying the transport of the atoms in these spin-dependent lattices.

- D. Jaksch, C. Bruder, J.I. Cirac, C.W. Gardiner and P. Zoller, Phys. Rev. Lett. 81, 3108 (1998); M. Greiner, O. Mandel, T. Esslinger, T.W. Hänsch and I. Bloch, Nature 415, 39 (2002)
- [2] M. Machholm, A. Nicolin, C.J. Pethick, and H. Smith, Phys. Rev. A 69, 043604 (2004)
- [3] W.D. Li and A. Smerzi, Phys. Rev. E, in press
- [4] B. Wu and Q. Niu, Phys. Rev. A 64, 061603(R) (2001); M. Machholm, C.J. Pethick and H. Smith, Phys. Rev. A 67, 053613 (2003)
- [5] See Research Line on "Breakdown of coherence in optical lattices"
- [6] M. Krämer, C. Menotti, L. Pitaevskii, S. Stringari, Eur. Phys. J. D 27, 247 (2003)
- [7] C. Menotti, M. Kraemer, L. Pitaevskii and S. Stringari, Phys. Rev. A 67, 053609 (2003)
- [8] C. Menotti, M. Krämer, A. Smerzi, L. Pitaevskii, and S. Stringari, condmat/0404272, accepted for publication in PRA

- [9] J.Dziarmaga, A.Smerzi, W.H.Zurek, and A.R.Bishop, cond-mat/0403607
- [10] in collaboration with Nandini Trivedi
- [11] F.S. Cataliotti, S. Burger, C. Fort, P. Maddaloni, F. Minardi, A. Trombettoni, A. Smerzi, M. Inguscio, Science 293, 843 (2001)
- [12] M. Krämer, L. Pitaevskii and S. Stringari, Phys. Rev. Lett. 88, 180404 (2002)
- [13] A. Smerzi and A. Trombettoni, Chaos 13, 766 (2003)
- [14] C. Fort, F. S. Cataliotti, L. Fallani, F. Ferlaino, P. Maddaloni, and M. Inguscio, Phys. Rev. Lett. 90, 140405 (2003)
- [15] see Research Line on "Excitations in Bose-Einstein condensates"
- [16] D. Culcer, A. MacDonald, Q. Niu, Phys. Rev. B 68, 045327 (2003)
- [17] J. Sinova, D. Culcer, Q. Niu, N. A. Sinitsyn, T. Jungwirth, and A. H. MacDonald, Phys. Rev. Lett. 92, 126603 (2004)
- [18] A. M. Dudarev, R. B. Diener, I. Carusotto, Q. Niu, Phys. Rev. Lett. 92, 153005 (2004)
- [19] I. Carusotto and E. J. Mueller, J. Phys. B (At. Mol. Opt. Phys.) 37, S115 S125 (2004)
- [20] V. Ahufinger, A. Sanpera, P. Pedri, L. Santos, M. Lewenstein, cond-mat/0310042

BREAKDOWN OF COHERENCE IN OPTICAL LATTICES

The dynamics of a Bose-Einstein condensate in optical lattices is of main interest for the BEC community. Among the theoretical and experimental challenges, we have, at first, to reconsider the meaning of phenomena which are well understood in homogeneous systems, like superfluidity, for instance. Yet, the high level of accuracy reached in the experimental manipulation of weakly coupled Bose-Einstein condensates is encouraging to dream about two far reaching goals for the next decade: the possibility to test foundational aspect of quantum mechanics, like the role of decoherence in the classical-quantum correspondence principle, and the creation of new technological devices (see Research Lines on "Quantum information applications" and "Interferometry with matter waves"). It is quite appropriate to expect that the possibility to explore these two directions will keep BEC at the forefront of international research for quite long yet.

It did not last long to realize that, despite of the diluteness of a BEC, *nonlinearity* is crucial for the understanding of its equilibrium and dynamical properties. There is a further key aspect that must be taken into account when the condensates are trapped in optical lattices: *discreteness*. The interplay between the *discrete* translational invariance of the periodic potential and the *nonlinearity* arising from interatomic interactions raises a new class of highly non-trivial phenomena which disappear when the lattice is turned off. Recognizing and exploiting such interplay has been a main effort of the Trento BEC group. We have mainly focused on a region of parameters such that the BEC ground state stands deeply in the superfluid phase, with the dynamics governed by the Gross-Pitaevskii equation (GPE). Because of the discrete translational invariance, the excitation spectrum of the system exhibits a band structure which has several analogies with the electron Bloch bands in metals. On the other hand, the coexistence of Bloch bands and nonlinearity allows, for instance, for solitonic structures and dynamical instabilities which do not have an analog neither in metals, nor in Galilean invariant systems.

Discrete nonlinear dynamics

The Gross-Pitaevskii equation in presence of an external, deep, periodic potential can be mapped in a discrete non-linear equation [1, 2]. The basic assumption is to write the order parameter as a sum of wave functions localized in each well $\Psi(\vec{r},t) =$ $\sum_{\ell} \psi_{\ell}(t) \Phi_{\ell}(\vec{r},t)$, where ℓ is the well index. This approximation takes into account only the lowest band dynamics of the system. The atoms can tunnel through the barriers, the tunneling rate depending on the overlap of the spatial wavefunctions localized in each well. The dynamical variables are the number of atoms in each site and the corresponding phases $\psi_{\ell}(t) = \sqrt{N_{\ell}(t)} \exp(i\phi_{\ell}(t))$. The temporal dependence of the Φ_{ℓ} arises from the change in shape of the spatial wavefunction due to the variation of the atom number in well ℓ during the dynamics. Such dependence is caused by the nonlinearity of the system, and vanishes in the limit of noninteracting bosons. The crucial observation is that the temporal dependence of $\Phi_{\ell}(\vec{r}, t)$ is adiabatic when higher bands excitations can be neglected, and does not contribute directly to the current-phase dynamics.

A consequence of the adiabatic approximation is that the nonlinearity, arising from the two-body interatomic interaction, is not quadratic but depends on the effective geometry of each well. In most experimental realizations, each well has a typical pancake shape, the long radius being in the transverse direction of the periodic potential. This trap geometry realizes an effective two-dimensional BEC. Since the chemical potential crucially depends on the effective dimensionality of the BEC, the resulting nonlinearity goes as the square root of the density $\sim |\psi_{\ell}|$. This simple fact has several important consequences. For instance, the sound velocity will depend on the effective geometry of each well, as well as the frequencies of low-lying excitations (a part the dipole oscillations frequency which does not depende on the interaction).

The resulting dynamical equations and part of the phenomenology are similar to those of an array of superconducting Josephson junctions. There are important differences, though. First, trapped BEC can have large density fluctuations, which in superconducting systems are strongly inhibited by the Coulomb energy. Moreover, discrete BEC can be quite easily driven with linear or harmonic external fields. Several new phenomena have been predicted in the context of BECs.

Superfluid dynamics

A prototypical experiment demonstrating the superfluid flow of a condensate in a periodic potential was published in 2001 [3]. For completeness, and because some of the concepts exposed in the paper are continuing to stimulate the discussion about the meaning of superfluidity in discrete systems, we briefly overview the theory underlying that experiment. A condensate trapped in a periodic potential was driven by an external harmonic field. In the superfluid regime, with all atoms coherently tunnel trough the barriers, the relative phases across the junctions $\phi_{\ell+1} - \phi_{\ell} \equiv \phi(t)$ remain locked together to the same (oscillating) value. As a consequence, the phase-current relation of the full array of weakly condensates can be expressed in the very simple form $d\xi(t)/dt \sim \delta \sin \Delta \phi(t)$; $d\phi(t)/dt \sim \xi(t)$, which, in analogy with the case of a su-

perconducting Josephson junction (in the resistively shunted junction model) and with the case of ${}^{3}He$, is a pendulum equation with the relative phase $\Delta\phi$ corresponding to the angle to a vertical axis and the center of mass ξ being the corresponding angular momentum [3, 1]. The interatomic tunneling rate $\delta/2$ is related to the effective mass of the system. The previous formula is interesting mainly because it allows to measure experimentally tunneling rates in terms of oscillation frequencies. We conclude noticing the single-particle nature of the phenomenology we have discussed, which persists, however, at large atomic densities, where non condensate atomic systems behave as normal gases.

Energetic instabilities

It is well known that a uniform condensate is stable against energetic instabilities when the velocity of the fluid is smaller than the sound velocity. This is the celebrated Landau criterion for superfluidity, which has been demonstrated experimentally (with some caveats) in superfluid helium and alkaly condensates. It is natural to ask how the Landau criterion would be modified by the presence of the periodic potential [4, 2]. To be more precise, let's consider a condensate in a Bloch state, with a given quasimomentum. The question is if the condensate will continue to flow without dissipation in the presence of a defect. The solution of such a problem can be obtained in the context of the Bogoliubov analysis, studying the quasi-particle excitations created on top of the large amplitude condensate wave. The result is that it is still possible to cast the Landau criterion in a similar form as in the uniform case. To this aim, we define the group velocity, different from the local velocity given by the gradient of the phase, as $v_g \equiv \partial E/\partial k \propto \delta \sin(\pi k/q_B)$ and the sound velocity $c(k) = c_0 \sqrt{\cos(\pi k/q_B)}$. Here k the quasimomentum of the system, q_B the Bragg momentum and c_0 the sound velocity for the condensate at rest in the reference frame of the lattice (see Research Line on "Dynamics of BEC in optical lattices"). The condensate flows without dissipating kinetic energy if the group velocity of the fluid is smaller than the sound velocity $v_q^2 < c^2$. Despite the formal similarity, the Landau criterion in the discrete case has a nontrivial dependence on the quasimomentum k, which brings to a quite different phenomenology as compared with a Galilean invariant system. We stress, indeed, that not only the group velocity, but also the sound velocity depends on the quasimomentum of the condensate.

The sound velocity decreases for increasing quasimomentum k and it can become immaginary, the consequences of which will be outlined in the next subsection.



Figure 1: (a) Bogoliubov spectrum for condensate with nonlinearity $gn = 0.5E_R$ and $k = q_B$ in the uniform system; $\omega = 0$ for $\bar{q} = -1.75q_B$ (red dot); (b) Time evolution of the coefficients of the Bogoliubov amplitudes in the presence of a δ -like defect in the linear regime: $|c_{\bar{q}}|^2$ grows like t^2 , while the other coefficients oscillate in time. An analogous behaviour is found in the presence of a lattice.

Modulational and Parametrical Instabilities

The presence of the lattice introduces a new important dissipation mechanism which does not have an analog in the translational invariant case (when the scattering length is positive). The sound velocity depends on the quasimomentum of the condensate, and it becomes immaginary when $\cos(\pi k/q_B) < 0$ [6, 2, 4, 5]. Then, the system becomes dynamically unstable. The dissipation dynamics of the dynamical instability (DI) is quite different from the energetic instability (EI). The energetic instability is turned on when a stationary state of the system with group velocity larger than the sound velocity hits a defect. Then, the amplitudes of the quasiparticles grow polynomially in time [7]. In the absence of the defect, small amplitude density and phase fluctuations continue to remain small during the dynamics. In a dynamically unstable system, instead, any small density fluctuation would grow exponentially fast, even in the absence of any external defect.

The breakdown of superfluidity due to the dynamical instability is so rapid and



Figure 2: Disruption of the dipole oscillation of a condensate in an external harmonic driving field (green line) for large enough initial displacement: (a) density of the condensate (blue line) and initial occupied region (shaded area); (b) randomization of the relative phase ϕ .

effective, that it can almost istantaneously drive the system to an effective insulator state. Such state is characterized by a de-locking of the interwell phases ϕ which start to "run" at different velocities. As a consequence, the system cannot support a coherent flow of atoms. If an harmonic potential is superimposed to the optical lattice, the DI localizes the atoms in one side of the trap. More specifically, with a parabolic confinement $\epsilon_{\ell} = \Omega \ell^2$, the DI occurs for a critical initial displacement $\xi_{cr}^2(0) = \delta/\Omega$. The experimental evidences of these effects have been reported in [8]. The onset of DI is predicted also for shallow lattices, i.e. beyond the validity of the discretized approach, as shown by the solution of a one dimensional GPE in Fig.(2) [2]. The role of transverse degrees of freedom in the dynamical instability of elongated 3D condensates has been recently investigated in [9, 10].

When some of the parameters of the DNLS (like, for instance, the tunneling rate) are periodically modulated in time, an additional *parametric* instability becomes possible. These domains of instability appear due to parametric resonance but, in contrast to an ordinary resonance, where the growth is proportional to the time variable, here the growth time is exponential as in the case of the modulational instability [11].

To conclude, we mention that the dynamical instability we have discussed can occur in the absence of optical lattices when the condensate has a negative scattering length and the strength of the interaction is, roughly speaking, larger than the gap between the ground and the first excitation energy. The quantum analysis of the problem, however, reveals that the condensate is *always* unstable. The growing time of excitations



Figure 3: Inhibition of expansion of an interacting condensate in a periodic potential: (a) density of the condensate (blue line) and initial occupied region (shaded area); (b) randomization of the relative phase ϕ .

increases with the total number of atoms, so that GPE is still reliable when $N \sim 10^3$. However, close to the instability point, excitations grow only logarithmically with the number of atoms, $\sim ln(N)$, and become crucial even for relatively large condensates [12].

Dynamical localizations

Related with the above discussion on dynamical instabilite is the phenomenon of localization of a condensate wavepacket. A condensate wavepacket in free space will obviously spread with time if the interatomic scattering length is positive. A "discrete" wavepacket trapped in a horizontal periodic potential instead would not spread, if a control parameter, proportional to the ratio between the interation and the tunneling energies, is above a critical value (see [2] and previous works by some of the authors). Above such a value, the wavepacket remains localized, namely, its width remains constant. This phenomenon is also accompained by a randomization of the relative phases between condensates trapped in different wells, as shown in Fig.3. First experimental evidences on the self-trapping with weakly coupled BECs are reported in [13, 14].

Defect-mediated finite-temperature transition in a planar lattice

At a critical temperature lower than the temperature T_{BEC} at which condensation in each well occurs, 2D lattices of BECs undergo a phase transition to a superfluid regime where the phases of the single-well condensates are coherently aligned [15]. This phenomenon is closely related to the Berezinskii-Kosterlitz-Thouless (BKT) transition



Figure 4: Lattice Fourier transform with: (A) a single lattice vortex; (B) a lattice vortex-antivortex pair; (C) $T = 0.5\delta/k_B$; (D) $T = 1.1\delta/k_B$. Figures (C) and (D) are snapshot Monte Carlo after reaching equilibrium. The lattice is 20×20 .

in the two-dimensional XY model, to which the finite temperature Hamiltonian of the BEC maps under specific conditions. The experimentally measurable quantity which provides the signature of this transition is the central peak of the interference pattern obtained after removing the confining trap see Fig.(4), which is the direct analog of the magnetization. In the low-temperature phase, characterized by the presence of bound vortex-antivortex pairs, the spatial correlations exhibit a power-law decay; above a critical temperature T_{BKT} , the decay is exponential and there is a proliferation of free vortices.

