Istituto Nazionale per la Fisica della Materia
Consiglio Nazionale delle Ricerche

Research and Development Center on
Bose-Einstein Condensation
Trento, Italy

Scientific Report
June 2006 - May 2008
Cover:

* Probability distributions of multiparticle entangled states. This quantity contains the information on the value of the phase shift in a Mach-Zehnder interferometer. The size of the relevant substructures, which here is of the order of the inverse of the total number of particles, indicates the sensitivity of the phase estimation (taken from L. Pezzé and A. Smerzi, to be published)
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Overview

The scientific mission

The INFM-CNR Research and Development Center on Bose-Einstein Condensation (BEC) is aimed to promote theoretical research on the various phenomena related to Bose-Einstein condensation and to the physics of cold atomic gases. Since the first observation of BEC in cold gases in 1995, the study of ultracold gases has become an emerging area of research at the crossing point of several disciplines, including atomic and molecular physics, quantum optics, statistical mechanics and condensed matter physics. BEC has been already achieved in many atomic species and an impressive number of experimental and theoretical groups worldwide is actively working in the field of ultracold atomic gases.

In the last years the field has further grown and developed in several important directions. On the one side the availability of advanced optical techniques has allowed to create new trapping configurations in the form of optical lattices. This has opened new frontiers of research where atomic physics meets important features of condensed matter and solid state physics. These include the superfluid-Mott insulating transition, the implementation of new geometrical configurations where even the dimensionality of the system can be controlled, the study of disorder and localization effects, the occurrence of Bloch and Josephson oscillations, just to mention a few of them.

The availability of Fano-Feshbach resonances characterizing the interaction between several atomic species has opened the fascinating possibility of tuning the two-body interactions by simply changing the external magnetic field, with the consequence that the value and even the sign of the scattering length can be changed in controlled way, yielding the challenging and unique possibility of studying strongly correlated configurations with dilute atomic gases. Important achievements concern the study of ultracold Fermi gases. The initial cooling limitations due to Pauli blocking have been overcome and now the field of ultracold Fermi gases is growing in an impressive way.
The scientific mission

with stimulating perspectives, also in view of the deep analogies between such systems and other Fermi systems, like high Tc superconductors and neutron stars. The Bose-Einstein condensation of molecules formed starting from a Fermi sea and the superfluid phase have been already realized and the long sought BCS-BEC crossover is now object of intense theoretical and experimental investigations. In particular, intense research activities have been devoted to the study of the hydrodynamic behavior (expansion and collective excitations), the vortical configurations, the thermodynamic properties, the radio-frequency transitions and the effects of polarization.

Also the field of mixtures of different atomic species is becoming a rich area of research. The perspective of generating new molecular configurations with permanent dipole moment and unexplored many-body quantum phases is now becoming realistic. Finally ultracold atomic physics is providing new perspectives of application to quantum information processes and precision measurements. These include, among others, the implementation of logical operations with the help of the new available trapping configurations, the advanced development of interferometric techniques based on matter waves and the study of the surface-atom forces.

At the same time the theoretical techniques have developed in a significant way and more sophisticated quantum many-body approaches have been proposed in order to study the complexity of the new configurations. The theoretical tools have been mainly based, in the first years, on the use of Gross-Pitaevskii theory for the order parameter and have proven quite successful, especially to predict the behaviour of trapped Bose-Einstein condensates at low temperature, both at equilibrium and out of equilibrium. More recent developments include, among others, quantum simulations based on Monte Carlo techniques, quantum models of solid state physics like the Hubbard model and new statistical approaches to study fluctuations and correlations. In general the theoretical approaches in the last few years have become more and more interdisciplinary. A significant example is given by the recent progress in the study of the surface-atom Casimir-Polder force stimulated by the availability of highly precise measurements of the oscillations of Bose-Einstein condensates. At the same time many of the ideas of dilute quantum gases are becoming of central importance also in solid state physics and quantum optics. A significant example is given by the physics of exciton-polaritons, where the concepts of Bose-Einstein condensation and superfluidity are gaining more and more importance.

The growth of the field has benefited in a crucial way by the cooperative efforts of experimental and theoretical groups in many laboratories. The aim of the BEC Center is to reinforce the interdisciplinary links of the theoretical research as well as the
links between theoretical and experimental activities, establishing direct and systematic collaborations with the main laboratories in the world.

The BEC Center

The BEC Center was established by the Istituto Nazionale per la Fisica della Materia (INFM) in Trento in June 2002, following a selection made by an international panel. The Center is hosted by the Department of Physics, University of Trento. Scientists belonging to the BEC Center include INFM-CNR researchers as well as personnel from the University, together with PhD students and post-doctoral fellows, who are partly funded by INFM-CNR and partly by the University. At present the scientists active in the Center are about 20. The budget of the BEC Center is provided by the Consiglio Nazionale per le Ricerche (CNR) and by the Provincia Autonoma di Trento (PAT). The research activity of the Center is also supported by the European Union and by the Italian Ministry of Research. In the last years the Center has significantly strengthened the links with the Atomic Physics team of the European Laboratory for Non-Linear Spectroscopy (LENS) in Florence, by reinforcing the scientific collaborations and by promoting joint institutional initiatives. Starting from 2006, the financial support of INFM-CNR to the Florence team is supplied and managed within a joint administrative body.

The BEC Center has significantly contributed to the worldwide development of the field of ultracold gases through a long series of scientific publications, the reinforcement and the creation of international collaborations, the organization of workshops and conferences and through the training of young scientists. The impact of the scientific work carried out by the Trento team is recognized by the Essential Science Indicators of Thomson Scientific where at the special topics “Bose-Einstein Condensates” (http://www.esi-topics.com/bose/) and “Superfluids” (http://www.esi-topics.com/superfluids/) the papers of the Trento BEC team rank at the top positions.
The articles written by the researchers of the BEC center are collecting more than 1000 citation/year, according to the ISI-Thomson database, with an average of about 25 citations per article. The study of surface-atom Casimir forces within a joint JILA-Trento collaboration, the theoretical predictions for the polarized phase of a resonant Fermi gas experimentally confirmed at MIT, the joint theory-experiment work done on vortices in a superfluid of polaritons observed at EPFL, the proposal and implementation of a new strategy for phase detection in Mach-Zehnder interferometers, are a few examples of the most important achievements of the last two years. Moreover, in 2008, the group published a new review article in Reviews of Modern Physics on the theory of ultracold atomic Fermi gases, as a natural continuation of a previous article on the Theory of Bose-Einstein condensation in trapped gases, published in 1999, which has been cited more than 2000 times to date.

Up-to-date information on the activity of the BEC Center can be found at the website: http://bec.science.unitn.it

October, 2008
Organization

Management

Director
* Sandro Stringari

Scientific board
* Jean Dalibard
* Chris Pethick
* William D. Phillips
* Gora Shlyapnikov
* Peter Zoller

Secretariat
* Laura Bianchini (from January 2006 to June 2007)
* Beatrice Ricci (from January 2008)

Research staff

Personnel of University of Trento
* Franco Dalfovo
* Stefano Giorgini
* Lev P. Pitaevskii
* Sandro Stringari
INFM-CNR researchers

* Augusto Smerzi
* Iacopo Carusotto
* Chiara Menotti
* Alessio Recati
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Visiting Scientists

* Nikolai Prokof’ev (Univ. Massachusetts, Amherst, USA), Dec. 2005 - Aug. 2006
* Massimo Boninsegni (Univ. Alberta, Edmonton, Canada), Sept. 2006 - Jan. 2007
* Dörte Blume (Washington State Univ., Pullman, USA), July 2006 - Dec. 2006

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* Michele Modugno
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* Michiel Wouters
Graduate Students

* Mauro Antezza (PhD thesis defended in November 2006)
* Luca Pezzè (PhD thesis defended in February 2007)
* Sebastiano Pilati (PhD thesis defended in February 2008)
* Francesco Bariani
* Ingrid Bausmerth
* Francesco Piazza
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* Francesco Bariani (Thesis, University of Insubria, Como, 2006)
* Francesco Piazza (Thesis, University of Bologna, 2007)

Technical Staff

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Research

This report provides an overview of the main scientific activities on ultracold atoms and related topics, with reference to the period June 2006 - May 2008. The scientific work carried out at the Center can be naturally classified according to the following research lines:

- Fermionic superfluidity and BCS-BEC crossover
- Polarized Fermi gases
- Quantum Monte Carlo
- Rotating gases and quantized vorticity
- Nonlinear dynamics and solitons
- Ultracold gases in optical lattices
- Dipolar gases
- Semiconductor microcavities and exciton-polaritons
- Quantum optics and quantum fields
- Casimir forces
- Matter-waves interferometry
- Quantum information processing
FERMIONIC SUPERFLUIDITY AND BCS-BEC CROSSOVER

An impressive amount of experimental and theoretical work has characterized the investigation of ultra cold Fermi gases in the last few years. The Trento team has contributed significantly to the field. In particular a significant effort has been devoted to writing a long review paper [1], published in the journal Reviews of Modern Physics. In this section we report a selection of significant achievements concerning the study of the elementary excitations along the crossover and of the single-particle excitations by stimulated Raman spectroscopy [2]. Other contributions of the Trento team concern of the study trapped Fermi gases with unequal masses [3], the study of the fluctuations of the number of particle both in Bose and Fermi gases [4] and the study of the Josephson relations along the crossover [5]. The following sections will be instead devoted to the effects of polarization on fermionic superfluidity and to the problem of rotating Fermi gases and to the implementation of Monte Carlo approaches.

Collective oscillations and Landau critical velocity

In [6] we have investigated the behaviour of the collective excitations of a superfluid Fermi gas along the BEC-BCS crossover and their interplay with the single-particle excitations in the determination of Landau’s critical velocity. The calculation was carried out using a natural generalization of the Bogoliubov de-Gennes equations which accounts for the self consistent treatment of the collective oscillations. Actually the BdG equations, in their traditional form, account for the description of the single-particle excitations (including the pairing gap), but not for the phonon mode (the so called Bogoliubov-Anderson mode). The description of this latter mode requires the development of a kinetic theory which was implemented in our work along the whole BEC-BCS crossover. Special attention has been devoted in this paper to the calculation of the Landau’s critical velocity whose behaviour is very different in the various regimes. In the molecular BEC regime the phonon mode provides the proper mechanism for Landau’s instability in the presence of a moving frame, while single particle excitations are not important, being located at very high energies. In the BEC regime the sound velocity vanishes with the square root of the scattering length, and hence gives rise to small values of the critical velocity. In the opposite BCS regime of negative scattering lengths the relevant excitations are instead of single-particle nature and are characterized by the pairing gap whose value becomes smaller and smaller as the scattering length tends to zero. One then expects that the critical velocity will take
its maximum values near unitarity, where both phonon and single particle excitations play an important role in characterizing the dynamic behaviour of the system. Near unitarity the critical velocity is actually of the order of the Fermi velocity. The main results for the excitation spectrum and for the value of the critical velocity along the crossover are reported in Figs. 1-2.

The critical velocity has been recently measured in the MIT laboratory [7] using a moving one-dimensional optical lattices. The experimental results confirm that the critical velocity is largest near unitarity (see Fig.3), in agreement with the predictions of theory.

**Raman spectroscopy and $d$-wave pairing**

New diagnostic techniques that are sensitive to the peculiar correlations that appear in strongly interacting Fermi gases are expected to play a central role in future experimental investigations that aim to characterize the exotic phases that may appear in such systems. As it was demonstrated by the success of angle-resolved photoemission spectroscopy (ARPES) from solid materials, a measurement of the one-body Green’s function can provide detailed information e.g. on the pairing mechanism that is responsible for a superconducting behaviour. Taking inspiration from this physics, we have investigated the promise of stimulated Raman spectroscopy to obtain information on the Fermi surface and the quasiparticle spectrum of a strongly interacting Fermi gas in an optical lattice [8]; features related to a $d$-wave pairing pseudo-gap are pointed out. A first experimental implementation of such a technique has been recently reported in [9]: the evolution of the momentum resolved photoemission signal is followed along the BEC-BCS crossover and a signal consistent with the presence of a large pairing gap is
Figure 2: Landau critical velocity calculated along the BCS-BEC crossover. The dashed line is the sound velocity.

Figure 3: Critical velocity through the BEC-BCS crossover. A pronounced maximum was found at resonance. The solid line is a guide to the eye. From [7]
obtained in the resonance regime at temperatures around $T_c$.


POLARIZED FERMI GASES

The behavior of spin-polarized Fermi systems has been a fascinating subject of research for over fifty years since it can lead to the interplay of superfluidity and magnetism. In particular a Fermi superfluid is resistant to spin polarization and so one can naturally raise the question of what happens when we attempt to polarize it.

An example occurs in a superconducting metal when we apply a magnetic field. Under certain conditions the coupling to the orbital motion is negligible and the important effect is the coupling to the electron spins. The field can lower the energy of the spin-polarized normal state and, if it is strong enough, make the normal state energetically more favourable than the superconducting spin-singlet state. The value of the field at which this transition takes place is known as the Chandrasekhar-Clogston limit and, in a BCS superconductor, requires that the field be larger than $\Delta/\sqrt{2}$ where $\Delta$ is the gap. Crucially, this estimate assumes that the change in energy of the normal state due to polarization is only kinetic in origin. In a strongly interacting system the value of the Chandrasekhar-Clogston critical field is expected to depend crucially also on the interactions in the normal phase.

This situation has arisen in recent experiments carried out in the strongly interacting unitary limit of two-component atomic Fermi gases [1, 2, 3, 4, 5]. In these systems the role of the polarizing magnetic field is played by the chemical potential difference between the two atomic species. There is now clear experimental evidence that such systems are characterized by a phase separation between a normal and a superfluid component. From a theoretical point of view it is then crucial to have an accurate knowledge of the equation of state in the two phases in order to make reliable predictions for the Chandrasekhar-Clogston limit. Actually most of the theoretical work have so far neglected interaction effects in the normal phase, thereby providing a poor description of the experimental results.

The Trento team has given an important contribution to the field analysing in detail the role of the interactions and consequently in establishing the phase diagram of the unitary spin-polarized system. These studies have provided a succesful description of the critical polarization and of the density profiles in the presence of harmonic trapping as well as useful predictions for the collective oscillations.

Chandrasekhar-Clogston limit and density profiles

The Trento approach [6, 7] is based on the description of the normal state energy at unitarity through an energy functional which consists of an expansion in the concentra-
Figure 1: Density profiles for a polarization $P = 44\%$. Theory: solid black line (dashed red line) is the spin-$\uparrow$ (spin-$\downarrow$) density. Experiment: the black (red) line is the spin-$\uparrow$ (spin-$\downarrow$) density as reported in [3]. The density jump in the $\downarrow$ component is clearly visible.

Polarized Fermi gases

...
approximation. We found that the superfluid core disappears for values of the total polarization $P = (N_\uparrow - N_\downarrow)/(N_\uparrow - N_\downarrow)$ larger than the critical value $P_c = 0.77$, in agreement with the experimental results of the MIT group [2, 3]. Moreover, both the in situ column and density profiles of the two spin components agree very well with the experimental values (see Fig. 1). It is worth noticing that the proper inclusion of interaction effects in the normal phase is crucial to correctly describe the superfluid/normal quantum phase transition. The BCS mean-field approach would predict the incorrect value $P_c \simeq 1$ for the critical polarization.

**Collective oscillations and collisional effects**

The functional (2) for the normal phase is particularly well suited to study the highly polarized limit where the number of particles in the spin-down component is much smaller than in the spin-up component ($N_\downarrow \ll N_\uparrow$). In this limit the collective oscillations of a trapped gas can be classified into two categories: the in-phase oscillations, where the motion is basically dominated by the majority component, and the out-of-phase (spin) oscillations where the minority component moves in the trap, the majority one being practically at rest. The spin dipole frequency is given by the simple formula [6]

$$\omega_{D}^{(s)} = \omega_i \sqrt{\frac{m}{m^*} \left(1 + \frac{3}{5} A\right)}$$  \(2\)
showing that even in the extreme limit of a single impurity interactions affect the spin frequency in a sizable way by increasing the value by a factor $\sim 1.23$ with respect to the ideal gas prediction. Its measurement, together with the determination of the radii and/or the RF spectrum [13], would consequently provide unique information on the value of the relevant interaction parameters in the normal phase. It is worth noticing that differently from other quantities – e.g., radii and the RF gap[7] – the spin frequency (2) is independent of the number of atoms.