- A. Smerzi and A. Trombettoni, Chaos, Focus issue on "Nonlinear localized modes: fundamental concepts and applications", 13, 766 (2003); A. Smerzi and A. Trombettoni, Phys. Rev. A68, 023613 (2003)
- [2] C. Menotti, A. Smerzi and A. Trombettoni, New. J Phys. 5, 112 (2003)
- [3] F.S. Cataliotti, S. Burger, C. Fort, P. Maddaloni, F. Minardi, A. Trombettoni, A. Smerzi, and M. Inguscio, Science 293, 843 (2001)

- [4] B. Wu and Q. Niu, Phys. Rev. A 64, 061603(R) (2001); M. Machholm, C.J. Pethick and H. Smith, Phys. Rev. A 67, 053613 (2003)
- [5] The MI instability in the context of the DNLS has been predicted by Yu.S. Kivshar and M. Peyrard, Phys. Rev. A 46, 3198 (1992).
- [6] A. Smerzi, A. Trombettoni, P.G. Kevrekidis, and A.R. Bishop, Phys. Rev. Lett. 89, 170402 (2002).
- [7] S. Ianeselli, C. Menotti and A. Smerzi, unpublished.
- [8] F. S. Cataliotti, L. Fallani, F. Ferlaino, C. Fort, P. Maddaloni, and M. Inguscio, New Journal of Physics 5, 71 (2003)
- [9] F. Nesi, M. Modugno, cond-mat/0310659
- [10] M. Modugno, C. Tozzo, F. Dalfovo, cond-mat/0405653
- [11] Z. Rapti, P.G. Kevrekidis, A. Smerzi and A.R. Bishop, J. Phys. B: At. Mol. Opt. Phys. 37, S257 (2004)
- [12] G. Berman, A. Smerzi and A.R. Bishop, Phys. Rev. Lett. 88 (2002) 120402
- [13] O. Morsch, M. Cristiani, J.H. Müller, D. Ciampini, and E. Arimondo, Phys. Rev. A 66, 021601 (2002)
- [14] M. Oberthaler, private communication
- [15] A. Trombettoni, A. Smerzi and P. Sodano, cond-mat/0404381; A. Smerzi, P. Sodano ad A. Trombettoni, J. Phys. B37 s265 (2004)

ULTRACOLD FERMI GASES

In the last two years impressive progress has been achieved in the physics of ultracold Fermi gases. Major goals are the achievement of Fermi superfluidity and the investigation of the long sought BEC-BCS crossover, first proposed to understand the behaviour of high T_c superconductors. The tunability of the two-body scattering length and the corresponding possibility of realizing highly correlated configurations are stimulating new experimental and theoretical work. In particular in the so-called unitarity regime, corresponding to values of the scattering length much larger than the relative wavelength of the scattering atoms, the many-body wave function of the system cannot be described neither in terms of a simple condensate of molecules, nor using standard BCS theory. At present (May 2004) Bose-Einstein condensation of molecules built with pairs of fermions has been experimentally achieved in a few laboratories and first measurements of the physical properties of these systems have already become available. A stringent test of the achievement of the superfluid phase is however still missing. Some significant contributions made by the Trento team in this rapidly evolving field of research are briefly described below (see also the discussion about the rotation of Fermi gases in the Research Line on "Rotating quantum gases" and the implementation of a qubit register discussed in the Research Line on "Quantum Information applications").

Expansion of an ultracold Fermi gas

The expansion of an atomic gas following the sudden removal of the confining trap is known to provide valuable information on the state of the system and on the role of interactions. In particular in the superfluid phase, where the macroscopic dynamics of the gas are governed by the equations of hydrodynamics, one predicts anisotropic expansion if the gas is released from an anisotropic trap. This feature was explored in [1] in the case of a superfluid Fermi gas and compared with the prediction of a non superfluid gas in the collisionless regime where the expansion is ballistic and hence asymptotically isotropic. The difference between the two predictions is striking (see Fig.1) and provides a first important criterion for exploring the superfluid behaviour of the gas.

One should however recall that the observation of anisotropic expansion (see Fig.2) is not a sufficient criterion for superfluidity [2]. In fact the anisotropic expansion might be caused by frequent collisions giving rise to a hydrodynamic regime also in the normal phase.

In [3] we have investigated the role of collisions and their consequences on the



Figure 1: Aspect ratio as a function of time for the expansion of the normal collisionless (lower blue curve) and superfluid phase (upper green curve) for $\lambda = \omega_z/\omega_\perp = 0.1$.



Figure 2: Anysotropic expansion observed in [2].



Figure 3: (a) Temperature $\tau = T/T_F$ after expansion as a function of the parameter $\xi = (\lambda N)^{1/3} (k_F a)^2$ for different values of $\lambda = \omega_z / \omega_\perp$. The gas prior to expansion is at zero temperature; (b) Column density after expansion as a function of radial distance for $\lambda = 0.03$ and $\xi = 1$ (black line), compared to the result of expansion in the collisionless (purple) and hydrodynamic (light blue) limits. Both axes are in arbitrary units.

expansion of an ultracold Fermi gas. Actually even at very low temperature collisions can be effective in a Fermi gas [4], consistently with the Pauli principle. This is due to the appearence of large deformations in momentum space during the expansion from a very deformed trap. By solving the Boltzmann equations with the method of the averages we have calculated the density profile of the expanding cloud. We find that in a dilute gas collisions do not modify significantly the aspect ratio with respect to the predictions of ballisitic expansion. However they are responsible for a significant increase of entropy, associated with a thermal broadening of the density profile (see Fig.3). This effect, which is predicted to be larger and larger as the trap deformation is increased, has not yet been investigated experimentally and might provide a new useful criterion to identify the transition from the normal to the superfluid phase. In fact in a superfluid, collisions are suppressed with the result that the gas will not exhibit any thermal broadening during the expansion, differently from what we predict for the normal phase.

Collective oscillations of superfluid Fermi gases

Collective oscillations in a superfluid are governed by the equations of hydrodynamics. In the case of Bose-Einstein condensed gases the predictions of hydrodynamics [5] have



Figure 4: Radial compressional frequency in an elongated trapped Fermi gas close to a Feshbach resonance as a function of the dimensionless parameter $(N^{1/6}a/a_{ho})^{-1}$. The mean field theory in the molecular BEC regime predicts $\omega/\omega_{\perp} = 2$, while in the unitarity limit one expects $\omega/\omega_{\perp} = \sqrt{10/3} \sim 1.83$. The full line for a > 0corresponds to the expansion accounting for beyond mean field (m.f.) corrections in the molecular condensate with $a_M = 0.6a$, while the full line for a < 0 corresponds to the expansion accounting for the mean field correction in the BCS phase. The dot-dashed line is a schematic interpolation between the two asymptotic regimes.

been accurately tested experimentally both concerning surface and compression modes. The frequency of the compression modes depends explicitly on the equation of state which, in the case of dilute Bose gases, is characterized by a simple linear density dependence in the chemical potential. Viceversa surface modes are independent of the equation of state, but differ significantly from the predictions of the non-interacting model. In a Fermi gas the equation of state varies significantly across the BEC-BCS crossover and consequently the study of these oscillations can provide a unique indicator of the equation of state. In [6] this problem has been addressed and explicit results for the collective frequencies have been obtained in relevant asymptotic regimes, like the unitarity regime, the molecular BEC condensate and the BCS asymptotic regime. The results, shown in Fig.4, exhibit a non trivial non monotonous dependence of the radial compression frequency as a function of the value of the scattering length.

Recent experiments [7] have partially confirmed these predictions, but several open questions remain to be understood. In particular while the frequency of the axial compression mode well agrees with the theoretical value in the unitarity limit, in the experiment carried out at Innsbruck a value smaller than the theoretical prediction was



Figure 5: Figure from [8]: Measured frequency Ω_r and damping rate Γ_r of the radial compression mode, normalized to the trap frequency (sloshing mode frequency) ω_r . The vertical dotted line marks the resonance position at 837(5) G. The star indicates the theoretical expectation of $\Omega_r/\omega_r = \sqrt{10/3}$ in the unitarity limit. A striking change in the excitation frequency occurs at ~910 G (arrow) and is accompanied by anomalously strong damping.

observed for radial mode. Furthermore no evidence of the beyond mean field effect predicted by theory was found in the molecular BEC regime. Finally in the case of gases interacting with negative scattering lengths one observes a transition which is likely a transition to the collisionless normal Fermi regime where the frequency is close to the non-interacting value (see Fig.5).

Fermi gases in 1D optical lattices

In the presence of a periodic optical potential quantum gases exhibit new interesting features (see Research Lines on "Dynamics of a BEC in optical lattices" and "Breakdown of coherence in optical lattices"). So far most experimental and theoretical works have been devoted to the case of bosons. In a series of recent papers we have provided a first investigation of the dynamic behaviour of a harmonically trapped 3D Fermi gas in the presence of a 1D optical potential. A major motivation of these studies is to understand whether a non superfluid Fermi gas can exhibit center of mass oscillations

around the minimum of the harmonic well. In the case of a Bose-Einstein condensate the oscillation is ensured by the superfluid nature of the gas and is associated with a coherent tunnelling through the barrier separating two consecutive wells (Josephson-like effect). In a joint experimental-theoretical collaboration with the team of M. Inguscio in Florence [9] we have investigated the problem in the case of a single spin species where interactions between atoms are negligible and the dynamical description is correctly provided by the collisionless picture. In this paper we have shown that if the Fermi energy E_F is larger than the bandwidth 2δ the gas exhibits an insulating regime. This is the consequence of the fact that, at low temperature, the relevant single-particle orbits in the $z - p_z$ phase diagram are not closed and cannot consequently drive the oscillation of the center of mass through the minimum of the harmonic potential (Fig.6). This peculiar feature has been explicitly checked experimentally, thereby proving the insulating nature of the Fermi gas for relatively large values of E_F . The role of collisions between atoms belonging to two different spin species has been investigated in [10]. In particular in this work we have proven that if the gas is in the collisional hydrodynamic regime the center of mass oscillation is overdamped if $E_F \geq 2\delta$ because of the occurrence of umklapp processes. Umklapp collisions are scattering events which do not conserve the momentum of the colliding atoms, part of the momentum being transferrd to the lattice. Our results on the collisionless and collisonal regime suggest that if $E_F \geq 2\delta$ the center of mass oscillation of a Fermi gas is possible only in the

presence of superfluidity.

Microscopic theory of the BEC-BCS crossover

In a dilute Fermi gas, the scattering length a_F can be varied from negative to positive values by tuning the magnetic field close to a Feshbach resonance. This constitutes, in principle, a perfect playground for the study of the crossover between BCS and BEC superfluidity. From the theoretical point of view, the crossover problem has been investigated originally by tuning the single parameter $k_F a_F$ (with k_F being the Fermi wavenumber), and tracking the behavior of the pair correlation functions [11]. The resulting theory correctly shows the link between BCS pairing (for $a_F < 0$ and $k_F |a_F| \ll 1$) and Bose-Einstein condensation of two-fermion dimers (for $a_F > 0$ and $k_F a_F \ll 1$). At T = 0 the theory is fairly simple and allows one to extract semianalytically important, although approximate, information on the microscopic state of the system. In [12] we used this approach to calculate the momentum distribution of a harmonically trapped gas in the crossover region. We showed that the relevant parameter to describe the crossover for a harmonically trapped gas is $N^{1/6} a_F/a_{ho}$,



Figure 6: Phase trajectories for a trapped 1-D Fermi gas in a lattice at T = 0, just before and after the displacement of the trap (figs. A and B, respectively), and their dynamical evolution (figs. C and D). The ordinate and abscissa are in units of $\tilde{p}_z \equiv p_z d/2\hbar$ and $\tilde{z} \equiv \sqrt{m\omega_z^2 z^2/4\delta}$.

with a_{ho} being the harmonic oscillator length and N the number of atoms. By varying this parameter from small and negative through infinity to small and positive, the momentum distribution changes smoothly from that of a virtually non-interacting Fermi gas in the BCS limit, to the one arising from the atomic momentum distributions inside the molecules, $n(k) \propto 1/(1 + k^2 a^2)$, in the BEC limit. The results are shown in Fig. 7.

Recently it has been shown that the above theory has a major flaw in that it predicts an incorrect value for the molecule-molecule scattering length in the BEC limit ($a_B = 2a_F$). A direct solution of the four-body Schrödinger equation in a vacuum yields $a_B \approx 0.6a_F$ [13], and also the first experimental data collected were consistent with the $0.6a_F$ result [8]. This raised a serious problem on what should be done to reproduce the correct molecule-molecule interaction. In the past, in order to give a more convenient description of the resonance, two-channel models had been proposed, which included explicitly a bosonic state in the Hamiltonian [14]. The scattering length a_F was then determined by the coupling between the bosonic closed channel and the fermionic open channel, and by the position of the bound state with respect to the scattering continuum. However, even in the two-channel models, if one keeps only two-body correlations, similarly as one does in traditional crossover theories, the wrong moleculemolecule interaction is obtained. In a recent work [15], we have shown that one has to



Figure 7: Momentum distribution of a trapped gas in the BEC (solid line), unitarity (long dashed line) and BCS regime (short dashed line). The plot shows, for the molecular BEC regime $n(k) \propto 1/(1 + k^2a^2)$ with $k_F^0a = 0.5$, and for the BCS regime the free fermion distribution. The momentum distributions are multiplied by k^2 to emphasize the large k behavior, and they are normalized so that $\tilde{k} = k/k_F$ and $\int d^3k n(k) = 1$.

include at least correlations between a composite boson and two fermions, in order to obtain the full four-fermion scattering physics and thus the correct molecule-molecule interaction in the BEC limit. As a consequence, this allows to determine correctly all the quantities sensitive to interactions in the BEC limit, such as the equation of state, the spatial extent of the cloud, the collective modes, the momentum distribution and the vortex core structure. We pointed out that the inclusion of those higher order correlations does not only bring quantitative changes in the results, but a qualitative change in the whole picture of the crossover.

Pairing fluctuations in trapped Fermi gas

In a Fermi gas with a weak attractive interaction, mean-field BCS theory predicts the appearence of a non-vanishing order parameter $\Delta_{BCS}(T)$ below a critical temperature T_c which depends strongly both on the interaction strength and on the geometry of the system. For low number of atoms in the trap, or in deep optical lattices in which few atoms are trapped in each lattice site, thermal fluctuations of the order parameter close to the critical temperature become important. They are precursors of the superfluid transition and extend up to 2 to 3 times T_c . In Ref. [16] we have studied this problem

for a harmonically confined gas, since the harmonic confinement well represents both the usual traps used in experiments and the potential felt in each site of a deep lattice. We have shown that: i) Pairing fluctuations are inhomogeneous due to the presence of the trap, and they are enhanced towards the center of the cloud. ii) An analytic approximation can be found for the spatial form and amplitude of the fluctuations which very well reproduces the full numerical calculations. iii) There is a parity effect, i.e. pairing fluctuations show a maximum or a minimum at the centre of the trap depending on the value of the last occupied oscillator shell being even or odd. iv) The pairing fluctuations can be observed, for instance, by measuring anomalous correlations between the densities at opposite ends of the cloud after the free expansion which follows the removal of the trap. These correlations are non-vanishing only due to BCS pairing. Mean-field theory predicts that they appear below T_c , while they are also present above T_c because of pairing fluctuations. An analytical expression can be found also for the anomalous correlations. v) The fluctuations are expected to be large and measurable in current experiments with fermions in deep optical lattices.

- [1] C. Menotti, P. Pedri and S. Stringari, Phys. Rev. Lett. 89, 250402 (2002)
- [2] K.M. O'Hara, S.L. Hemmer, M.E. Gehm, S.R. Granade, J.E. Thomas, Science 298, 2179 (2002)
- [3] B. Jackson, P. Pedri, and S. Stringari, cond-mat/0404175, Europhys. Lett., in press
- [4] S. Gupta, Z. Hadzibabic, J.R. Anglin, and W. Ketterle, Phys. Rev. Lett. 92, 100401 (2004)
- [5] S. Stringari, Phys. Rev. Lett. 77, 2360 (1996)
- [6] S. Stringari, Europhys. Lett. 65, 749 (2004)
- [7] J. Kinast, S.L. Hemmer, M.E. Gehm, A. Turlapov, J.E. Thomas, Phys. Rev. Lett. 92, 150402 (2004);
- [8] M. Bartenstein, A. Altmeyer, S. Riedl, S. Jochim, C. Chin, J. Hecker Denschlag, R. Grimm, Phys. Rev. Lett. 92, 203201 (2004)

- [9] L. Pezzé, L. Pitaevskii, A. Smerzi, S. Stringari, G. Modugno, E. DeMirandes, F. Ferlaino, H. Ott, G. Roati, M. Inguscio, cond-mat/0401643, Phys. Rev. Lett., in press
- [10] G. Orso, L.P. Pitaevskii and S. Stringari, cond-mat/0402532, Phys. Rev. Lett., in press
- [11] A. J. Leggett, in Modern Trends in the Theory of Condensed Matter (Springer-Verlag, Berlin, 1980), pp. 13–27; P. Nozières and S. Schmitt-Rink, J. Low Temp. Phys. 59, 195 (1985); C. A. R. Sá de Melo, M. Randeria, and J. R. Engelbrecht, Phys. Rev. Lett. 71, 3202 (1993)
- [12] L. Viverit, S. Giorgini, L. P. Pitaevskii, and S. Stringari, Phys. Rev. A 69, 013607 (2004)
- [13] D. S. Petrov, C. Salomon, G. V. Shlyapnikov, cond-mat/0309010
- M. Holland, S. J. J. M. F. Kokkelmans, M. L. Chiofalo, and R. Walser, Phys. Rev. Lett. 87, 120406 (2001); E. Timmermans, K. Furuya, P. W. Milonni, A. K. Kerman, Phys. Lett. A 285, 228 (2001); Y. Ohashi and A. Griffin, Phys. Rev. Lett. 89, 130402 (2002)
- [15] M. J. Holland, C. Menotti, and L. Viverit, cond-mat/0404234
- [16] L. Viverit, G. M. Bruun, A. Minguzzi, and R. Fazio, cond-mat/0402620

BOSE-FERMI MIXTURES

To date boson-fermion mixtures have come to the attention of the cold gases community mostly as a functional tool for sympathetic evaporative cooling of a fermionic gas to its quantum degenerate state [1, 2, 3]. They were not yet given serious direct experimental attention as an interesting system in its own right, with the exception of the first experimental data collected from the group at LENS in Florence. Nevertheless mixtures have extremely interesting physical properties and one can expect more and more experiments to come. As a matter of fact much experimental and theoretical work was carried out on mixtures of ³He and ⁴He fluids (see for instance [4] and references therein). Dilute quantum gases, however, present several advantages over the helium fluids, namely the simplicity of the interaction, the gaseous (weakly correlated) state of the system, and a much larger region of stability, which allows to explore the properties of mixtures over a wide range of possible concentrations [5, 6, 7]. Below we report the work that has been done in this context by the Trento group, part of which was carried out in collaboration with the group in Florence.

Stability properties of K-Rb mixtures

The basic problem of determining the stability diagram of a boson-fermion mixture was investigated both experimentally and theoretically in Ref. [6] for the case of harmonically trapped ⁴⁰K and ⁸⁷Rb. The interaction between a potassium and a rubidium atom is attractive $(g_{BF} < 0)$. This leads to a collapse of the mixture for sufficiently high densities. The region of instability can be calculated theoretically by simultaneously solving the Gross-Pitaevskii (GP) equation for the Bose gas and the Thomas-Fermi (TF) equation for the Fermi gas. The two equations are coupled by the mean-field interaction terms, which for a non-uniform gas are $g_{BF}n_F(\mathbf{r})$ and $g_{BB}n_B(\mathbf{r})$, with $n_F(\mathbf{r})$ and $n_B(\mathbf{r})$ being the fermion and boson densities respectively. Experimentally a small gravitation sag causes the minima of the trapping potentials felt by the two clouds to be slightly displaced. This was accounted for in the calculations. When the selfconsistent iterations fail to converge the cloud is expected to collapse. The position of the instability threshold depends very sensitively on the boson-fermion interaction (see Fig. 1), and thus on the atom-atom s-wave scattering length a_{BF} [8]. For instance, the critical boson number scales as $N_B^{crit} \sim |a_{BF}|^{-12}$. The value of a_{BF} can be determined independently from collisional rate measurements within a 20% accuracy. The strong dependence of the stability threshold on a_{BF} , however, further allows a four-fold improvement in the estimate of the scattering length, which is predicted to



Figure 1: Region of stability of the Fermi-Bose mixture, as a function of the number of atoms. The black dots are the experimental points; lines are the theoretical prediction for the boundary between the stable (left) and collapse (right) regions, for three values of the inter-particle scattering length (in units of the Bohr radius a_0): $a_{BF} = -380 \ a_0$ (dotted line), $a_{BF} = -395 \ a_0$ (continuous line), $a_{BF} = -410 \ a_0$ (dashed-dotted line). The marked dots are found very close to the instability (see text).

be $a_{BF} = -395 \pm 15 \ a_0$.

The measurements reported in Ref. [6] were carried out virtually at T = 0. For this reason the coupled zero-temperature GP and TF equations were sufficient to analyze the experimental observations. In Ref. [7] the effects of finite temperature on the stability were theoretically considered. The system was studied using a Hartree-Fock-Bogoliubov-Popov theory especially developed for a Fermi-Bose mixture. It was found that temperature tends to stabilize the mixture. For fixed boson-boson and bosonfermion scattering lengths, the instability line moves towards higher and higher values of N as the temperature increases (see Fig. 1). The physical reason for the effect is that the higher kinetic energy due to thermal motion contrasts the fermion-boson attraction which leads to the collapse.

Ground-state properties of mixtures

Whenever a boson and a fermion cloud are allowed to overlap, the system really becomes a single fluid, but it is useful to consider how the properties of the isolated gases are affected in the solution. In Ref. [9] binary mixtures were systematically investigated, with particular attention to the change in the properties of the Bose condensate due to the presence of fermions. The most straightforward consequence is that an interaction energy arises, leading to a modified thermodynamics. Also the microscopic and macroscopic properties of the Bose condensate are affected, however. In [9] the Green's function formalism was applied to obtain the effects to second order in the Bose-Fermi interaction on the spectrum of phonon excitations, the boson momentum distribution, the fraction of Bose-Einstein condensate and the superfluid density. The extensive details can be found in the publication, here we only highlight the most important features. The crucial quantities which determine the character of the mixture are the ratio $w = m_B/m_F$ and the product $k_F \xi_B$. Here k_F is the Fermi wavenumber which depends on the fermion density n_F as $(6\pi^2)^{1/3}n_F^{1/3}$, while ξ_B is the boson coherence length given by $\xi_B = 1/\sqrt{8\pi n_B a_{BB}}$. By varying the above parameters in all possible ways within the stability region one finds several striking features. The most notable are: i) The boson momentum distribution is strongly affected by the fermions resulting in a suppression of the occupation at low momenta and in a long tail at large momenta (see Fig. 2). ii) For large values of w one predicts a decrease in the condensate depletion as compared to the Bose gas alone. iii) For small values of w, the fermions act as nearly static impurities and the superfluid density may become smaller than the condensate fraction at large fermion concentrations (see Fig. 3).

In this work the fermions were assumed to be in the normal state. It should be noted that, at sufficiently low temperatures, the fermions may also become superfluid via *p*-wave pairing due to the attraction induced by the exchange of phonons of the Bose-Einstein condensate (background polarization). This possibility was considered in Ref. [10]. The associated critical temperature is sufficiently low to justify ignoring the effect in a first approximation.

Critical temperature of Bose-Einstein condensation of a trapped Bose-Fermi mixture

When bosons and fermions are harmonically trapped together, the critical temperature for Bose-Einstein condensation of the Bose gas is modified. This is because the critical temperature is related to the central density of the gas, which in turn is affected by the presence of fermions. If the boson-fermion interaction is weak, and one is therefore far from the instability threshold, the shift in T_c can be evaluated to first order in a_{BF} . The calculation was carried out in Ref. [11], where it was found that

$$\left(\frac{\delta T_c}{T_c^0}\right)_{BF} = -\frac{2^{5/3}}{3^{5/6}\pi\zeta(3)} \left(\frac{m_F}{m_B} + 1\right) \frac{a_{BF}}{\ell_F} N_F^{1/6} \cdot F(\tilde{T}_F, \alpha) .$$



Figure 2: Boson momentum distribution of a ${}^{87}\text{Rb}$ - ${}^{40}\text{K}$ mixture (solid line). The momentum distribution of the pure boson system (short dashed) and the Bose-Fermi contribution (long dashed) are also shown. In the axes labels x stands for $k\xi_B$.



Figure 3: Condensate depletion (solid line) and normal fluid fraction (dotted line) of a hypothetical mixture with mass ratio $m_B/m_F=0.1$ as a function of the fermion density.

 $\ell_F = \sqrt{\hbar/m_F \omega_F}$ is the fermionic oscillator length, N_F the number of fermions, and $T_C^0 \simeq 0.94\hbar\omega_B N_B^{1/3}$ the BEC critical temperature for a non-interacting Bose gas in a harmonic trap. The function F has a complicated expression which can be found in the reference. The important feature is that it depends on the two parameters $\alpha = m_B \omega_B^2/m_F \omega_F^2$ and $\tilde{T}_F = T_F/T_c^0$, with T_F being the Fermi temperature and m_i and ω_i the masses and trapping frequencies respectively. In general the function F must be determined numerically, but analytic expressions can be found in the two limits $\tilde{T}_F \gg 1$ (Thomas-Fermi) and $\tilde{T}_F \ll 1$ (Boltzmann). The predicted shift should be compared with the omologous one arising from the boson-boson interaction alone: $(\delta T_c/T_c^0)_{BB} = -1.33 (a_{BB}/\ell_B) N_B^{1/6}$ [12]. It is interesting to notice the formal analogy between the two shifts. From a quantitative point of view, their ratio depends strongly on the values of α , \tilde{T}_F (and thus N_F/N_B), and a_{BF}/a_{BB} .

- F. Schreck, L. Khaykovich, K. L. Corwin, G. Ferrari, T. Bourdel, J. Cubizolles, and C. Salomon, Phys. Rev. Lett. 87, 080403 (2001).
- [2] G. Roati, F. Riboli, G. Modugno, and M. Inguscio, Phys. Rev. Lett. 89, 150403 (2002).
- [3] Z. Hadzibabic, C. A. Stan, K. Dieckmann, S. Gupta, M. W. Zwierlein, A. Görlitz, and W. Ketterle, Phys. Rev. Lett. 88, 060402 (2002).
- [4] G. Baym and C. J. Pethick, Landau Fermi-liquid Theory: Concepts and Applications (John Wiley, New York, 1991).
- [5] L. Viverit, C. Pethick and H. Smith, Phys. Rev. A 61, 053605 (2000).
- [6] M. Modugno, F. Ferlaino, F. Riboli, G. Roati, G. Modugno, and M. Inguscio, Phys. Rev. A 68, 043626 (2003).
- [7] X.-J. Liu, M. Modugno, and H. Hu, Phys. Rev. A 68, 053605 (2003).
- [8] The interaction is related to the scattering length by $g_{BF} = 2\pi \hbar^2 a_{BF}/m_R$, with $m_R = m_B m_F/(m_B + m_F)$ being the reduced mass.
- [9] L. Viverit and S. Giorgini, Phys. Rev. A 66, 063604 (2002).
- [10] D. V. Efremov and L. Viverit, Phys. Rev. B 65, 134519 (2002).

- [11] A. P. Albus, S. Giorgini, F. Illuminati, and L. Viverit, J. Phys. B 35, L511 (2002).
- [12] S. Giorgini, L. P. Pitaevskii, and S. Stringari, Phys. Rev. A 54, R4633 (1996).