A crucial point to discuss is the role of collisions which tend to damp out the spin oscillation. Indeed the spin dipole mode described above is well defined only in the collisionless limit $\omega_D^{(s)} \tau_P \gg 1$, where $\tau_P$ is the collisional time, while it becomes overdamped in the hydrodynamic regime $\omega_D^{(s)} \tau_P \ll 1$ since the spin current is not conserved by collisions. The collisional time $\tau_P$ is expected to be particularly small at unitarity due to the strong interactions, unless one considers extremely cold systems where collisions are quenched by Pauli blocking. We have investigated this problem in close collaboration with the team of the Niels Bohr Institut [14]. The most relevant result is shown in Fig. 2 where we report the quantity $1/\omega_D^{(s)} \tau_P$ as a function of the amplitude of the oscillation in units of the majority cloud radius $R_\uparrow$ and for different values of the temperature. While, as expected, at zero temperature the “small” amplitude oscillations are well defined, by increasing $T$ the effects of collisions become soon important due to the small value of the collective frequency. Less damping is of course expected if one considers the higher frequency modes of the gas.


Quantum Monte Carlo methods are particularly well suited theoretical tools to investigate the properties of physical systems in which correlations and fluctuations play a major role. An important example is provided by a two-component Fermi gas where the interaction strength is tuned by means of a Feshbach resonance. At low temperatures the state of the gas changes in a continuous way from a BCS superfluid of large Cooper pairs to a Bose-Einstein condensate of tightly bound molecules (BCS-BEC crossover). The equation of state along the crossover and the properties of correlation functions, such as the momentum distribution, the two-body density matrix, the pair correlation function and the static structure factor, have been calculated by the Trento group in a number of publications using the fixed-node diffusion Monte Carlo (FN-DMC) method [1]. This technique is applicable at zero temperature and is based on an ansatz for the nodal surface of the many-body wavefunction which provides a rigorous upper bound for the ground-state energy of the gas. Even though the FN-DMC method is not “exact”, as it relies on the fixed-node approximation to overcome the fermionic sign problem, it is known to be highly accurate to describe the properties of gas-like states.

A direction of research which in the last couple of years has become extremely interesting because of the experimental activity carried out first at MIT and Rice University and now in many other laboratories, concerns two-component gases with imbalanced Fermi energies, either by a different mass or a different population of the two species. In particular, in the case of polarized systems, one predicts a quantum phase transition from a normal to a superfluid state by changing the interaction strength for fixed polarization. The Trento group has carried out an extensive study, using the FN-DMC method, of the phase diagram of polarized Fermi gases along the BCS-BEC crossover and of the nature of the superfluid to normal quantum phase transition.

Another example of a class of many-body problems that can only be addressed using numerical techniques concerns the critical behavior and the determination of the superfluid transition temperature. In this case critical fluctuations dominate the physical properties of the system and, in general, do not allow for a treatment in terms of mean-field or perturbation theory. We have addressed, using the exact Path Integral Monte Carlo (PIMC) method, the problem of the superfluid transition temperature of a Bose gas in three and two spatial dimensions. In 3D we improved on previous calculations by using a recently proposed algorithm [2] which allows for an efficient calculation of the superfluid density of very large systems. In 2D we investigated...
the critical behavior of a gas of hard disks and we calculated the Kosterlitz-Thouless transition temperature as a function of the interaction strength.

In the following we describe in some more details the main results obtained from the quantum Monte Carlo simulations mentioned above.

**Normal state of a polarized Fermi gas at unitarity**

We study the polarized normal phase in the unitary limit by adopting an approach inspired by the theory of dilute solutions of $^3$He in $^4$He. We assume that the majority species ($\uparrow$) forms a background experienced by the minority species ($\downarrow$) and that the latter behaves as a gas of weakly interacting fermionic quasiparticles even though the $\uparrow - \downarrow$ atomic interaction is very strong.

We begin by writing the expression for the energy $E$ of a homogeneous system in the limit of very dilute mixtures and at zero temperature. The concentration of $\downarrow$ atoms is given by the ratio of the densities $x = n_\downarrow/n_\uparrow$ and we will take it to be small. If only $\uparrow$ atoms are present then the energy is that of an ideal Fermi gas $E(x = 0) = 3/5 E_{F\uparrow} N_\uparrow$, where $N_\uparrow$ is the total number of $\uparrow$ atoms and $E_{F\uparrow} = \hbar^2/2m(6\pi^2 n_\uparrow)^{2/3}$ is the ideal gas Fermi energy. When we add a $\downarrow$ atom with a momentum $p$ ($|p| \ll p_{F\uparrow}$), we shall assume that the change in $E$ is given by

$$\delta E = \frac{p^2}{2m^*} - \frac{3}{5} E_{F\uparrow} A.$$  \hspace{1cm} (1)

When we add more $\downarrow$ atoms, creating a small finite density $n_\downarrow$, they will form a degenerate gas of quasiparticles at zero temperature occupying all the states with momentum up to the Fermi momentum $p_{F\downarrow} = \hbar(6\pi^2 n_\downarrow)^{1/3}$. The energy of the system can then be written in a useful form in terms of the concentration $x$ as (see also the Section on Polarized Fermi gases in this report):

$$\frac{E(x)}{N_\uparrow} = \frac{3}{5} E_{F\uparrow} \left(1 - Ax + \frac{m}{m^*} x^{5/3} + Bx^2 \right).$$ \hspace{1cm} (2)

Eq.(2) is valid for small values of the concentration $x$, i.e. when interactions between $\downarrow$ quasiparticles as well as further renormalization effects of the parameters can be neglected.

We calculate $A$ and $m^*$ using a fixed-node diffusion Monte Carlo (FN-DMC) approach [3]. For a single $\downarrow$ atom in a homogeneous Fermi sea of $\uparrow$ atoms the trial wave function $\psi_T$, which determines the nodal surface used as an ansatz in the FN-DMC
Figure 1: Equation of state of a normal Fermi gas as a function of the concentration $x$ (circles). The solid line is a polynomial best fit to the FN-DMC results. The dashed line corresponds to Eq. (2). The dot-dashed line is the coexistence line between the normal and the unpolarized superfluid states and the arrow indicates the critical concentration $x_c$ above which the system phase separates. For $x = 1$, both the energy of the normal and of the superfluid (diamond) states are shown.

calculation, is chosen to be of the form

$$
\psi_{\mathbf{q}}(\mathbf{r}_{\downarrow}, \mathbf{r}_{\uparrow 1}, ..., \mathbf{r}_{\uparrow N}) = \exp(i\mathbf{q} \cdot \mathbf{r}_{\downarrow}) \prod_{i=1}^{N_{\uparrow}} f(r_{\uparrow i}) D_{\uparrow}(N_{\uparrow}),
$$

where $r_{\downarrow}$ denotes the position of the $\downarrow$ atom and $r_{\uparrow i} = |\mathbf{r}_{\downarrow} - \mathbf{r}_{\uparrow i}|$. In this equation the plane wave $\exp(i\mathbf{q} \cdot \mathbf{r}_{\downarrow})$ corresponds to the impurity travelling through the medium with momentum $\hbar \mathbf{q} = (n_x, n_y, n_z)2\pi\hbar/L$, where $L$ is the length of the cubic box and the $n_i$ are integers describing the momentum in each coordinate. Furthermore, $D_{\uparrow}(N_{\uparrow})$ is the Slater determinant of plane waves describing the Fermi sea of the $N_{\uparrow}$ atoms and the Jastrow term $f(r)$ accounts for correlations between the impurity and the Fermi sea. We obtain the following values: $A = 0.97(2)$ and $m^*/m = 1.04(3)$.

A relevant question is to understand whether the equation of state (2) is adequate to describe regimes of large values of $x$ where interaction between quasiparticles and other effects might become important. To answer this question we have carried out a FN-DMC calculation [3] of the equation of state at finite concentrations $x = N_{\downarrow}/N_{\uparrow}$.
using the trial wave function

\[ \psi_T(r_1',...,r_{N_1}',r_1,...,r_{N_1}) = \prod_{i,i'} f(r_{ii'}) D_\uparrow( N_\uparrow ) D_\downarrow( N_\downarrow ) , \]

where \( i \) and \( i' \) label, respectively, \( \uparrow \) and \( \downarrow \) atoms. The nodal surface of the wave function \( \psi_T \) is determined by the product of Slater determinants \( D_\uparrow( N_\uparrow ) D_\downarrow( N_\downarrow ) \) and coincides with the nodal surface of a two-component ideal Fermi gas. As a consequence, the wave function in Eq. (4) is incompatible with off-diagonal long-range order (ODLRO) and describes a normal Fermi gas. The results for the equation of state of the normal Fermi gas are shown in Fig. 1 from which one also estimates the coefficient \( B \) in Eq. (2). The first order transition between the polarized normal and the unpolarized superfluid phase takes place at the critical concentration \( x_c = 0.44 \) and the coexistence curve is shown in Fig. 1.