NUMERICAL SIMULATIONS IN QUANTUM GASES

Stochastic numerical simulations provide us with a very powerful tool to investigate systems with many degrees of freedom. In particular, quantum Monte Carlo techniques have been widely applied to the study of systems at low temperatures, where the effects of quantum degeneracy are important. At zero temperature the Variational Monte Carlo (VMC) and Diffusion Monte Carlo (DMC) solve the many-body Schroedinger equation for the ground state and for excited states of the system. The VMC method was first introduced in the seminal work by McMillan [1] to study the ground state properties of liquid ⁴He. The VMC technique calculates the expectation value of the Hamiltonian of the system on a trial wavefunction, which is parametrized in terms of a set of variational parameters. The high-dimensional integral corresponding to this expectation value is performed with Monte Carlo techniques and the value obtained gives an upper bound to the exact ground state energy of the system. The variational estimate can be improved through minimization with respect to the parameters of the wavefunction. The ground state energy of a system of bosons can be calculated using the DMC method, which solves the time-independent Schroedinger equation by evolving in imaginary time the wavefunction of the many-particle system. Apart from statistical errors, the DMC method is exact. However, it can not be used directly to calculate excited states or the ground state of a system of fermions. In this case, due to the presence of nodal surfaces, the wavefunction can not be interpreted as a probability distribution and the DMC algorithm can not be applied. Upper bounds to the energy of excited states or fermionic ground states can be obtained using the fixed node DMC method, where the sign problem is overcome by imposing a given nodal constraint and by applying the DMC algorithm in the regions of configuration space separated by the nodal surface. The Green's function Monte Carlo method, which is equivalent to the DMC one, was used in Ref. [2] to calculate the ground-state properties of liquid 4 He. The thermodynamic properties and correlation functions of a system of bosons at finite temperature can be calculated exactly using the path integral Monte Carlo method (PIMC). One of the most impressive applications of this technique is the study of the superfluid transition in liquid ⁴He [3]. The first experimental realizations of trapped quantum gases have been obtained in large three-dimensional traps in conditions of strong diluteness. Simple mean-field approaches provide a very accurate description of these systems and there is no need to resort to more sophisticated techniques. Nevertheless, quantum Monte Carlo methods have been applied to dilute quantum gases to investigate beyond mean-field effects at zero temperature [4], the temperature dependence of the condensate fraction in trapped systems [5] and the highly non trivial

problem, which can not be addressed using mean-field methods, of the Bose-Einstein transition temperature in homogeneous systems with weak repulsive interactions [6]. In more recent years, quantum gases have been realized in restricted geometries, such as one- or two-dimensional configurations and in deep optical lattices. Furthermore, the strength of the interactions has been varied over a very wide range utilizing Feshbach resonances of the scattering amplitude. These experimental achievements opened the possibility of realizing strongly correlated systems where mean-field approaches are no longer valid and the application of quantum Monte Carlo methods is very useful. Quantum Monte Carlo techniques have been applied to the study of the superfluid-Mott insulator quantum phase transition in systems of cold atoms in optical lattices [7] and of the unitarity limit of fermionic atoms close to a Feshbach resonance [8]. The research activity of the Trento BEC-INFM group in this field has been devoted to the study using DMC of systems with disorder and of one-dimensional systems and to the development of a new quantum Monte Carlo method based on stochastic evolution of classical fields. Some of the most relevant contributions are discussed in more details below.

Superfluidity versus Bose-Einstein condensation in a Bose gas with disorder

The study of disordered Bose systems has attracted in the recent past considerable attention both theoretically and experimentally. The problem of boson localization, the superfluid-insulator transition and the nature of elementary excitations in the presence of disorder have been the object of theoretical investigations [9] and Monte-Carlo numerical simulations [10], both based on Hubbard or equivalent models on a lattice. Disordered Bose systems are produced experimentally in liquid ⁴He adsorbed in porous media, such as Vycor or silica gels (aerogel, xerogel). The suppression of superfluidity and the critical behavior at the phase transition have been investigated in these systems in a classic series of experiments [11]. In this work [12] we investigate the effects of disorder on BEC and superfluidity in a Bose gas at zero temperature. As a model for disorder a uniform random distribution of static impurities is assumed. This choice provides us with a reasonable model for ⁴He adsorbed in porous media and might also be relevant for trapped Bose condensates in the presence of heavy impurities. In addition, the quenched-impurity model allows us to derive analytical results in the weak-disorder regime using the Bogoliubov model and can be implemented in a quantum Monte-Carlo simulation. By using the Diffusion Monte-Carlo method we calculate the superfluid and the condensate fraction of the system as a function of density and strength of disorder. The presence of disorder depletes the condensate and reduces the



Figure 1: Superfluid fraction ρ_s/ρ (solid symbols) and condensate fraction N_0/N (open symbols) as a function of the gas parameter na^3 for different values of the strength of disorder R. Solid lines and dashed lines correspond respectively to the superfluid density and the condensate fraction calculated within the Bogoliubov model.

superfluidity of the system. In the regime of weak disorder we find that our results are in agreement with the predictions of the Bogoliubov model. For strong disorder the system enters a regime where the superfluid fraction is depleted below the value of the condensate fraction (see Fig. 1), as it was first suggested in Ref. [13].

Beyond Tonks-Girardeau: strongly correlated regime in quasi-one-dimensional Bose gases

In this work [14] we consider a homogeneous 1D Bose gas with contact interactions and large attractive coupling constant. This system can be realized in tight waveguides by exploiting a confinement induced resonance of the effective 1D scattering amplitude (see section on "Quantum gases in low dimensions" for further details). By using a variational *ansatz* for the many-body wavefunction, we show that for small densities the gas-like state is stable and the corresponding equation of state is well described by a gas of hard rods (*i.e.* impenetrable bosons with finite range hard-core repulsion). By calculating the compressibility of the system, we provide an estimate of the critical density at which the gas-like state becomes unstable against cluster formation. Within the hard-rod model we calculate the one-body density matrix and the static structure factor of the gas. The results show that in this regime the system is more strongly correlated than a Tonks-Girardeau gas: the static structure factor exhibits a pronounced peak at distances of the order of the interparticle spacing and the particle-particle correlation



Figure 2: Square of the lowest breathing mode frequency, ω^2 , as a function of the coupling strength Na_{1D}^2/a_z^2 for the Lieb-Liniger Hamiltonian $(g_{1D} > 0)$ and in the super-Tonks regime $(g_{1D} < 0)$. The dashed line is an analytical result obtained from the hard-rod equation of state

function decays at large distances faster than in the Tonks-Girardeau regime. We also calculate as a function of the interaction strength the frequency of the lowest breathing mode for harmonically trapped systems. We predict that in the super-Tonks regime the ratio ω/ω_z of the frequency of the collective mode to the longitudinal harmonic oscillator frequency is larger than 2, the value expected for a Tonks-Girardeau gas (see Fig. 2).

Coherence and correlation properties of a one-dimensional attractive Fermi gas

The main topic of the long-standing collaboration of I. Carusotto with Y. Castin (LKB-ENS, France) has been the development and the application of new Quantum Monte Carlo techniques for the numerical study of interacting many-particle systems [15]. Recently [16], techniques of this kind have been applied to study the condensation of pairs in a one-dimensional Fermi gas in a regime of relatively strong interactions, so to characterize the consequences of the transition on the different observables of the system and identify specific features which may represent unambiguous experimental signatures of the onset of condensation of pairs. Numerical results have been extensively compared to the predictions of a perturbative expansion in the interaction coupling constant (see Fig. 3), and of existing approximate approaches. Several among the most relevant correlation functions of the Fermi gas have been considered, in particular the first-order pair coherence function $g_{\text{pair}}^{(1)}(x) = \langle \Psi_{\downarrow}^{\dagger}(x) \Psi_{\uparrow}^{\dagger}(x) \Psi_{\downarrow}(0) \Psi_{\downarrow}(0) \rangle$ whose long distance limit is strictly related to the order parameter of the phase transition in a Landau-Ginzburg theory.



Figure 3: Normalized pair coherence function for two different temperatures $T = 0.56 T_F$ (red) and $T = 0.056 T_F$ (blue). Circles: Monte Carlo results. Solid lines: perturbative results. N = 12 atoms on a $\mathcal{N} = 16$ points lattice. Notice the appearance of long-range order at the lower temperature: this is an unambiguous signature of the onset of a BCS state.

Other contributions to the field of numerical simulations in quantum gases involving members of the Trento BEC-INFM group are listed below:

- Quantum Monte Carlo study of the three- to one-dimensional crossover for a trapped Bose gas [17]
- Correlation functions and momentum distribution of one-dimensional Bose systems [18]
- [1] W.L. McMillan Phys. Rev. 138, 442 (1965)
- [2] P.A. Whitlock, D.M. Ceperley, G.V. Chester and M.H. Kalos, Phys. Rev. B 19, 5598 (1979)
- [3] D.M. Ceperley and E.L. Pollock, Phys. Rev. Lett. 56, 351 (1986)
- [4] S. Giorgini, J. Boronat and J. Casulleras, Phys. Rev. A 60, 5129 (1999)
- [5] W. Krauth, Phys. Rev. Lett. 77, 3695 (1996)
- [6] P. Grüter, D. Ceperley and F. Laloë, Phys. Rev. Lett. 79, 3549 (1997); M. Holzmann and W. Krauth, Phys. Rev. Lett. 83, 2687 (1999)
- [7] G.G. Batrouni *et al.*, Phys. Rev. Lett. **89**, 117203 (2002); V.A. Kashurnikov, N.V.
 Prokof'ev and B.V. Svistunov, Phys. Rev. A **66**, 031601(R) (2002)
- [8] J. Carlson, S.-Y. Chang, V.R. Pandharipande and K.E. Schmidt, Phys. Rev. Lett. 91, 050401 (2003)
- [9] M.P.A. Fisher *et al.*, Phys. Rev. B **40**, 546 (1989)
- [10] R.T. Scalettar *et al.*, Phys. Rev. Lett. **66**, 3144 (1991); W. Krauth *et al.*, Phys. Rev. Lett. **67**, 2307 (1991)
- [11] For a review see J.D. Reppy, J. Low Temp. Phys. 87, 205 (1992)
- [12] G.E. Astrakharchik, J. Boronat, J. Casulleras and S. Giorgini, Phys. Rev. A 66, 023603 (2002)
- [13] K. Huang and H.-F. Meng, Phys. Rev. Lett. 69, 644 (1992)
- [14] G.E. Astrakharchik, J. Boronat, J. Casulleras and S. Giorgini, preprint condmat/0405225
- [15] I. Carusotto, Y. Castin, Phys. Rev. Lett. **90**, 030401 (2003)
- [16] Y. Castin and I. Carusotto, preprint physics/0404025
- [17] G.E. Astrakharchik and S.Giorgini, Phys. Rev. A 66, 053614 (2002)
- [18] G.E. Astrakharchik and S. Giorgini, Phys. Rev. A 68, 031602(R) (2003)

QUANTUM INFORMATION APPLICATIONS

The experimental realization of a diluted trapped gas of condensed bosons (BEC) has triggered an enormous interest towards this form of quantum matter. Condensates are characterized by: 1) coherence, which allows for the observation of novel quantum phenomena at the macroscopic scale; 2) versatility: condensates can be manipulated to a high accuracy by means of magnetic and optical potentials. In the last years there have been crucial progresses, both experimental and theoretical, in the understanding of these systems. Efforts are now focusing on the possibility of using the macroscopic quantum properties of the condensates to develop new technological applications. At Trento BEC Centre, among other possible research directions, the development of quantum computers is being explored, based on cold atoms [1] trapped in optical lattices (Fig. 1) or on atom chips (Fig. 2). Despite rapid progress in this field, no experimental method has yet proven to be perfectly ideal for the realization of QIP. An urgent need thus remains for developing new technologies for a scalable implementation of quantum computation.



Figure 1: Parallel quantum logical operations in an optical lattice [2]

Trapping and cooling of neutral atoms by means of optical and magnetic forces has opened new research directions in atomic physics. At the macroscopic level, it allows for realizing Bose-Einstein condensation in many-particle systems, where both condensed-matter and quantum mechanical phenomena can be studied. On the other hand, in microscopic traps few atoms can be manipulated in principle at the individual level. This opens the possibility of "quantum engineering", i.e., of preparing and modifying quantum states in a controlled fashion. Different technologies have been recently developed to this aim, from optical lattices [4] to atom chips [5]. In the first case, the trapping force is given by the spatial modulation of light intensity due to the interference of counter-propagating laser beams. In this system a superfluid-Mott insulator transition has been observed [4], and parallel operations leading to controlled collective entanglement have been realized [6]. In the case of atom chips, several trapping configurations for atoms close to surfaces have been achieved, based on microstructure-generated magnetic fields [5] or on light fields modulated by microlenses [7].

The first highlights in this newly started research direction at the Trento BEC Centre are:



Figure 2: Atom chip for atomic micromanipulation via magnetic fields [3].

Optimal gate control on atom chips

In [8], we examined the performance of a quantum phase gate implemented with cold neutral atoms in switchable microtraps (Fig. 3), when anharmonic traps are employed and the effects of finite temperature are also taken into account. Both the anharmonicity and the temperature are found to pose limitations to the performance of the quantum gate. We presented a quantitative analysis of the problem and showed that the phase gate has a high quality performance for the experimental values that are presently or in the near future achievable in the laboratory.



Figure 3: QIP in switching microtraps: a) before, b) during gate operation [8].

High-fidelity quantum register initialization with fermions

In [9], we showed that fermionic atoms have crucial advantages over bosonic atoms in terms of loading in optical lattices for use as a possible quantum computation de-vice. After analyzing the change in the level structure of a non-uniform confining potential as a periodic potential is superimposed to it, we showed how this structure combined with the Pauli principle and fermion degeneracy can be exploited to create unit occupancy of the lattice sites with very high efficiency (Fig. 4).



Figure 4: Fermions in an optical lattice: level structure (top) and density profile (bottom) for two different lattice depths [9].

Marker qubits and molecular interactions for quantum computing with atoms in optical lattices

In [10], we developed a scheme for quantum computation with neutral atoms, based on the concept of "marker" atoms, i.e., auxiliary atoms that can be efficiently transported in state-independent periodic external traps to operate quantum gates between physically distant qubits. This allows for relaxing a number of experimental constraints for quantum computation with neutral atoms in microscopic potential, including singleatom laser addressability. We discussed the advantages of this approach in a concrete physical scenario, namely in two-component optical lattices, whereby atom transport is achieved via adiabatic transfer between motional states in neighboring lattice wells (Fig. 5), and the mechanism used to mark atoms is the molecular interaction responsible for Feshbach resonances, which are currently a subject of intense experimental research in the field of cold atoms. In other words, our proposal relies on techniques that are presently being developed, and represents therefore a feasible candidate for the implementation of quantum information processing with neutral atoms in optical lattices.

Entanglement in a condensate

In [11] we analyze the entanglement properties of the Bogoliubov vacuum, which is obtained as a second order approximation to the ground state of an interacting Bose–Einstein condensate. We work on one and two dimensional lattices and study the entanglement between two groups of lattice sites as a function of the geometry of the configuration and the strength of the interactions (see Fig. 6). As our measure of entanglement we use the logarithmic negativity, supplemented by an algorithmic check for bound entanglement where appropriate. The short-range entanglement is found to grow approximately linearly with the group sizes and to be favoured by strong interactions. Conversely, long range entanglement is favoured by relatively weak interactions. Working with periodic boundary conditions we find some surprising finite size effects for the very long range entanglement. No examples of bound entanglement are found.

Two-atom collisions in tightly confined anisotropic geometries

In [12], we solve the problem of two atoms interacting through regularized delta potential and confined in a harmonic trap. We derive exact analytical results for the energy spectra, in the case when the ratio of axial to radial trap frequency is an integer (cigarshape traps), or the inverse of an integer (pancake-shape traps). We demonstrate that



Figure 5: Adiabatic transfer between neighboring wells in a lattice potential for gate operation [10].



Figure 6: Definition of subsystems used in [11] to study entanglement in a BEC.

in very elongated traps the lowest excited states of the system can be calculated from the one-dimensional theory with renormalized scattering length. On the other hand, the properties of the ground-state in elongated traps depend solely on the frequency of the tight confinement, and must be calculated from the full three-dimensional theory. The same is true in the opposite regime, when the ratio of axial to radial frequency is very small. We derive analytical expressions for the wave functions of the system, valid for arbitrary ratio of axial to radial trap frequency. Finally, we analyze the energy spectrum of two interacting atoms in the vicinity of a Feshbach resonance, utilizing a model with an energy-dependent scattering length.

- [1] J. I. Cirac and P. Zoller, *Physics Today* 57, 38 (2004).
- [2] D. Jaksch, T. Calarco and P. Zoller, *Physik in unserer Zeit* **31**, 260 (2000).
- [3] J. Schmiedmayer, R. Folman, and T. Calarco, Journal of Modern Optics 49, 1375 (2002).
- [4] M. Greiner, O. Mandel, T. Esslinger, T.W. Hansch, and I. Bloch, Nature 415, 39 (2002).
- [5] R. Folman, P. Krueger, J. Schmiedmayer, J. Denschlag, and C. Henkel, Advances in Atomic, Molecular and Optical Physics 48, 263 (2002).
- [6] O. Mandel, M. Greiner, A. Widera, T. Rom, T. W. Hänsch, and I. Bloch, *Nature* 425, 937 (2003).

- [7] F. Buchkremer, R. Dumke, M. Volk, T. Muther, G. Birkl, and W. Ertmer, Laser Physics 12, 736 (2002).
- [8] A. Negretti, T. Calarco, M. A. Cirone, A. Recati, quant-ph/0312066.
- [9] L. Viverit, C. Menotti, T. Calarco, A. Smerzi, cond-mat/0403178.
- [10] T. Calarco, U. Dorner, P. Julienne, C. Williams, P. Zoller, quant-ph/0403197 (in print on Phys. Rev. A).
- [11] U. V. Poulsen, T. Meyer, M. Lewenstein, (to be submitted).
- [12] Z. Idziaszek, T. Calarco, (to be submitted).

INTERFEROMETRY AND SENSORS WITH COLD GASES

Ultracold gases are ideal tools for interferometry, atomic clocks and in general precision measurements (see, for example, [1, 2, 3, 4]). In particular they might become efficient sensors to test inertial effects, forces at the micron scale etc.. The Trento team has recently started new scientific activities on these themes in collaboration with the Los Alamos National Laboratory (interferometry with coherent matter waves) and with Jila, Boulder (test of the Casimir-Polder force).

Mach-Zehnder interferometer

A main goal of the experimental realization of a BEC interferometer is to achieve a phase sensitivity beating the shot-noise limit, where the uncertainty is limited by $\sim 1/\sqrt{N}$, N being the total number of atoms. The common wisdom indicates, as ultimate theoretical sensitivity, the Heisenberg limit, with uncertainty scaling as $\sim 1/N$ [5]. This goal can be achieved with a Mach-Zehnder interferometer having at the input ports Foch, Schroedinger-cat or squeezed states. We are studying [6] the interference pattern distribution in a serie of identical Mach-Zender experiments realized with a BEC trapped in a double-well trap, see Fig.(1). Let us consider a phase shift $\theta_0 = E_0 \tau$ due to the interaction with an external weak field E_0 for a time τ , and a general input state $|\psi\rangle = \sum_{j=0}^{\infty} \sum_{n=-j}^{j} C_{j+n}^{1} C_{j-n}^{2} |n,j\rangle$, where $C^{1,2}$ are the amplitudes of the two different input states. The final state at the output ports of the Mach-Zehnder is $|\psi\rangle_{out} = exp[i\theta_0(a^{\dagger}b - b^{\dagger}a)]|\psi\rangle_{inp}$. The phase measurement is based on the measurement of the total and the relative number of atoms at the output ports. The information about the phase shift θ_0 and its uncertainty is contained in the output phase distribution P_{out} obtained after projecting the output state on the Susskind-Glogover phase states $|\phi,j\rangle = 1/(2j+1)\sum_{m=j}^{j} exp(im\phi)|m\rangle$. The final expression can be written in terms of the Jacoby polynomials, and it is easy to recover the $1/\sqrt{N}$ or the 1/N limit depending on the chosen quantum states at the input ports. The final goal of this line of research is to include the coupling with an environment (thermal atoms, noise, etc.) in order to understand how robust is the -quantum- Heisenberg precision of the measuseremnt and how it reduces to the -classical- shot-noise limit.

Double-slit interferometer

A most important problem arising in the experimental realizations of matter wave interferometers is the creation of efficient beam splitters. Recently, a BEC beam splitter has been realized in [7]. A single, cigar-shaped condensate was trapped in a double-well



Figure 1: (a) Scheme of a Mach-Zehnder interferometer. Two coherent input fields are split by a half-trasparent mirror (beam splitter), and recombined after one of them has interacted with an external force potential (Phase Shifter). Counting the number of photons/atoms in the detectors allows measurement of the intensity of the external field. (b) A Bose-Einstein condensate Mach-Zehnder interferometer can be created by trapping the condensate in a double well potential. The action of the beam splitters can be induced by modulating the height of the tunneling barrier. The external force induces an energy difference, E_0 , which can be extracted by the interference pattern of the two overlapping condensates obtained after removing the trap.

trap having a barrier much smaller than the BEC chemical potential. The condensate was therefore split along the axial direction by increasing linearly the distance between the two wells and the height of the interwell barrier. The final distance between the two condensates was large enough to allow for *individual* addressing and manipulation. After holding the two condensates in the respective traps for a variable time, the confining field was turned off. The interference pattern of the two overlapping condensates was reproducible in different realizations of the experiment, but only for short holding times. The loss of coherence between the two condensates cannot be blaimed to quantum fluctuations (which, in any case, we have predicted to occur at much larger time scales), and should be reconduct to a source of "classical" noise. In order to recognize the origin of such noise, and maximize the performance of the beam BEC splitter, we have studied the adiabaticity time scales of the system. We have solved [8] the dynamical GPE in the full three dimensional space, Fig.(2), in order to take in account all possible Bogoliubov excitations in the splitting process. We have pointed out an avoided crossing in the Bogoliubov energy levels as a function of the separation between the two condensates. This occurs close to a critical point when the chemical potential



Figure 2: Contour plots of the probability density integrated over the y-coordinate $P_y(x, z, t)$ from 3D simulations for $t_{hold} = 10.6$ ms and $t_{free} = 5.3$ ms, as a function of the ramping time t_{ramp} : a) 2.7 ms, b) 5.3 ms, c) 10.6 ms, and d) 21.2 ms.

is of the order of the interwell barrier energy, and it appears to be a crucial factor in the loss of adiabaticity of the BEC beam-splitter. The best strategy to restore adiabaticity is, therefore, to consider an adattable splitting process, which should slow down close to the critical point. Our analisys also applies to a generic Y-shaped wave-guides which can be implemented experimentally in microchips.

Casimir-Polder force on a Bose-Einstein condensate

The Casimir-Polder force [9] is a fundamental manifestation of the fluctuations of the electromagnetic field. It plays a crucial role at the micron and submicron scale. This force has been the object of systematic research activity in the last decades both at the experimental and theoretical level. So far the Casimir-Polder effect has been observed for forces between metallic surfaces [10, 11, 12] where, however, the theoretical description exhibits non trivial difficulties at finite temperature [13, 14]. The thermal fluctuations of the field become important at distances comparable to the photon thermal wavelength $\lambda_T = \hbar c/k_B T$ where the force is weak and difficult to detect. The purpose of this research [15], carried out in collaboration with the experimental team of Eric Cornell at Boulder, is to investigate the effects of the Casimir-Polder force on



Figure 3: Relative shift of the frequency of the center of mass motion of a Bose-Einstein condensate of rubidium atoms as a function of the distance from a sapphire substrate. The total size of the condensate in the direction orthogonal to the substrate is 5 microns. Results are given at zero and room temperature.

the frequency of the collective oscillations of a Bose-Einstein condensate trapped close to the surface of a dieletric. The shift of the collective frequency with respect to the unperturbed value ω_0 depends on the size of the condensate, on the temperature of the substrate, on the optical proprieties of the substrate and of the Bose-Einstein condensed atoms as well as on the distance of the condensate from the substrate. We predict effects of the order of 10^{-4} for the relative shifts at distances where thermal effects become important at room temperature. A typical prediction is given in Fig.3 where the frequency shifts of the center of mass motion are reported at zero and room temperature as a function of the distance of a rubidium condensate from a sapphire substrate.

- [1] B.P. Anderson and M.A. Kasevich, Science **282**, 1686 (1998).
- [2] G. Roati et al., cond-mat/0402328 (Phys. Rev. Lett., in press).
- [3] Ch. Salomon et al., C.R. Acad.Sci. Paris, T2 Série 4, 1313 (2001)
- [4] Atom Interferometry, Ed. P.R. Berman, Academic press (1997)

- [5] B. Yurke, et. al., Phys. Rev, A 33 4033 (1985); M.J. Holland, K. Burnett, Phys. Rev. Lett. 71 9 (1993)
- [6] L. Pezze' *et al.*, work in progress
- [7] Y. Shin et. al., Phys. Rev. Lett.92 050405 (2004)
- [8] L. Collins et al., cond-mat/0404149
- [9] I.E. Dzyaloshinskii, E.M. Lifchitz and L.P. Pitaevskii, Advances in Physics 10, Part 38, 165 (1961)
- [10] S.K. Lamoreaux, Phys. Rev. Lett. 78, 5 (1997)
- [11] G. Bressi *et al.*, Phys. Rev. Lett. **88**, 041804 (2002)
- [12] R.S. Decca et al., Phys Rev. Lett. 91, 050402 (2003)
- [13] B.W. Ninham and J. Daicic, Phys. Rev. A 57, 1870 (1998)
- [14] V.B. Bezerra et al., Phys. Rev. A 69, 022119 (2004)
- [15] M. Antezza, L. Pitaevskii and S. Stringari, in preparation

Projects and scientific collaborations

Projects

During the period June 2002 - May 2004 the members of the BEC Center have been involved in national projects supported by the Italian Ministry of Research and by INFM. Among them:

MIUR PRIN 2000 on "Theory of coherent fluids: Bose gases, superfluids and superconductors" (national coordinator: Franco Dalfovo)

MIUR PRIN 2001 on "Coherence effects in trapped quantum gases" (national coordinator: Sandro Stringari)

MIUR PRIN 2002 on "Fault tolerance, control and stability in quantum information processing" (national coordinator: Giulio Casati)

MIUR PRIN 2003 on "Superfluidity and Fermi atomic gases" (national coordinator: Massimo Inguscio)

INFM PRA 2001 on "Photonmatter: Coherent Light and Coherent Matter" (national coordinator: Massimo Inguscio)

Scientific Collaborations

The scientific activity carried out at the BEC Center is the result of numerous national and international collaborations. Some of the most significant ones are briefely described below:

• European Laboratory for Nonlinear Spectroscopy (LENS, Florence). The Trento team has a long and fruitful experience of collaboration with LENS, also in the framework of national projects. Recent joint experimental-theoretical work has concerned the study of the insulating behaviour of Fermi gases trapped in optical lattices [1]. Michele Modugno from LENS is collaborating with the BEC Center on the study of the dynamics and stability diagrams of condensates and Bose-Fermi mixtures in harmonic traps and optical lattices by means of the Gross-Pitaevskii theory [2]. Iacopo Carusotto is actually collaborating with Gabriele Ferrari on the development of a novel optical frequency synthetizer by means of difference frequency generation of the pulsed field of a mode-locked laser [3].

- L.Pezzè, L.Pitaevskii, A. Smerzi, S.Stringari, G. Modugno, E. DeMirandes, F. Ferlaino, H. Ott, G. Roati and M. Inguscio, cond-mat/0401643, Phys. Rev. Lett., in press
- [2] M. Modugno, C. Tozzo and F. Dalfovo, cond-mat/0405653
- [3] G. Ferrari and I. Carusotto, submitted to Phys. Rev. Lett. (2004).
- Ecole Normale Superieure (ENS, Paris). A joint experimental-theoretical collaboration with the team of Jean Dalibard has concerned the scissors mode in rotating condensates [1]. Theoretical collaborations have concerned the study of Kelvin modes in vortex lattices [2] and the role of collisions on the expansion of atomic gases [3]. Roland Combescot, Christophe Salomon and Sandro Stringari have co-organized the recent workshop on Ultracold Fermi gases (Levico, March 2004). Sandro Stringari will spend part of the next academic year at the College de France in close collaboration with the teams of ENS. Iacopo Carusotto is at present at ENS. He started collaborating with Yvan Castin(LKB-ENS) in 1999 on the application of new Quantum Monte Carlo techniques for the numerical study of low-dimensional many-particle systems, in both the bosonic and fermionic cases [4]. Luca Giorgetti, Trento PhD student, has recently joined this collaboration. Iacopo Carusotto has also started a collaboration with Cristiano Ciuti (LPA-ENS) on the subject of the many-body physics of exciton-polariton gases in semiconductor microcavities [5].
 - M. Cozzini, S. Stringari, V. Bretin, P. Rosenbusch and J. Dalibard, Phys. Rev. A 67, 021602(R) (2003)
 - [2] F. Chevy and S. Stringari, Phys. Rev. A 68, 053601 (2003)
 - [3] P. Pedri, D. Guéry-Odelin and S. Stringari, Phys. Rev. A 68, 043608 (2003)
 - [4] I. Carusotto and Y. Castin, Comptes Rendus Physique 5, 107-127 (2004); physics/0404025
 - [5] Iacopo Carusotto and Cristiano Ciuti, cond-mat/0404573
- *Boulder* (JILA and University of Colorado, Boulder). The Trento team has fruitful collaborations with JILA. In February 2003 Chiara Menotti spent three weeks in the group of Murray Holland. During this collaboration, the resonance field

theory was implemented for the first time in an inhomogeneous system and applied to study the collapse of a Bose-Einstein condensate [1]. Murray Holland from JILA is presently a long term visitor of the Center and is collaborating with Chiara Menotti and Luciano Viverit at the study of the BCS-BEC crossover [2]. Mauro Antezza, Lev Pitaevskii and Sandro Stringari are presently collaborating with the team of Eric Cornell on a joint experimental-theoretical project on the effects of the Casimir-Polder force on Bose-Einstein condensates. Starting from next September Stefano Giorgini will spend a sabbatical year at JILA.