The density profiles obtained from the above equation of state for a gas in harmonic trap are discussed in the Section on Polarized Fermi gases in this report.

**Phase diagram of a polarized Fermi gas along the BCS-BEC crossover**

Recent experiments, carried out on harmonically trapped configurations, investigate superfluidity and Bose-Einstein condensation of fermionic pairs in these systems by varying the strength of interactions, exploring the BCS-BEC crossover, the temperature of the gas and the degree of polarization [5]. Evidences of phase separation between a superfluid core and a normal external shell are reported for systems close to the unitary
limit [6] and on both sides of the Feshbach resonance [7]. On the theoretical side, the phase diagram of a polarized Fermi gas with tunable interactions is the subject of a number of studies both at zero [8] and at finite temperature [9]. These studies, which refer to uniform systems and are based on a mean-field approach, predict the existence of a normal phase for large enough polarization on the BCS side of the crossover and of a polarized superfluid phase on the BEC side, separated by a region where the two phases coexist in equilibrium. However, if interactions are not weak, the mean-field theory fails to describe correctly the nature of the phase separated state and to provide a reliable estimate of the critical value of polarization where phase separation occurs. The key ingredient, that is missing in the mean-field description, is the proper account of interaction effects in the normal phase [10].

We carried out a quantitative study of the phase diagram at zero temperature along the BCS-BEC crossover [11] using fixed-node diffusion Monte Carlo (FN-DMC) simulations, which have proven very accurate in the investigation of strongly correlated ultracold Fermi gases. We determine the equation of state of the homogeneous normal and superfluid phase as a function of interaction strength and population imbalance. From the phase equilibrium conditions we identify three different phase separated states corresponding to: (a) the polarized superfluid coexisting with the fully polarized normal gas, (b) the polarized superfluid coexisting with the partially polarized normal gas and (c) the unpolarized superfluid coexisting with the partially polarized normal gas. The state (a) and (c) are respectively separated from the homogeneous superfluid and normal phase by a first order phase transition, while state (a) and state (b) as well as
Quantum Monte Carlo methods

Critical temperature of interacting Bose gases in two and three dimensions

The theoretical determination of the superfluid transition temperature in homogeneous, interacting Bose systems is a fine example of a many-body problem that can be quantitatively addressed only by “exact” numerical techniques. This fact is well understood in the case of strongly interacting quantum fluids, such as liquid $^4$He, but at first glance is surprising in the case of dilute gases. However, in three dimensions (3D) the presence of any finite interaction changes the universality class of the transition from the Gaussian complex-field model, corresponding to the ideal gas Bose-Einstein condensation (BEC) temperature $T_c^0$, to that of the XY model. Thus, the critical temperature $T_c$ can not be obtained from $T_c^0$ perturbatively. In two dimensions (2D) the superfluid transition, which belongs to the Berezinskii-Kosterlitz-Thouless (BKT) universality class, is induced by interaction effects and there is no unperturbed critical temperature to start with.

In a 3D weakly repulsive gas the critical temperature shift is fixed by the $s$-wave scattering length $a$ ($a > 0$), which characterizes interatomic interactions at low temperature [12, 13],

$$T_c = T_c^0 \left[ 1 + c(a r^{1/3}) \right].$$  (5)
Quantum Monte Carlo methods

Figure 5: (color online). Critical temperature in 3D as a function of the gas parameter $na^3$. The symbols labeled by PRA04 correspond to the results of Ref. [16], the ones labeled by PRL97 correspond to Ref. [12]. The dashed line (green) is the expansion (5) of Ref. [14] and the dotted line (black) is the expansion of Ref. [15] including logarithmic corrections. The solid line (red) is a guide to the eye.

Here $n$ is the gas density and $T_c^0 = (2\pi h^2 / mk_B)[n / \zeta^{(3/2)}]^{2/3}$ with $m$ the particle mass and $\zeta^{(3/2)} \approx 2.612$. The numerical coefficient $c$ in Eq. (5) was calculated in Refs. [14] by solving the effective 3D classical $|\psi|^4$ model using lattice Monte Carlo simulations. The reported value is $c = 1.29 \pm 0.05$. The same classical model was employed in Ref. [15] to calculate the higher-order logarithmic correction to (5) (see Fig. 5).

Continuous-space studies of a gas of hard spheres, based on the conventional PIMC algorithm, were carried out in Refs. [12, 16]. Both calculations suffered from two shortcomings: first, the number of particles in the simulation was only few hundreds making the extrapolation to the thermodynamic limit difficult; second, the algorithm is known to be inefficient for simulations of the superfluid density. The results of Ref. [16] for $T_c$ agree with the asymptotic law (5) at small densities, in contrast to the significantly lower values obtained in Ref. [12] (see Fig. 5). Moreover, there is strong (about ten standard deviations!) discrepancy between Refs. [12] and [16] at higher densities calling for further investigation of the problem.

In 2D the BKT transition temperature of a weakly interacting gas is written in the form

$$T_{\text{BKT}} \approx \frac{2\pi h^2 n_{2D}}{mk_B} \frac{1}{\ln(\xi/4\pi) + \ln(1/n_{2D}a_{2D}^2)},$$

where $n_{2D}a_{2D}^2$ is the corresponding gas parameter, and $a_{2D}$ is the 2D scattering length.
Figure 6: (color online). Results for a 3D hard-sphere gas with $na^3 = 5 \times 10^{-3}$. Scaled superfluid fraction as a function of temperature for different system sizes. The inset shows the $N$ dependence of the intersection point between lines corresponding to pairs of consecutive system sizes.

Figure 7: (color online). Critical temperature of a 2D gas of hard disks as a function of the gas parameter $na^2$. The solid line is the result (6).
The numerical coefficient $\xi$ was calculated in Ref. [17] from lattice Monte Carlo simulations of the classical $|\psi|^4$ model, similarly to the 3D case, yielding the value $\xi = 380 \pm 3$.

An important question concerns the range of applicability of Eqs. (5)-(6) since they were derived for the limit of vanishingly small interaction strength. More broadly, one is interested in knowing up to what value of the gas parameter it is possible to express $T_c$ as a function of $na^3$ alone and ignore the dependence on the interaction potential details. These questions are particularly relevant for the 2D case where experimental determinations of the critical temperature in trapped configurations have become available [18, 19].

In collaboration with Nikolay Prokof’ev we carried out PIMC simulations to calculate the superfluid transition temperature in interacting Bose gases both in 3D and in 2D [20]. We performed large-scale simulations of homogeneous systems with up to $N = 10^5$ particles. The simulations were based on the worm algorithm, recently extended to continuous-space systems, which allows for a reliable and efficient calculation of the superfluid density. With new data the extrapolation of the critical temperature to the thermodynamic limit could be done with the level of accuracy that was unreachable in previous attempts (see Fig. 6). In 3D we determined the dependence of $T_c$ on the gas parameter $na^3$, reporting good agreement with the expansion (5) in the dilute regime and significant deviations from previous studies [12, 16] at higher densities. Furthermore, we carried out simulations with both a hard- and a soft-sphere interatomic potential to investigate the universal behavior in terms of the scattering length. In 2D we calculated $T_{\text{BKT}}$ for a hard-disk gas as a function of the interaction strength, finding results in excellent agreement with the prediction (6) up to regimes of surprisingly high density (see Fig. 7).


Quantum Monte Carlo methods


Rotational effects are one of the key issues in the physics of superfluids, due to the irrotational constraint imposed by the existence of the order parameter, which is at the origin of the quenching of the moment of inertia and of the formation of quantized vortices. In the last two years the research activity of the Trento team on rotating quantum gases has mainly focused on the behavior of superfluid Fermi gases. A fruitful collaboration with the Orsay team has also concerned the study of Bose-Einstein condensates in the one-dimensional ring geometry.

Adiabatic rotation of a Fermi superfluid

Due to the irrotationality constraint, a superfluid gas cannot rotate like a normal fluid. This implies the quenching of the moment of inertia at small angular velocities and the occurrence of quantized vortices at higher $\Omega$.

An interesting possibility is that the systems keeps the macroscopic conditions of irrotational flow, without the formation of vortices, even at high angular velocity, in a regime of metastability. This is known to happen in rotating Bose-Einstein condensates [1] and a similar possibility in the case of a Fermi superfluid at unitarity was investigated in [2]. It requires proper conditions of adiabaticity in the ramping of the angular velocity in order to avoid that the system jumps into the ground state vortical configuration. There is however a deep difference between a BEC and a Fermi superfluid at unitarity. In fact a unitary superfluid is very fragile near the border where the superfluid energy gain vanishes like $n^{2/3}$ and atoms may like to enter a normal phase where, in the presence of a rotating trap, they gain energy due to the centrifugal effect. In [3] we have investigated this effect by imposing the equilibrium between the superfluid and the rotating normal phase. This is obtained by imposing that, at equilibrium, the superfluid and normal phase should have the same pressure. We predict the occurrence of a bimodal configuration with a non rotating superfluid core surrounded by a rotating normal shell. A typical form for the density distribution is shown in Fig. 1. The density change (first order phase transition), going from the superfluid core and the normal outer shell, is evident. The density jump does not depend on the angular velocity and the ratio between the normal and superfluid density is 0.85. These results follows from the knowledge of the equation of state available from Monte Carlo calculations in the normal [4] and superfluid [5] phase. The resulting bimodal distribution shares important analogies with the phase separation exhibited by – non-rotating – polarized Fermi gases (see section on Polarized Fermi gases in this report), where an unpolarized
superfluid is in equilibrium with a polarized normal gas.

In a more recent paper [6] we have studied the consequences of the rotation on the phase separation of a polarized Fermi gas. The main result in this case is that the weight of the normal phase is favoured with respect to the non rotating configuration. Furthermore the jump in the densities is not constant on the interface, but depends on both the value of the total polarization and on the angular velocity. A further interesting feature emerging from these calculation is the occurrence of a spontaneous breaking of isotropic symmetry in the rotating plane. This effect, already known in the case of rotating BEC’s [1] exhibits different features in the case of the rotating Fermi gas due to the new role played by the interface between the superfluid and the normal component.

**Dynamics of a Fermi superfluid with a vortex lattice**

In [7] we have focused on the dynamical behavior of the system in the presence of vortical configurations, when the rotation gives rise to a triangular vortex lattice. At a macroscopic level, the system can be described by the equations of rotational hydrodynamics. This is the diffused vorticity approximation, which is valid provided the typical length scales of the considered dynamics are significantly larger than the intervortex spacing. In this framework, we have studied the breathing modes of a rotating Fermi gas close to the unitary regime. These oscillations are sensitive to both the equation of
Rotating quantum gases

Figure 2: Frequencies of the axial and radial breathing modes $\omega_\pm$ calculated for a BEC gas containing a vortex lattice in the vorticity approximation, as a function of the rotation frequency $\Omega_0$. We consider a cigar-shape trapping ($\omega_\perp/\omega_z = 10$).

state and the angular momentum carried by the system and hence provide a valuable tool to investigate the BEC-BCS crossover in the presence of vortices. To this aim, a polytropic equation of state was assumed, i.e., $\mu \propto n^\gamma$, where $\mu$ is the chemical potential, $n$ is the density, and $\gamma$ is a polytropic index. In the BEC regime $\gamma = 1$, while at unitarity $\gamma = 2/3$.

By considering an axisymmetric configuration and solving the equations of rotational hydrodynamics for the density and the velocity variations of the lowest compressional modes, we have found two solutions, corresponding to the coupling between the radial and axial breathing motion (see Fig. 2). In the derivation of this result it is crucial to include the changes in the rotational term of the velocity flow, due to the motion of vortex lines, in order to ensure the conservation of angular momentum. We have studied the resulting equation for the compressional frequencies in different limiting cases of physical interest: the centrifugal limit where the rotation rate $\Omega$ approaches the radial trapping frequency $\omega_\perp$; the case of isotropic trapping where the axial trapping frequency $\omega_z$ coincides with the radial one; the cigar-shaped configuration where $\omega_z \ll \omega_\perp$. Finally, a detailed discussion has been devoted to the differences with respect to the case of BECs in the centrifugal limit. Indeed, it turns out that the conditions to reach the lowest Landau level (LLL) regime, where the vortex core size becomes comparable with the intervortex distance and the Thomas-Fermi approximation breaks down, are much more demanding than in the BEC case.

While the rotational hydrodynamic equations explicitly account for the presence of
vortices through the average vorticity, within this description it is impossible to discuss the effects of vortex-vortex interaction in the lattice dynamics. Such phenomena are at the basis of Tkachenko modes, corresponding to the shear distorsions of the lattice. Their frequency, being driven by the elastic forces of the vortex array, is typically much smaller than the frequency of the hydrodynamic modes. The Tkachenko oscillations have been already studied in the case of BEC, both theoretically and experimentally. The usually adopted theoretical framework is given by the elasto-hydrodynamic equations, which generalize the rotational hydrodynamic equations by the inclusion of an elastic energy term. The averaged density and velocity (including the rotational term corresponding to average vorticity) are hence coupled to a displacement field, accounting for the average displacement of vortices from the equilibrium position in the lattice.

The study of Tkachenko modes in Fermi gases is particularly appealing due to the possibility of generating very large vortex arrays in these systems. The number of vortices can be almost an order of magnitude larger than in the bosonic case, due to the larger size of the trapped configurations. This offers the possibility of investigating Tkachenko modes at relatively small rotation rates, where the oscillation frequency is significantly enhanced with respect to the high rotation regime and experimental detection can be more accurate. In [8] the Tkachenko oscillations were investigated for an axisymmetric configuration of a Fermi gas at unitarity.

**Bose-Einstein condensate in ring geometry**

In the recent years, many laboratories started developing ring traps for ultra-cold atoms. The interest lies not only in the possibility of providing a direct manifestation of superfluidity, but also in the potential applications as high precision gyroscopes, based on matter-wave rather than light-wave interferometry. The predicted better precision and stability are especially interesting for the aerospace industry.

In the framework of a collaboration with the experimental group of Philippe Bouyer and Alain Aspect, Sylvain Schwartz, under the supervision of Iacopo Carusotto and Chiara Menotti, has devoted an important part of the PhD thesis to the theoretical study of tight closed-loop waveguides for atoms. Using a Born-Oppenheimer point of view, we have reduced the full 3D experimental situation in the presence of strong transverse confinement and rotation to an effective 1D problem [9]. We obtain extra effective potential terms with respect to the ones previously known in the literature, which arise due to the spatial variation of the curvature and the transverse confinement. We have checked the validity of our analytical results comparing successfully with 2D numerical simulations (see Fig. 3). This provides us with a simple but accurate theory,
useful to predict the response of the matter-wave gyroscope to a global rigid rotation.

Further developments of this research line include the study of the robustness of supercurrent states against the effect of potential barriers placed along the ring and, in particular, the estimation of their lifetime.

**Rotation in 2D**

In the wake of our old interest on stochastic methods for the numerical study of interacting quantum gases, we have developed a semiclassical field method for the weakly interacting Bose gas at finite temperature. Contrary to standard classical field models, this semiclassical method does not suffer from ultraviolet divergences and provides exact results for the thermodynamics of non-interacting gases. For interacting systems, the method has been shown to give accurate predictions as long as the temperature is higher than the chemical potential.

This method has been applied in [10] to the study of thermal vortices in spatially homogeneous, two-dimensional gases. As compared to previous work, our calculations fully include density fluctuations in the gas and therefore take into account the smooth transition from large amplitude phonons to vortex pairs.

Numerical results for the vortex density and the vortex pair distribution function have been obtained for temperatures spanning across the Kosterlitz-Thouless transition.
At low temperatures, we have found that the vortex density follows an activation law of the form \( \exp(-\Delta/k_BT) \) with an activation energy \( \Delta \) weakly depending on temperature in both the ideal and the interacting cases. Concerning the spatial correlations between the positions of vortices, we have found that no qualitative difference appears between the ideal and the interacting cases at high temperatures, while in the activation regime vortex pairs in the ideal gas have a much longer size.

NONLINEAR DYNAMICS AND SOLITONS

Sound waves, Bogoliubov quasiparticles, topological and nonlinear excitations, such as vortices and solitons, in Bose-Einstein condensates have been the objects of a very intense activity in the last decade. This is also a traditional field of research for the group in Trento. Here we briefly report on recent advances on solitons in both Bose and Fermi gases, as well as on the flow of a superfluid past an obstacle.