- [1] J.N. Milstein, C. Menotti and M.J. Holland, New J. Phys. 5, 52 (2003)
- [2] M. J. Holland, C. Menotti and L. Viverit, cond-mat/0404234
- Innsbruck. The Institute for Theoretical Physics of Innsbruck University has a significant tradition of scientific collaboration with the BEC Theory Group in Trento, started before the establishment of the CRS-BEC Centre (including joint scientific meetings started in 1997). Alessio Recati, a Trento graduate student, prepared his doctoral thesis under the supervision of Peter Zoller and is actively collaborating with his group [1, 2, 3, 4, 5]. Before joining the Trento team Chiara Menotti was in Innsbruck during her PhD and as postdoc. Tommaso Calarco has a tradition of scientific collaboration on quantum information with the Innsbruck team where several of the main theoretical schemes in this field have been developed [6]. Among the partners of Trento's QIP research are included most European groups working with atom chips (J. Schmiedmayer in Heidelberg, T. Hänsch in Munich, E. Hinds in London, A. Aspect in Paris, to name a few), assembled since 2000 in an EC project whose theoretical activity is coordinated by Tommaso Calarco in collaboration with ECT* in Villazzano.
 - A. Recati, T. Calarco, P. Zanardi, J. I. Cirac and P. Zoller, Phys. Rev. A 66, 032309 (2002)
 - [2] A. Recati, P.O. Fedichev, W. Zwerger, J. von Delft and P. Zoller, condmat/0212413
 - [3] Uwe R. Fischer, Petr O. Fedichev, A. Recati, and P. Zoller, condmat/0212419
 - [4] A. Recati, P.O. Fedichev, W. Zwerger and P. Zoller, Phys. Rev. Lett. 90, 020401 (2003); J. Opt. B: Quantum Semiclass. Opt. 5, S55 (2003)

- [5] P. O. Fedichev, Uwe R. Fischer and A. Recati, Phys. Rev. A 68, 011602 (2003)
- [6] T. Calarco, U. Dorner, P. Julienne, C. Williams and P. Zoller, quantph/0403197
- Weizmann Institute, Rehovot. Since 2002 we are in close collaboration with the experimental group of Nir Davidson, at the Department of Physics of Complex Systems, Weizmann Institute of Science, Rehovot, Israel. Joint efforts have been devoted to understanding the response of elongated Bose-Einstein condensates to light (Bragg) pulses. The group of Trento performed Gross-Pitaevskii simulations and developed semi-analytic models in order to interpret and stimulate the experiments done at Rehovot, obtaining significant results in the observation and characterization of Bogoliubov excitations [1, 2].
 - J.Steinhauer, N.Katz, R.Ozeri, N.Davidson, C.Tozzo and F.Dalfovo, Phys. Rev. Lett. 90, 060404 (2003)
 - [2] N. Katz, R. Ozeri, N. Davidson, C. Tozzo and F. Dalfovo, condmat/0405222
- *MPI-Goettingen.* Since several years the group of Trento is in contact with experimental groups working on liquid helium nanodroplets. Among them, a profitable collaboration continues with J.Peter Toennies and his co-workers, at the Max-Planck-Institute für Strömungsforschung in Göttingen. The last joint work is about the expansion of solid ${}^{4}He$ into vacuum, where one observes surprising phenomena, likely due to the peculiar motion of defects and vacancies in this quantum solid [1].
 - R. E. Grisenti, M. Kaesz, J. P. Toennies, G. Benedek and F. Dalfovo, cond-mat/0403642
- *Hannover*. With the University of Hannover the Trento team has a PhD co-tutelle program. Paolo Pedri is preparing the doctoral thesis under the joint supervision of Maciecj Lewenstein and Sandro Stringari. He is actively collaborating with the other members of the Hannover team on the physics of low dimensional gases [1, 2, 3, 4].

- [1] P. Pedri and L. Santos, Phys. Rev. Lett. **91**, 110401 (2003)
- [2] P. Pedri, L. Santos, P. Ohberg and S. Stringari, Phys. Rev. A 68, 043601 (2003)
- [3] V. Ahufinger, A. Sanpera, P. Pedri, L. Santos and M. Lewenstein, condmat/0310042
- [4] M. Rodriguez, P. Pedri, P. Törmä and L. Santos, cond-mat/0310498
- Los Alamos National Laboratory (LANL). There is a long standing collaboration of the Trento group with LANL. Augusto Smerzi is presently spending at LANL a sabbatical year as staff member, developing projects in collaboration with Gennady Berman, Alan Bishop, Wojeck Zurek and Lee Collins. Collaborations include the study of the classical (GPE) and the quantum dynamics of instabilities, quantum chaos at the onset of fermionization in a one-dimensional gas of bosons, Kibble-Zurek scenario at the onset of the Mott-insulator superfluyd transition, GPE discrete dynamics in presence of defects, adiabaticity time scales on double well dynamics [1, 2, 3, 4]. Smerzi is co-author of two large projects which have been recently funded by the DOE regarding technological applications with BEC's, with emphasis on the development of a new generation of interferometers. Within these projects, Luca Pezzè, Ph.D student in Trento, has received a one year fund to work at the LANL and Sara Ianeselli, undergraduate student in Trento, will visit the lab in 2004 for two months to complete her undergraduate thesis.
 - G.P. Berman, F.Borgonovi, F.M. Izrailev and A.Smerzi, Phys. Rev. Lett. 92, 030404 (2004)
 - [2] J.Dziarmaga, A.Smerzi, W.H.Zurek and A.R.Bishop, cond-mat/0403607
 - [3] Z. Rapti, P.G. Kevrekidis, A. Smerzi and A.R. Bishop, Phys. Rev. E 69, 017601 (2004)
 - [4] Z. Rapti, P.G. Kevrekidis, A. Smerzi and A.R. Bishop, J. Phys. B: At. Mol. Opt. Phys. 37, S257 (2004)
- *Barcelona*. The collaboration between the Trento BEC group and the Universitat Politecnica de Catalunya (UPC) started many years ago. In 1995 Stefano Giorgini spent one year as a PostDoc associate at the Departament de Fisica i Enginyeria Nuclear of UPC and since then he actively collaborated with the Quantum Monte

Carlo group leaded by J. Boronat and J. Casulleras on numerical simulations of quantum gases [1, 2]. A new collaboration is presently being started involving the student Sebastiano Pilati on a project of developing a path-integral quantum Monte Carlo algorithm.

- G.E. Astrakharchik, J. Boronat, J. Casulleras and S. Giorgini, Phys. Rev. A 66, 023603 (2002)
- [2] G.E. Astrakharchik, J. Boronat, J. Casulleras and S. Giorgini, condmat/0405225

Publications

The following list includes all the papers published or submitted for publication in the period June 2002 - May 2004, having at least one of the authors with the Trento BEC-INFM affiliation.

Books

Bose-Einstein Condensation; L. Pitaevskii and S. Stringari, Oxford University Press, International Series of Monographs on Physics, **116** (2003)

Papers

2004 (and preprins)

Role of transverse excitations in the instability of Bose-Einstein condensates moving in optical lattices; M. Modugno, C. Tozzo and F. Dalfovo, cond-mat/0405653

High sensitivity phonon spectroscopy of Bose-Einstein condensates using matter-wave interference; N. Katz, R. Ozeri, N. Davidson, C. Tozzo and F. Dalfovo, cond-mat/0405222

Beyond Tonks-Girardeau: strongly correlated regime in quasi-one-dimensional Bose gases; G.E. Astrakharchik, J. Boronat, J. Casulleras and S. Giorgini, cond-mat/0405225

Modulational and Parametric Instabilities of the Discrete Nonlinear Schrödinger Equation; Z. Rapti, P.G. Kevrekidis, A. Smerzi and A.R. Bishop, J. Phys. B: At. Mol. Opt. Phys. 37, S257 (2004)

Variational Approach to the Modulational Instability; Z. Rapti, P.G. Kevrekidis, A. Smerzi and A.R. Bishop, Phys. Rev. E 69, 017601 (2004)

Probing microcavity polariton superfluidity through resonant Rayleigh scattering; I. Carusotto and C. Ciuti, cond-mat/0404573

Quasi-particle properties and Cooper pairing in trapped Fermi gases; L. Giorgetti, L. Viverit, G. Gori, F. Barranco, E. Vigezzi and R. A. Broglia, cond-mat/0404492

On the Nonlinear Krönig-Penney model; WeiDong Li and A. Smerzi, Phys. Rev. E, in press.

On the Observable Signature of a Defect-Mediated Finite-Temperature Transition in a Bosonic Planar Lattice; A. Trombettoni, A. Smerzi and P. Sodano, cond-mat/0404381

Propagation of sound in a Bose Einstein condensate in an optical lattice; C. Menotti,M. Kramer, A. Smerzi, L. Pitaevskii and S. Stringari, cond-mat/0404272

The role of boson-fermion correlations in the resonance theory of superfluids; M. J. Holland, C. Menotti and L. Viverit, cond-mat/0404234

Collisions and expansion of an ultracold dilute Fermi gas; B. Jackson, P. Pedri and S. Stringari, cond-mat/0404175

Coherence and correlation properties of a one-dimensional attractive Fermi gas; Y. Castin and I. Carusotto, physics/0404025

Double-Slit Interferometry with a Bose-Einstein Condensate; L.A. Collins, L. Pezze', A. Smerzi, G.P. Berman and A.R. Bishop, cond-mat/0404149

Observation of dynamical instability for a Bose-Einstein condensate in a moving 1D optical lattice; L. Fallani, L. De Sarlo, J. E. Lye, M. Modugno, R. Saers, C. Fort and M. Inguscio, cond-mat/0404045

Quantum computations with atoms in optical lattices: marker qubits and molecular interactions; T. Calarco, U. Dorner, P. Julienne, C. Williams and P. Zoller, quant-ph/0403197

Expansion of Solid He into Vacuum: the Geyser Effect; R. E. Grisenti, M. Kaesz, J. P. Toennies, G. Benedek and F. Dalfovo, cond-mat/0403642

Non-equilibrium Mott transition in a lattice of Bose-Einstein condensates; J.Dziarmaga, A.Smerzi, W.H.Zurek and A.R.Bishop, cond-mat/0403607

Efficient and robust initialization of a qubit register with fermionic atoms; L. Viverit, C. Menotti, T. Calarco and A. Smerzi, cond-mat/0403178

Pairing fluctuations in trapped Fermi gases; L. Viverit, G. M. Bruun, A. Minguzzi and R. Fazio, cond-mat/0402620

Publications

Umklapp collisions and center of mass oscillation of a trapped Fermi gas; G. Orso, L.P. Pitaevskii and S. Stringari, cond-mat/0402532

Insulating Behavior of a Trapped Ideal Fermi Gas; L.Pezzè, L.Pitaevskii, A. Smerzi, S.Stringari, G. Modugno, E. DeMirandes, F. Ferlaino, H. Ott, G. Roati and M. Inguscio, cond-mat/0401643

Tkachenko oscillations and the compressibility of a rotating Bose gas; M. Cozzini, L. P. Pitaevskii and S. Stringari, cond-mat/0401516

Phonon evaporation in freely expanding Bose-Einstein condensates; C.Tozzo and F. Dalfovo, Phys. Rev. 5, 053606 (2004)

Momentum distribution of a trapped Fermi gas with large scattering length; L. Viverit, S. Giorgini, L. P. Pitaevskii and S. Stringari, Phys. Rev. A **69**, 013607 (2004)

Collective oscillations of a trapped Fermi gas near a Feshbach resonance; S. Stringari, Europhys. Lett. 65, 749 (2004)

Interacting fermions in quasi-one-dimensional harmonic traps; G.E. Astrakharchik, D. Blume, S. Giorgini and L.P. Pitaevskii, cond-mat/0312538

Superfluidity of the 1D Bose gas; I. Carusotto and Y. Castin, Comptes Rendus Physique 5, 107 (2004)

Spin-orbit coupling and Berry phase with ultracold atoms in 2D optical lattices; A. M. Dudarev, R. B. Diener, I. Carusotto and Q. Niu, cond-mat/0311356

Quantum Monte Carlo study of quasi-one-dimensional Bose gases; G. E. Astrakharchik,D. Blume, S. Giorgini, and B. E. Granger, J. Phys. B: At. Mol. Opt. Phys. 37, S205 (2004)

Imaging of spinor gases; I. Carusotto and E. J. Mueller, J. Phys. B: At. Mol. Opt. Phys. **37**, S115 (2004)

Loss and revival of phase coherence in a Bose-Einstein condensate moving through an optical lattice; F. Nesi and M. Modugno, J. Phys. B: At. Mol. Opt. Phys. **37**, S101

(2004)

On the observable signature of a defect-mediated finite-temperature transition in a bosonic lattice; A. Trombettoni, A. Smerzi and P. Sodano, preprint

Scissors modes of two-component degenerate gases: Bose-Bose and Bose-Fermi mixtures; M. Rodriguez, P. Pedri, P. Törmä and L. Santos, cond-mat/0310498

Creation of discrete solitons and observation of the Peierls-Nabarro barrier in Bose-Einstein condensates; V. Ahufinger, A. Sanpera, P. Pedri, L. Santos and M. Lewenstein, cond-mat/0310042

Irregular dynamics in a one-dimensional Bose system; G.P. Berman, F.Borgonovi, F.M. Izrailev and A.Smerzi, Phys. Rev. Lett. **92**, 030404 (2004)

Quasi-one-dimensional Bose gases with large scattering length; G.E. Astrakharchik, D. Blume, S. Giorgini, and B.E. Granger, Phys. Rev. Lett. **92**, 030402 (2004)

Motion of a heavy impurity through a Bose-Einstein condensate; G.E. Astracharchik and L.P. Pitaevskii, cond-mat/0307247