Solitons in two-dimensional Bose-Einstein condensates

Bose-Einstein condensates of ultracold atoms are ideal systems for exploring matter wave solitons. For most purposes, these condensates at zero temperature are well described by the Gross-Pitaevskii (GP) equation which has the form of a Nonlinear Schrödinger Equation, where nonlinearity comes from the interaction between atoms. In 1D, the GP equation for condensates with repulsive interaction admits solitonic solutions corresponding to a local density depletion, namely gray and dark solitons. Such solitons have already been created and observed in elongated condensates with diverse techniques. Also multidimensional solitons in condensates have attracted much attention [1]. An interesting type of excitation in a 2D condensate is represented by a self-propelled vortex-antivortex pair which is a particular solitonic solution of the GP equation [2, 3]. In the low momentum limit, the relation between the energy \( E \) and momentum \( P \) of the soliton approaches the dispersion law of Bogoliubov phonons, \( \epsilon = cq \), from below. In this limit, when the 2D soliton moves at a velocity \( V \) close to the Bogoliubov sound speed \( c \), the phase singularities of the vortex-antivortex pair disappear and the soliton takes the form of a localized density depletion, also called rarefaction pulse. Moreover, if \( V \) is close to \( c \) the GP equation can be rewritten in a simpler form, known as Kadomtsev-Petviashvili (KP) equation [4].

In [5] we studied the excitations of the condensate in the presence of the soliton by linearizing the KP equation around the stationary solution. By looking at the shape of the eigenfunctions we found excitations localized near the soliton, having shape and dispersion law similar to those of the transverse oscillations of a 1D gray soliton in a 2D condensate. We also used a stabilization method in order to obtain a better determination of the dispersion law of the localized states. The same method allows us to visualize the coupling between the localized states and the free states, i.e., the Bogoliubov phonons of the uniform condensate. The occurrence of a coupling between bound and unbound states, which is visible in the avoided crossings in the stabilization diagram, is worth stressing. The width of the avoided crossings is directly related to
the lifetime of the bound (resonant) states associated with their decay into Bogoliubov sound modes. Our analysis suggests that the bound states of the 2D soliton have a finite lifetime. Our results can be of interest for the investigation of ultracold bosonic gases in disk-shaped confining potentials, where vortex pairs are known to play a crucial role [6]. The observation of solitons in these systems would represent a nice manifestation of nonlinear dynamics in low dimensional superfluids.

**Flow of a Bose-Einstein Condensate Past an Obstacle**

The Čerenkov effect was first discovered in the electromagnetic radiation emitted by charged particles traveling through a dielectric medium at a speed larger than the medium’s phase velocity. The concept of Čerenkov radiation can be generalized to any system where a source is uniformly moving through a homogeneous medium at a speed larger than the phase velocity of some elementary excitation to which the source couples. In [7] we studied the density modulation that appears in a Bose-Einstein condensate (BEC) which flows against a localized obstacle at rest with the experimental images taken by the JILA group [8]. Modulo a Galilean transformation, the physics of a moving source in a stationary medium is in fact equivalent to the one of a uniformly moving medium interacting with a stationary defect. The experiment has been performed by letting a BEC expand at hypersonic speed against the localized optical potential of a far-detuned laser beam. The observed density profiles are successfully reproduced by numerically
solving the time-dependent Gross-Pitaevskii equation and physically interpreted by a simple model of Čerenkov emission of Bogoliubov excitations by a weak defect.

In [9] we have investigated the stability of dark solitons generated by supersonic flow of a Bose-Einstein condensate past an obstacle, as in Fig. 2. The figures illustrates the results of a numerical simulation of a two-dimensional flow past a disk-shaped obstacle. We see that the Mach cone (imaginary lines drawn from the origin at angles \( \pm \arcsin(1/M) \) with respect to the \( x \) axis, \( M = 2 \) being the supersonic flow velocity at \( |x| \to \infty \) ) separates regions with wave patterns of different nature. Outside the Mach cone there is a stationary pattern arising due to waves with a Bogoliubov dispersion law radiated by the obstacle, as those studied in [7]. Inside the Mach cone nonlinear waves occur. In case of large enough obstacles these nonlinear waves form a dispersive shock waves considered [10], but for obstacles with the size about a healing length, as in Fig. 1, just one soliton is formed in each symmetrical “shock”. At large enough time of evolution and far enough from the obstacle, the profile of a dark soliton tends to a stationary state described by the solution of the GP equation. This means that dark solitons generated by a fast enough flow of a BEC past an obstacle are stable in the reference frame of the obstacle, contrary to the above mentioned instability of dark solitons in the reference frame where the fluid at infinity is at rest. This happens because the instability of solitons in the obstacle reference frame is “convective” only: the disturbances are convected from the region at finite distance behind the obstacle and, hence, the solitons there become stable. This phenomenon explains the surprising stability of the flow picture that has been observed in numerical simulations [11].
Solitons in Fermi gases

In three dimensions a dark soliton is characterized by a real order parameter which changes sign at a planar node (a point node in 1D). In the BEC regime the soliton is a solution of the GP equation for the order parameter of the condensate with repulsive interaction. While in the BEC case the node of the order parameter causes a notch in the density distribution, in a BCS superfluid the density is almost unaffected by the presence of the node. The situation is similar to that of vortices. As in that case, the behavior of the density along the BCS-BEC crossover is expected to be interesting and rather nontrivial, as a result of the delicate balance of coherence and nonlinear interactions. In [12] we investigated this problem by using a mean-field theory for a 3D Fermi gas at zero temperature, based on the solution of the Bogoliubov - de Gennes (BdG) equations [13].

As shown in Fig. 3b, we found the occurrence of a deep depletion of the density at
unitarity (solid line) with a $\simeq 80\%$ contrast, comparable to the one of the BEC regime (dash-dotted). On the BCS side (dashed), conversely, the contrast is only $\simeq 30\%$ at $(k_Fa)^{-1} = -1$ and becomes exponentially small in the limit $k_F|a| \ll 1$. These results are consistent with those obtained for the profile of a vortex core [14]. In the same work we also discussed the existence of fermionic states which are bound to the soliton (Andreev states).


ULTRACOLD GASES IN OPTICAL LATTICES

Dynamical instability and dispersion management of an attractive condensate

The properties of Bose-Einstein condensates propagating in optical lattices have been the object of an intense experimental and theoretical activity in recent years [1]. These studies have been mainly focused on the case of repulsive interactions. Since BECs with tunable attractive interactions are becoming available in current experiments [2], this opens now interesting perspectives for the investigations of the behaviour of BECs in optical lattices in unexplored regimes.

Motivated by this, we have investigated the properties of an attractive BEC moving in a 1D optical lattice [3], finding a remarkable behaviour compared to the case of repulsive interactions investigated so far. There are at least three points that make the attractive case particularly interesting. i) Increasing the lattice velocity stabilizes the system: attractive condensates are dynamically unstable at low quasimomenta, but can be stabilized above a certain critical velocity (see Fig. 1). This is just the opposite behaviour of that found for repulsive BECs [4, 5], and can be naively explained with the fact that changing the sign of interactions is equivalent to a change of sign in the effective mass. ii) For low quasimomenta and shallow lattices DI takes place via long wavelength excitations. In this regime the excitation modes are not sufficient to produce a site-to-site dephasing, but are rather expected to induce collective oscillation of the system, producing density modulations that extend over several sites of the lattice. iii) Negative effective mass and DI appear in separate regimes, thus opening the possibility for being investigated in a more clear way.

These effects have been discussed by comparing the predictions of the Bogoliubov linear stability analysis with the solution of a time dependent Gross-Pitaevskii model for the waveguide expansion of the condensate in the presence of a moving lattice [3]. We have shown that in the unstable regime breathing-like oscillations are produced for low velocities, and that the absence of dynamical instability for higher velocities allows for the control of condensate dispersion when the effective mass is switched from positive to negative values [6, 7].

BECs in quasiperiodic lattices

One of the most intriguing phenomena in physics is the localization of waves in disordered media, originally predicted by P. W. Anderson fifty years ago, in the context
Figure 1: Stability diagrams as a function of the quasimomenta \( p \) and \( q \) of the condensate and the excitations, for different values of the lattice intensity \( s \) and interaction strenght \( g \). Colored regions indicate where the system is dynamically unstable. The colour scale corresponds to the growth rate of the unstable modes for a given \( p, q \). The solid and dashed lines in (a), (b) and (c) correspond to analytic predictions for \( s = 0 \), whereas the bold line in (g), (h) and (i) refer to the case \( g = 0 \) (see [3]). Light shaded area: energetic instability. Dotted region in (g), (h), and (i): regimes of DI for the repulsive case.
of transport of electrons in crystals [8]. The combination of ultracold atoms and optical potentials offers a novel platform for the study of disorder-related phenomena [9] where most of the relevant physical parameters, including those governing interactions, can be controlled. In particular, quasiperiodic lattices, obtained by superimposing a primary optical lattice with a weak secondary lattice of incommensurate wavelength, makes them excellent candidates for the investigation of disorder in atomic systems. Indeed, due to the presence of the secondary lattice that breaks the discrete translational invariance, such systems display a “metal-insulator” phase transition from extended to localized states as for Anderson localization in case of pure disordered systems.