Dissipative spin-boson model and Kondo effect in low dimensional quantum gases; A. Recati, P.O. Fedichev, W. Zwerger, J. von Delft and P. Zoller, cond-mat/0212413

Vortex liquids and vortex quantum Hall states in trapped rotating Bose gases; U.R. Fischer, P.O. Fedichev and A. Recati, J. Phys. B **37**, S301-S310 (2004)

2003

Superfluid dynamics of a Bose-Einstein condensate in a period potential; C. Menotti, A. Smerzi and A. Trombettoni, New J. Phys. 5, 112 (2003)

Finite temperature effects on the collapse of trapped Bose-Fermi mixtures; Xia-Ji Liu,M. Modugno and Hui Hu, Phys. Rev. A 68, 053605 (2003)

Phase fluctuations and coherence of an almost ideal condensate; L.P. Pitaevskii and S. Stringari, Annales Henri Poincaré 4, S793 Suppl.2 (2003)

Dynamics of a classical gas including dissipative and mean field effects; P. Pedri, D. Guéry-Odelin and S. Stringari, Phys. Rev. A **68**, 043608 (2003)

Kelvin Modes of a fast rotating Bose-Einstein Condensate; F. Chevy and S. Stringari, Phys. Rev. A 68, 053601 (2003)

Bose-Einstein condensates in 1D optical lattices: compressibility, Bloch bands and elementary excitations; M. Kraemer, C. Menotti, L. Pitaevskii and S. Stringari, Eur. Phys. J. D 27, 247 (2003)

Mean-field analysis of the stability of a K-Rb Fermi-Bose mixture; M. Modugno, F. Ferlaino, F. Riboli, G. Roati, G. Modugno, and M. Inguscio, Phys. Rev. A 68, 043626 (2003)

Fermi Gases in Slowly Rotating Traps: Superfluid vs Collisional Hydrodynamics; M. Cozzini and S. Stringari, Phys. Rev. Lett. **91**, 070401 (2003)

Landau damping in trapped Bose-condensed gases; B. Jackson and E. Zaremba, New J. Phys. 5, 88 (2003)

Violation of self-similarity in the expansion of a 1D Bose gas; P. Pedri, L. Santos, P. Ohberg and S. Stringari, Phys. Rev. A 68, 043601 (2003)

Discrete nonlinear dynamics of weakly coupled Bose-Einstein condensates; A. Smerzi and A. Trombettoni, Chaos 13, 766 (2003)

Nonlinear tight-binding approximation for Bose-Einstein condensates in a lattice; A. Smerzi and A. Trombettoni, Phys. Rev. A 68, 023613 (2003)

Three-dimensional quasi-Tonks gas in a harmonic trap; P. Pedri and L. Santos, Phys. Rev. Lett. **91**, 110401 (2003)

Zero-temperature damping of Bose-Einstein condensate oscillations by vortex-antivortex pair creation; P.O. Fedichev, U.R. Fischer, and A. Recati Phys. Rev. A 68, 011602 (2003)

Feshbach resonances and collapsing Bose-Einstein condensates; J. N. Milstein, C. Menotti,

and M. J. Holland, New J. Phys. 5, 52 (2003)

Bogoliubov spectrum and Bragg spectroscopy of elongated Bose-Einstein condensates; C. Tozzo and F. Dalfovo, New J. Phys. 5, 54 (2003)

Spin-charge separation in ultra-cold quantum gases; A. Recati, P. O. Fedichev, W. Zwerger and P. Zoller, Phys. Rev. Lett. **90**, 020401 (2003)

Correlation functions and momentum distribution of one-dimensional Bose systems; G.E. Astrakharchik and S. Giorgini, Phys. Rev. A **68**, 031602 (2003).

Fermi 1D quantum gas: Luttinger liquid approach and spin-charge separation; A. Recati, P.O. Fedichev, W. Zwerger and P. Zoller, J. Opt. B: Quantum Semiclass. Opt. 5, S55 (2003)

Dynamic structure factor of a Bose Einstein condensate in a 1D optical lattice; C. Menotti, M. Kraemer, L. Pitaevskii and S. Stringari, Phys. Rev. A 67, 053609 (2003)

Macroscopic dynamics of a Bose-Einstein condensate containing a vortex lattice; M. Cozzini and S. Stringari, Phys. Rev. A 67, 041602(R) (2003)

Scissors mode of a rotating Bose-Einstein condensate; M. Cozzini, S. Stringari, V. Bretin, P. Rosenbusch and J. Dalibard, Phys. Rev. A **67**, 021602(R) (2003)

Bragg spectroscopy of the multi-branch Bogoliubov spectrum of elongated Bose-Einstein condensates; J.Steinhauer, N.Katz, R.Ozeri, N.Davidson, C.Tozzo and F.Dalfovo, Phys. Rev. Lett. **90**, 060404 (2003)

Landau damping of transverse quadrupole oscillations of an elongated Bose-Einstein condensate; M. Guilleumas and L.P. Pitaevskii, Phys. Rev. A 67, 053607 (2003)

2002

The Quest for Superfluidity in Fermi Gases; L. Pitaevskii and S. Stringari, Science **298**, 2144 (2002)

Holonomic quantum computation with neutral atoms; A. Recati, T. Calarco, P. Zanardi, J. I. Cirac and P. Zoller, Phys. Rev. A **66**, 032309 (2002)

Critical temperature of Bose-Einstein condensation in trapped atomic Bose-Fermi mixtures; A.P. Albus, S. Giorgini, F. Illuminati and L. Viverit, J. Phys. B: At. Mol. Opt. Phys. **35**, L511 (2002)

Expansion of an interacting Fermi gas; C. Menotti, P. Pedri and S. Stringari, Phys. Rev. Lett. **89**, 250402 (2002)

Quantum Monte Carlo study of the three- to one-dimensional crossover for a trapped Bose gas; G.E. Astrakharchik and S. Giorgini, Phys. Rev. A 66, 053614 (2002)

Ground-state properties of a dilute Bose-Fermi mixture; L. Viverit and S. Giorgini, Phys. Rev. A **66**, 063604 (2002)

Macroscopic dynamics of a trapped Bose-Einstein condensate in the presence of 1D and 2D optical lattices; M. Kraemer, L. Pitaevskii and S. Stringari, Phys. Rev. Lett. 88, 180404 (2002)

Collective oscillations of a 1D trapped Bose gas; C. Menotti and S. Stringari, Phys. Rev. A 66, 043610 (2002)

Superfluidity vs Bose-Einstein condensation in a Bose gas with disorder; G.E. Astrakharchik, J. Boronat, J. Casulleras and S. Giorgini, Phys. Rev. A 66, 023603 (2002)

Finite size effects on the collective oscillations of a trapped Bose gas; F. Zambelli and S. Stringari, Laser Physics **12**, 240 (2002)

Vortex nucleation and quadrupole deformation of a rotating Bose-Einstein condensate; M. Kraemer, L. Pitaevskii, S. Stringari and F. Zambelli, Laser Physics **12**, 113 (2002)

Consequence of superfluidity on the expansion of a rotating Bose-Einstein condensate; M. Edwards, C. W. Clark, P. Pedri, L. Pitaevskii and S. Stringari, Phys. Rev. Lett. 88, 070405 (2002)

Talks at workshops and conferences

2004

Tommaso Calarco: "The basics of optical spin quantum computing" 317° Heraeus Seminar Spintronics, Bad Honnef (Germany), January 11-14 2004

Tommaso Calarco: "All-optical spin quantum computing" Symposium Cryptography and Quantum Information, Karpacz (Poland), January 14-17 2004

Murray Holland: "A theory of resonant interactions in dilute quantum gases" Workshop on Ultra-Cold Fermi Gases, Levico (Italy), March 4-6 2004

Sandro Stringari: "Collective oscillations in trapped superfluid Fermi gases" Workshop on Ultra-Cold Fermi Gases, Levico (Italy), March 4-6 2004

Tommaso Calarco: "Quantum computing via molecular interactions: 'natural' and 'artificial'"

International Conference Quantum information with atoms, ions and photons, La Thuile (Italy), March 6-12 2004

Marco Cozzini: "Tkachenko oscillations and the compressibility of a rotating Bose gas," Mathematical Problems in Modeling Generation and Dynamics of Vortices Verona, March 12-13 2004

Tommaso Calarco: "The basics of optical spin quantum computing" 2nd Workshop Semiconductor Quantum Optics, Rügen (Germany), April 14-17 2004

Stefano Giorgini: "Degenerate gases in quasi-1D harmonic traps" Aspects of Large Quantum Systems Related to Bose-Einstein Condensation, Aarhus, April 15-18 2004

Tommaso Calarco: "Entanglement: introduction" XXII Convegno Fisica teorica e struttura della materia, Fai della Raganella (Trento), April 18-21 2004

Murray Holland: "A theory of resonant interactions in dilute quantum gases"

Workshop on Ultra-Cold Fermi Gases, Levico (Italy), March 3-6 2004

Tommaso Calarco: "QC with atoms in optical lattices: marker atoms and molecular Interactions"

2nd Workshop Quantum Information with atoms and photons, Torino, April 26-27 2004

Sandro Stringari: "Expansion and collective oscillations in ultracold Fermi gases" Conference on Frontiers in Quantum Gases, Santa Barbara, May 10-14 2004

Murray Holland: "A theory of resonance superfluidity" NASA Workshop on Fundamental Physics, Solvang (California), April 20-22, 2004

Iacopo Carusotto: "Many-Body Physics with Exciton-Polaritons in Semiconductor Microcavities"

Quantum Gases Program at KITP, Santa Barbara (California), May 2004

Chiara Menotti: "The role of boson-fermion correlations in the resonance theory of superfluids"

Quantum Gases Program at KITP, Santa Barbara (California), May 2004

2003

Sandro Stringari: "Effects of superfluidity in trapped Fermi gases" Workshop on Quantum Optics, Obergurgl (Austria), February 23 - March 1 2003

Brian Jackson: "Dynamics of quantum-degenerate gases at finite temperature" BEC Center Inauguration meeting, Trento, March 14-15 2003

Stefano Giorgini: "Bose-Einstein condensates in low dimensions" BEC Center Inauguration meeting, Trento, March 14-15 2003

Augusto Smerzi: "Superfluid dynamics of BEC in periodic potentials" BEC Center Inauguration meeting, Trento, March 14-15 2003

Lev Pitaevskii: "Virial coefficient in the unitarity limit" BEC Center Inauguration meeting, Trento, March 14-15 2003

Sandro Stringari: "BEC in Trento"

BEC Center Inauguration meeting, Trento, March 14-15 2003

Chiara Menotti: "Linear dynamics of BEC with and without an optical lattice" BEC Center Inauguration meeting, Trento, March 14-15 2003

Franco Dalfovo: "Rotations and vortices in Bose superfluids" BEC Center Inauguration meeting, Trento, March 14-15 2003

Michele Modugno: "Collapse and expansion of a Fermi-Bose misture" BEC Center Inauguration meeting, Trento, March 14-15 2003

Luciano Viverit: "Superfluidity in atomic Fermi gases" BEC Center Inauguration meeting, Trento, March 14-15 2003

Meret Kraemer: "Bose-Einstein Condensates in optical lattices" XXII Convegno Fisica Teorica e Struttura della Materia, Fai della Paganella, March 20-23, 2003

Sandro Stringari: "Sum rules and collective oscillations of a quantum gas" (series of lectures)

School on Quantum Gases in Low Dimensions, Les Houches, April 15-25 2002

Marco Cozzini: "Fermi Gases in Slowly Rotating Traps: Superfluid vs Collisional Hydrodynamics"

Meeting on Cold Gases, Ringberg Castle (Germany), May 5-6 2003

Grigori Astrakharchik: "Quantum Monte Carlo Study of low dimensional Bose systems"

Meeting on Cold Gases, Ringberg Castle (Germany), May 5-6 2003

Augusto Smerzi: "BEC dynamics in optical lattices" DAMOP 2003, Boulder, May 20-24 2003

Brian Jackson: "Dynamics of quantum degenerate gases at finite temperatures" Theory of Quantum Gases and Quantum Coherence, Levico (Italy), June 12-14 2003

Marco Cozzini: "Atomic gases in rotating traps"

Stefano Giorgini: "Quasi -1D Bose gases with large scattering length"Workshop on Exploring the interface between cold atom and condensed matter physics:from strong correlation to entanglement, Aspen Center for Physics (Colorado), June 16 - July 6 2003

Chiara Menotti: "Excitations of a Bose Einstein condensate in a 1D optical lattice" International Workshop on Laser Physics, Hamburg, August 25-29 2003

Cesare Tozzo: "Bogoliubov Spectrum and Bragg Spectroscopy Of Elongated Bose-Einstein Condensates" International Workshop on Laser Physics, Hamburg, August 25-29 2003

Franco Dalfovo: "Bogoliubov Spectrum and Bragg Spectroscopy of Elongated Bose-Einstein Condensates" International Symposium on Quantum Fluids and Solids - QFS2003, Albuquerque, August 2003

Sandro Stringari: "Superfluidity of ultracold Fermi gases" Conference on Mysteries, Puzzles and Paradoxes in Quantum Mechanics, Gargnano (Italy), September 1-5 2003

Sandro Stringari: "Effects of superfluidity in rotating quantum gases" Symposium Quantum Challenges, Warsaw, September 4-7 2003

Sandro Stringari: "Effects of superfluidity in rotating quantum gases" Bose-Einstein Condensation : EuroConference on the New Trends in Physics of Quantum Gases San Feliu de Guixols (Spain), September 13-18 2003

Iacopo Carusotto: "Stochastic methods for interacting Bose and Fermi gases" Bose-Einstein Condensation : EuroConference on the New Trends in Physics of Quantum Gases San Feliu de Guixols (Spain), September 13-18 2003

Murray Holland: "Feshbach resonances and molecular condensates" Conference on Theoretical Concepts and Recent Experiments on Ultracold Molecules Volterra (Italy), September 22-28, 2003 Sandro Stringari: "Bose-Einstein Condensation" Ten years of ECT*: Achievement and Vision, Trento, October 10-11 2003

Sandro Stringari: "Superfluidity of ultracold Fermi gases" Symposium on Frontiere della Fisica, Milano, November 27 2003

2002

Franco Dalfovo: "Nucleation of vortices in rotating condensates" and "Quantized vortices in superfluid nanodroplets" Workshop on Vortices in Bose-Einstein Condensates, Les Treilles (Provence-France), June 2002

Franco Dalfovo: "Mean field effects in Bragg scattering on condensates" Workshop on Recoil Induced Effects in Light Scattering on BEC, Gargnano (Italy), June 2002