We have performed experimental and theoretical investigations of the properties of Bose-Einstein condensates in quasiperiodic lattices, considering both the regime where interactions play an important role in masking the effect of disorder [10], and the non-interacting regime, that has allowed for the first direct demonstration of Anderson localization of a matter wave [11].

**Effect of interactions on the localization of a Bose-Einstein condensate in a quasiperiodic lattice**

The transport of a repulsive BEC in the quasiperiodic lattice has a rather complex behaviour due to the interplay of disorder and interactions [10]. Indeed, the presence of repulsive interactions produces a screening of the quasiperiodic potential, preventing the occurrence of disorder induced localization. In the low density regime this screening makes the quasycrystal lattice an effective superlattice with periodicity corresponding to the beating of the two lattice wavelengths. This ultimately introduces mechanisms characteristic of nonlinear transport in periodic potentials, such as center-of-mass oscillations of reduced amplitude, due to a large effective mass, and long time damping, due to dynamical instability. By increasing the density, the screening takes place up to larger wavelengths, eventually masking the beat periodicity, making the transition from the superlattice back to the primary lattice. This produces an increase of the growth rate of dynamical instability, but with a delayed onset (due to an increased critical velocity) owing to the screening effects. These theoretical predictions, obtained in the framework of the Gross-Pitaevskii theory, have been experimentally confirmed for the dipole motion of a $^{87}$Rb condensate [10].
Figure 2: (a), (b) Experimental profiles and fitting function (thick line) \( f_\alpha(x) = A \exp\left(-|x-x_0|/l^\alpha\right) \), the exponent \( \alpha \) being a fitting parameter, for \( \Delta/J = 1 \) (a) and \( \Delta/J = 15 \) (b) (note the vertical log scale). The dotted line in (b) represents a gaussian fit, \( \alpha = 2 \). (c) Dependence of the fitting parameter \( \alpha \) on \( \Delta/J \), indicating a transition from a gaussian to an exponential distribution.

Anderson localization of a non-interacting Bose-Einstein condensate

The localizing properties of the disordered lattice can be exploited by tuning the condensate interactions down to zero. This has been achieved by considering a \(^{39}\)K Bose-Einstein condensate, whose s-wave scattering length can be tuned down to zero by means of a Feshbach resonance, independently of the other condensate parameters [2]. The resulting system – a non-interacting matter wave in a quasiperiodic lattice – constitutes a realization of the so called Aubry-André model, that displays a transition from extended to exponentially localized states analogous to the Anderson transition, but already in one dimension [12]. We have experimentally demonstrated this localization transition by investigating transport properties, spatial and momentum distributions, by making a direct comparison between experimental data and theoretical calculations [11].

For example, Fig. 2 shows the spatial distribution of the atoms after a fixed evolution time in the lattice: for a weak disorder the gaussian profile (in Fig. 2a) corresponds to a ballistic expansion, while for larger disorder \( \Delta \), where we observe no diffusion, the wavefunction is localized with exponentially decaying tails (Fig. 2b). Important information have also been extracted from the modification of the momentum distribution \( P(k) \) as the disorder strength \( \Delta \) is increased, see Fig. 3, and from the interference of multiple localized peaks [11]. These results constitutes the first experimental observation of Anderson localization for a non-interacting matter wave [13]. Since the interaction between atoms in the condensate can be controlled, this system represents a novel tool to solve fundamental questions on the interplay of disorder and interaction.
Figure 3: (a), (b) Experimental and theoretical momentum distributions $P(k)$ for increasing $\Delta/J$ (0, 1.1, 7.2 and 25, from top to bottom). The interference pattern of a regular lattice observed at $\Delta = 0$ is at first modified by the appearance of peaks at the beating between the two lattices, and then increasingly broadened. Momentum is measured along the horizontal axes in units of $k_1$. (c) Root-mean-squared size of the central peak of $P(k)$ versus $\Delta/J$, for three different values of $J$. The experimental data follow a unique scaling behaviour, as expected from theory (continuous line). The width of the peak is measured in units of $k_1$. (d) Visibility $V$ of the interference pattern versus $\Delta/J$. In both experiment and theory (continuous line), $V$ decreases abruptly for $\Delta/J \approx 6$, indicating localization on distances comparable to the lattice period.
and to explore exotic quantum phases.

**Unitary Fermi gas in a 1D optical lattice**

First experiments with Fermi superfluids in one-dimensional (1D) optical lattices [14] have focused on the study of the critical velocity along the crossover and revealed that superfluidity is particularly robust at unitarity. It is also well known that a periodic potential favors the formation of molecules in a two-component Fermi gas [15], providing an effective shift of the resonance and bound states in the two-body problem even at unitarity. A major problem is to understand the consequences of the molecular formation in the superfluid phase. Moreover, for large laser intensities, the lattice is expected to give rise to 2D effects. In [16] we investigated these properties by using a mean-field theory based on the Bogoliubov – de Gennes (BdG) equations. By numerically solving the BdG equations, we have first calculated the equation of state, the compressibility and the effective mass of the unitary Fermi gas in the lattice. The results have been used to obtain interesting predictions for observable quantities such as the sound velocity, the frequency of collective modes, and the density profile of trapped gases in typical experimental configurations. We have shown that the inclusion of a 1D optical lattice, by favoring the formation of molecular configurations and by inducing a band structure in the quasiparticle spectrum, has profound consequences on all these quantities.

Ultracold gases in optical lattices


DIPOLAR GASES

Stimulated by the recent achievement of Bose-Einstein condensation with $^{52}$Cr atoms [1] and the progresses in trapping and cooling of polar molecules [2, 3], in the last two years a new line of research on cold atomic dipolar gases has been opened. Contrary to the case of usual quantum degenerate bosonic gases, where $s$-wave scattering dominates and the real interatomic molecular potentials can be replaced for most purposes by a contact potential proportional to the scattering length $a$ of the atoms, the physics of dipolar gases is governed by the interaction potential being long-range and anisotropic [4, 5, 6, 7].

Particles interacting via dipole-dipole interaction are atoms or molecules with a permanent dipole moment (either magnetic or electric). The corresponding potential is

$$U_{dd}(r) = \frac{C_{dd}}{4\pi} \frac{1 - 3\cos^2 \theta}{r^3},$$

where $C_{dd}$ is the dipolar coupling constant ($C_{dd} = \mu_0 \mu^2$ for magnetic moments $\mu$, $C = d^2/\varepsilon_0$ for electric dipole moments $d$), and $\theta$ the angle between the orientation of the dipoles and the direction joining the two dipoles. To characterize the relative strength of the dipolar and contact interactions, usually one introduces the dimensionless parameter $\epsilon_{dd} \equiv \frac{C_{dd} m_1 m_2}{\pi \hbar^2 a}$. For the alkalis used in standard BEC experiments, the value of $\epsilon_{dd}$ is extremely small (for example, for $^{87}$Rb, one has $\epsilon_{dd} \simeq 0.007$), making the effects of the dipole-dipole interaction usually negligible. Only very recently dipolar effects have been observed in alkali atoms [8].

On the other hand, Chromium has a magnetic dipole moment of $6 \mu_B$ and a scattering length of about $100a_0$ ($a_0$ is the Bohr radius), which give $\epsilon_{dd} \simeq 0.16$. This is enough to observe dipolar effects at the perturbative level. A strong dipolar system can be created exploiting a Feshbach resonance, which allows tuning the $s$-wave scattering length almost to zero.

Polar molecules in their ground state have a large electric dipole moment, on the order of one Debye, which would allow them to act as truly strongly interacting dipoles ($\epsilon_{dd} \simeq 100$). A promising approach towards the creation of a quantum degenerate gas of polar molecules is the use Feshbach resonances in ultra-cold mixtures to create heteronuclear molecules. The crucial experimental step, i.e. their conversion to the lowest roto-vibrational state, has been recently successfully implemented [3].