Sandro Stringari: "Superfluidity of Bose-Einstein condensed gases" INFM Meeting, Bari, June 24 2002

Stefano Giorgini: "Quantum Monte Carlo study of dilute Bose gases in restricted geometries" Laser Physics Workshop, Bratislava, July 1-5 2002

Sandro Stringari: "Superfluidity of Bose-Einstein condensed gases" Vautherin symposium, Orsay, July 4-5 2002

Franco Dalfovo: "Bragg scattering on condensates" BEC Summer Programme, Trento, July 2002

Chiara Menotti: "Collective oscillations of a 1D trapped Bose gas", workshop on "Recent developments in the physics of cold atomic gases" Trento, July 15-18, 2002

Franco Dalfovo: "Quantized vortices in helium droplets" Workshop on The Physics of Quantum Fluid Clusters, Trento, September 2002

Sandro Stringari: "Vortices and Precession Phenomena in superfluid gases" Workshop on the Physics of Quantum Fluid Clusters, Trento, September 16-21 2002 Sandro Stringari: "Vortices and Precession Phenomena in superfluid gases" Workshop on Atom Optics and Interferometry, Lunteren, September 28 - October 2 2002

Augusto Smerzi: "BEC dynamics in periodic potentials" Workshop on Nonlinear Dynamics in Classical and Quantum Mechanics, Sammommè (Italy), October 10-11 2002

Visitors

Long term visitors

Murray Holland (JILA, Univ. Colorado, Boulder), Sept 2003 - August 2004

Short term visitors

Nandini Trivedi (TIFR, Bombay), March 5-15, 2004

Alexander Fetter (Univ. Stanford), February 15 - March 6, 2004

Josh Milstein (JILA and Univ. Colorado, Boulder), December 15-20, 2003

Simon Gardiner (JILA, Boulder), December 8-22, 2003

Patrick Navez (Bruxelles), December 1-3, 2003

Ilya G. Kaplan (UNAM, Mexico), September 8-12, 2003

Aurel Bulgac (Washington Univ., Seattle), July 5-8, 2003

Dörte Blume (Washington), June 4-12, 2003

David Guery-Odelin (ENS, Paris), May 6-11, 2003

Eugene Nikitin (Haifa), May 1st - June 1st, 2003

Georg Bruun (NORDITA, Copenhagen), April 8-12, 2003

Frederic Chevy (ENS, Paris), March 26-30, 2003

Yuri Kagan (Moskow), March 15 - April 15, 2003

Nicolas Pavloff (Orsay), March 14-18, 2003

Dörte Blume (Washington), February 1-28, 2003

Miguel Angel Cazalilla (ICTP - Trieste), November 12-15, 2002

Andrea Trombettoni (Perugia, Italy), September 26 - October 9, 2002

Events and outreach

Conferences organized by the BEC center



International Symposium on Quantum Fluids and Solids Trento, Italy, 5-9 July 2004

QFS2004 is an international symposium devoted to traditional topics of liquid and solid 4He, 3He, 3He-4He mixtures, Hydrogen, confined fermionic and bosonic gases, and other systems that exhibit long-range quantum order and quantum coherence. Emphasis is given to novel experimental techniques and recent theoretical advances. The meeting is a continuation of a long series of symposia, started in Sanibel (Florida) in 1975. The last editions were held at Konstanz and Albuquerque. QFS2004 will be hosted by the University of Trento. The BEC center is co-sponsoring the event and is strongly involved in the organization. The local organizing committe is composed by F.Dalfovo (Symposium chair), L.Reatto (Program chair), F.Pederiva (Publication chair), L.Pitaevskii, S.Stringari and S.Vitale. About 260 people are expected to attend the symposium.



Workshop on Ultracold Fermi Gases Levico (Trento - Italy), 3-6 March 2004

Organizers: Roland Combescot, Christophe Salomon and Sandro Stringari

The workshop on "Ultracold Fermi gases" involved 120 scientists actively working in the field of ultracold Fermi gases, representative of the most important laboratories in the world, including the Nobel laureates Wolfgang Ketterle and Antony Leggett. This workshop was particularly timely due to the recent impressive progress in this field of research.



Second International Workshop **Theory of Quantum Gases and Quantum Coherence** Levico (Trento, Italy), 12-14 June 2003

Organizers: Stefano Giorgini, Fabrizio Illuminati, Chiara Menotti and Anna Minguzzi

The second workshop on "Theory of quantum gases and quantum coherence" (the first edition was held in Salerno in 2001) was particularly intended to get together "young" researchers (at the level of PhD students, Postdocs and junior research associates). The workshop was attended by more than 100 scientists coming from Europe and overseas and the topics of discussion covered various theoretical aspects in the fields of Bose-Einstein condensation, degenerate Fermi gases and coherence properties in quantum many-body systems. Many contributions to the conference were collected in a special section of the issue 37-7 of the Journal of Physics B: Atomic, Molecular and Optical Physics.


INAUGURATION MEETING & CELEBRATION OF LEV PITAEVSKII'S 70TH BIRTHDAY Trento, March 14-15, 2003

The inauguration meeting was combined with the celebration of the the 70th birthday of Lev Pitaevskii. The meeting was attended by about 90 participants coming from different countries and provided a unique opportunity to present the scientific activity of the center. The celebration of Lev Pitaevskii's birthday was introduced by a general talk of Claude Cohen-Tannoudji.



Workshop on the Physics of Quantum Fluid Clusters Trento, 16-21 September 2002 hosted by ECT*

Organizers: M. Barranco, F. Dalfovo and J. Navarro

The main purpose of this workshop was to bring together experimentalists and theorists working with quantum fluid clusters and, more generally, with superfluidity in finite systems. This workshop was the next in a series of meetings which started in Trento in 1989 and continued at Ringberg Schloss, Bavaria, in 1994, 1997 and 2000. The main topics were: Structure and elementary excitations of quantum fluid clusters, Spectroscopy of embedded atoms and molecules, Scattering from and within quantum fluid clusters, Quasi-1D quantum fluids. About 40 people attended the workshop, which was held at ECT^{*}.



BEC2002 SUMMER PROGRAMME

Trento, 1 July - 9 August 2002

Organizers: Charles Clark, Franck Laloe and Sandro Stringari

The BEC summer programme represented an important opportunity for the international community working in the field of BEC to meet and interact on a relatively long term basis. The summer programme was hosted by the European Center for Theoretical Physics (ECT^{*}) in Trento. This initative followed previous long term programmes on similar topics organized in Santa Barbara (1998) and Leiden (2000). A focused conference was organized at the Physics Department of the University of Trento in the middle of the programme. The summer programme and the conference were attended by about 100 participants including the Nobel Laureates Eric Cornell and Wolfgang Ketterle.



Annual meeting Fisica teorica e struttura della materia Fai della Paganella (Trento)

Every year, in March or April, the group of Trento organizes a traditional three day meeting of the community of italian theorists working in the field of condensed matter physics. The meeting is sponsored by INFM. Most of the participants are young researchers, presenting their last results on quantum mechanics, statistical mechanics, quantum field theory, computational physics, etc., with applications in condensed matter, soft matter, electron systems and others fields. In the last two editions, held respectively on 20-23 March 2003 and 18-21 April 2004, and organized by F.Dalfovo and S.Giorgini, there were about 80 participants. The scientific advisory committee also involved L.Colombo, R.Fazio, U. Marini Bettolo Marconi, G.Senatore and A. Vulpiani.

Monday, 15 March 2004 Franco Dalfovo Adiabatic and non-adiabatic phonon evaporation in expanding BEC

Monday, 8 March 2004 Nandini Trivedi (TIFR - Mumbay) Superfluidity/ Superconductivity in strongly interacting systems: Some lessons from High Tc Superconductors

Friday, 27 Feb 2004 Meret Kraemer BEC in rotating traps and optical lattices

Tuesday, 24 Feb 2004 Andrea Perali (Univ. Camerino) BCS-BEC crossover in ultracold Fermi gases

Tuesday, 17 Feb 2004 Alexander Fetter (Univ. Stanford) Annular Structures in Rapidly Rotating Bose-Einstein Condensates

Tuesday, 10 February 2004 Zbigniew Idziaszek Microcanonical fluctuations of a Bose-Einstein condensate

Tuesday, 27 January 2004 Giovanna Morigi (Univ. Ulm) Eigenmodes spectrum of a Coulomb chain in a harmonic potential

Friday, January 23, 2004
Trento-Tübingen meeting
Franco Dalfovo: The fate of phonons in expanding condensates
Giovanni Cennini: All Optical Atom Laser
Chiara Menotti: Sound propagation in optical lattices

Sebastian Slama: Collective Atomic Recoil Laser
Luciano Viverit: Fermions in optical lattices for quantum computation
Sebastian Guenther: Degenerate mixtures of Rubidium and Lithium in tighly confining traps
Stefano Giorgini: Degenerate gases in quasi 1D traps
Jozsef Fortragh: Onchip laboratory for Bose Einstein condensates

Monday, 20 January 2004 C. Salomon (ENS Paris) Recent results on cold Fermi gases at ENS

Friday, 16 January 2004John D. Reppy (Cornell University)High Resolution Heat Capacity Measurements and Hyperuniversality

Tuesday, 13 January 2004 Lev Pitaevskii Kinetic energy of a Bose condensate

Tuesday, 16 December 2003 Josh Milstein (JILA, Boulder) The Nature of Superfluidity in Ultracold Fermi Gases Near Feshbach Resonances

Monday, 15 December 2003 Marco Cozzini Tkachenko modes and sum rules

Friday, 12 December 2003Simon Gardiner (JILA-Boulder and Oxford)Quantum Accelerator Modes in Atom Optics: Epsilon Classics and Semiclassics

Thursday, 4 December 2003 Cheng Chin (Innsbruck) Formation of BEC of ultracold molecules

Monday, 1 December 2003 Patrick Navez (Bruxelles) Collisionless dynamics of the condensate predicted in the random phase approximation

Monday, 24 November 2003 Luca Pezzè Ideal Fermi gas in a harmonic plus period potential

November-December 2003 Murray Holland Lectures on Resonance superfluidity

Tuesday, 18 November 2003 L.Viverit Efficient and robust initialization of a fermionic atom qubit register

Monday, 17 November 2003 M. Köhl (ETH-Zurich) Low-dimensional Bose gases in optical lattices

Friday, 15 November 2003 Alessio Recati Some aspects of cold atoms in reduced dimensionality: quantum computation and models of solid state physics

Monday, 10 November 2003 Uffe Poulsen Entanglement and spin chains

Monday, 3 November 2003 Marilù Chiofalo (Pisa) Resonant-tunnelling dynamics of ultracold atoms

Monday, 27 October 2003

Lev Pitaevskii 1D superfluidity

Friday, 24 October 2003 Mattia Jona-Lasinio (Pisa) Tunneling Landau-Zener asimmetrico nei condensati Bose- Einstein in un potenziale periodico

Monday, 6 October 2003 Michele Modugno BEC in optical lattices

Monday, 29 September 2003 Stefano Giorgini Cold gases in 1D

Thursday, 11 September 2003I.G. Kaplan (UNAM, Mexico)Pauli exclusion principle, fractional statistics and permutation symmetry of many-particle wave functions

Friday, 18 July 2003
Aurel Bulgac (Seattle)
Dilute atomic gases with a large positive scattering length

Wednesday, 9 July 2003 Oriol Bohigas (LPTMS-Orsay) Chaotic dynamics and quantum systems

Friday, 6 June 2003 Marckus Oberthaler (Konstanz) Dispersion management for atomic matter waves

Monday, 19 May 2003 Hui Hu (Trieste) Finite temperature excitations of a trapped Bose-Fermi mixture

Friday, 16 May 2003E. Nikitin (Haifa)High-precision molecular wave-packet interferometry with HgAr dimers

Friday, 9 May 2003
David Guery-Odelin (ENS)
Magnetic guided beam: a first step towards a continuous atom laser

Monday, 28 April 2003G. Blatter (Zurich)Commensurate- incommensurate transition of cold atoms in an optical lattice

Monday, 14 April 2003 Franco Dalfovo Linear and nonlinear response of a condensate to a Bragg pulse

Friday, 11 April 2003Georg Bruun (NORDITA-Copenhagen)Many-body properties of Fermi gases with resonant interactions

Monday, 7 April 2003 Yuri Kagan (Moskow) The nonlinear damping of Bose-Einstein condensate oscillations at ultra-low temperatures

Monday, 31 March 2003 Yuri Kagan (Moscow) Anomalous tunneling of phonon excitations in BEC

Tuesday, 18 March 2003 Iacopo Carusotto (ENS-Paris) Stochastic reformulation of the N body problem Monday, 17 March 2003 Nicolas Pavloff (Orsay) Flow of an atom laser past an obstacle

Friday, 7 March 2003Luigi Accardi (Univ. Roma2)Outline of the stochastic limit of quantum theory

Monday, 3 March 2003 Michele Modugno Theory and experiments in Florence

Monday, 17 February 2003 Dörte Blume (Washington Univ.) Formation of atomic tritium condensates and clusters

Monday, 10 February 2003 Eric Braaten (Ohio State University) Efimov Physics in Alkali Atoms

Monday, 3 February 2003 Brian Jackson Collective modes of condensates at finite temperature

Friday, 31 January 2003 Augusto Smerzi Discrete Gross-Pitaevskii equation for condensates in optical lattices

Monday, 27 January 2003 Luciano Viverit BCS superfluidity in trapped fermions and Bose-Fermi mixtures

Friday, 17 January 2003 Alessio Recati Quasi 2D superfluids and creation of vortex pairs

Friday, 15 November 2002
Miguel Angel Cazalilla (ICTP - Trieste)
Effective harmonic-fluid approach to 1D interacting bosons in a box

Friday, 8 November 2002 Marco Cozzini *Rotating vortex arrays*

Wednesday, 30 October 2002 Bruno Laburthe (NIST) BEC loaded into 1D, 2D, and 3D optical lattices

Monday, 28 October 2002 Chris Pethick (NORDITA) Some novel features of quantum Bose gases

Friday, 25 October 2002 Stefano Giorgini Superfluidity in Bose-Hubbard model

Wednesday, 23 October 2002 Oliver Morsch (Pisa) Dynamics and phase evolution of BEC in optical lattices

Friday, 4 October 2002 Andrea Trombettoni (Perugia) *BEC in optical lattices*

Friday, 27 September 2002 Li Weidong Coherent properties of Bose Josephson Junction

Education and training

The BEC Center has contributed to the PhD programme of the Physics Department of the University of Trento, by funding several fellowships. Several students are presently preparing their doctoral thesis under the supervision of members of the BEC center and/or in the framework of international collaborations.