The recent experiments with Chromium BEC have verified a series of theoretical predictions. The effect of the dipole-dipole interaction has been measured in the ex-
Dipolar gases

Pandoring cloud both in the perturbative [9] and in the strong dipolar regime [10]. The collapse of a dipolar BEC has been studied in detail as a function of the s-wave scattering length and trap anisotropy [11]. In particular it has been experimentally confirmed that a pancake trap can stabilize a dipolar condensate at $a = 0$. Strong effects of dipolar interactions have been observed in spinor condensates, like the Einstein-de Haas effect [12]. Other theoretical predictions for dipolar bosons concern the existence of the roton minimum in the excitation spectrum [13], the inelastic scattering of solitary waves in 2D systems [14], and the liquid to solid quantum phase transition at high densities [15].

Dipolar atoms in optical lattices

Our studies of dipolar atoms extend the well established research line of the BEC center on optical lattices. In lattices, striking effects of long-range interactions appear in the strongly correlated bosonic systems through the presence of novel quantum phases [16], like the supersolid phase, presenting the coexistence of superfluidity and periodic spatial modulation of the density. The existence of the supersolid phase in $^4$He has not yet been unambiguously proven experimentally [17]. In this respect, cold atoms with long range interactions are particularly appealing [18], bringing into play the extra feature of the anisotropy of the interaction.

Metastable states of dipolar atoms in optical lattices

The motivation behind our study of dipolar atoms in 2D lattices is the idea that one could exploit the long-range character of the interaction to create controllable states, robust against small local imperfections, aimed to be quantum memories. This idea is supported by the existence of a large number of metastable states in a system of dipoles in a 2D lattice, suggesting the analogy with classical complex systems such that neural networks or spin glasses. Other proposals for quantum gates and quantum memories with dipolar particles had been previously developed in [19].

In spite of the extensive literature on the subject, the existence of metastable states in a system of dipoles in a 2D lattice has been addressed for the first time in our work [20]. To define the metastable states, we use the same criterium as for the insulating states in lattice systems with on-site interaction only, namely the stability against particle-hole excitations: considering for simplicity only one repulsive nearest-neighbor interaction $U_{NN}$, at zero tunneling ($J = 0$) the checkerboard ordering of the atoms (as shown in Fig.1(GS)) is the ground state in the range of chemical potential $0 < \mu < 4U_{NN}$. A “double-checkerboard” ordering of the atoms (as shown in Fig.1(I)) is metastable in the range of chemical potential $U_{NN} < \mu < 3U_{NN}$, but with higher
Dipolar gases

energy (see Fig1(a)).

The phase diagram of the system for a $4 \times 4$ elementary cell and different cut-off of the interaction is shown in Fig.1. There are insulating lobes corresponding both to ground and metastable states, characterised by integer and fractional filling factors and a non uniform distribution of the atoms in the lattice. The filling factors allowed and the metastable configurations clearly depend strongly on the elementary cell chosen to describe the lattice and on the cut-off range of interactions [21]. In the region at larger tunneling, the system is either supersolid or superfluid, but we did not devote our attention to this part of the phase diagram in detail yet. This phase diagram is confirmed by the imaginary and real time evolution of the system: depending on the initial conditions the imaginary time evolution can converge to the metastable configurations, while in the real time evolution, their stability is reflected into typical small oscillations around local minima of the energy landscape.

We have studied the stability of the metastable states with a path integral formulation in imaginary time, which can describe the tunneling below a potential barrier (instanton effect). This analysis suggests that the metastable configurations are very stable when many sites must invert their population to reach another metastable state. However, especially in larger lattices, two metastable configurations might differ just by the occupation of few lattice sites (defects). This fact, and the corresponding small energy difference, should be carefully taken into account in an analysis at finite temperature for realistic experimental parameters.

Detection, initialisation and manipulation of the metastable states

In view of the possible application of such system as quantum memories, one should be able to initialize, manipulate and detect the state of the system [21].

 Initialization: In an experiment, it will be very hard to reach the ground state or a given metastable configuration without defects. We have checked that the presence of defects is strongly reduced in the outcome of the imaginary time evolution, when a local potential energy following the desired pattern is added to the optical lattice. Hence, one can use superlattices to prepare the atoms in configurations of preferential symmetry. The configurations obtained in such a way will also remain stable once the superlattice is removed, thanks to dipole-dipole interaction.

 Manipulation: Our aim is to transfer in a controlled way the system from one metastable configuration to another. We have studied the real time evolution of the system with varying appropriately the Hamiltonian parameters (tunneling coefficients and local chemical potentials) and shown the impossibility of mapping this problem
Figure 1: (Color online) (a, b, c) Phase diagram with a range of the dipole-dipole interaction cut at the first, second and fourth nearest neighbor respectively. The thick line is the ground state and the other lobes correspond to the metastable states, the same color corresponding to the same filling factor. In (a, b, c) the ground state filling factors are multiples of $1/2$, $1/4$ and $1/8$ respectively. In (c), the metastable state filling factors are $m/16$ ($\forall m \neq 1; 15$). Metastable configurations at filling factor $1/2$, appearing at the first (I) and second (IIa-IIb) nearest neighbor, and the corresponding ground state (GS); those metastable states remain stable for all larger ranges of the dipole-dipole interaction. Figure from [21].
onto a simple adiabatic transfer \[21\]. This is due to the fact that, in spite of the modification of the lattice parameters, the metastable states survive unchanged till the point where the stability condition is not fulfilled anymore. The transition to any other state is then abrupt. A way around this problem is to push the system into the superfluid region and then drive it back into a different insulating state. The transfer between two metastable states turns out to be a quantum controlled process, where the Hamiltonian parameters must be controlled with very high precision to obtain the desired result.

**Detection:** Spatially modulated structures can be detected via the measurement of the spatial noise correlations of the pictures produced after expansion \[20, 22\], as shown in Fig.2. From such a measurement, one could in principle reconstruct exactly the density distribution and recognize each single defect. For the moment, the average over a finite number of different experimental runs producing the same spatial distribution of atoms in the lattice is required to obtain a good signal to noise ratio, so that single defect recognition is beyond present experimental possibilities. However, this method should turn valuable when a reproducible initialization procedure is developed, as explained above.

![Figure 2: Spatial noise correlation patterns for configurations (IIa), (I) and (IIb) in Fig. 1, assuming a localised gaussian density distribution at each lattice site. Figure from \[20\].](image)

**Further developments**

Further developments in the study of 2D lattices of dipolar bosons include the comparison with quantum Monte Carlo studies, the study of more 2D lattice layers, a more detailed characterization of the superfluid region of the phase diagram, and the extension to spinorial dipolar particles.

Other topics of interest concern the physics of rotating and fermionic dipolar gases (see \[5\] and references therein). Dipole-dipole interaction in rotating bosonic samples
Dipolar gases modifies the vortex lattice structure and the quantum Hall physics. Dipolar rotating fermions present analogies with electronic systems, namely the fractional quantum Hall physics provided by the long-range interaction and the possibility of Wigner crystalisation at low densities (to be contrasted to the one at high densities in absence of rotation). Moreover, thanks to the filled Fermi sea, dipolar fermions usually present a better stability with respect to bosons against the attractive part of dipole-dipole interaction. The attractive part of the interaction can lead in suitable configurations to a BCS pairing in a polarised Fermi gas, where the anisotropy of the interaction is reflected in the pairing gap. The extension of those results to geometries of reduced dimensionality has been recently addressed in Trento [23].


SEMICONDUCTOR MICROCAVITIES AND EXCITON-POLARITONS

After a long quest, Bose-Einstein condensation in a solid state environment has recently been observed in a gas of exciton-polaritons in a planar semiconductor microcavity [1]. In addition to being a further verification of Einstein’s prediction in a completely new context, this observation has raised a number of theoretical questions as the Bose gas under investigation is an intrinsically non-equilibrium object, whose stationary state is not determined by a thermodynamical equilibrium condition, but rather by the balance between driving and dissipation. And also from the experimental point of view, polariton systems appear as very attractive thanks to the availability of simple all-optical schemes for the experimental diagnostic and manipulation of the polariton gas.

Polaritons are generated in the microcavity from the incoherent relaxation of electron-hole pairs that are created by the incident light. Polariton-polariton and polariton-phonon scattering are responsible for the transfer and the accumulation of polaritons into the bottom of the lower polariton branch. If the pumping is sufficiently strong, Bose stimulation sets in and a macroscopic fraction of the polaritons condense into a single coherent state before they are lost. Among the many decay channels that are available for polariton losses, a crucial role is played in the experiment by those radiative processes where an in-cavity polariton is emitted in the surrounding space as a propagating photon: thanks to these processes, a detailed information on the state of the polariton gas can be extracted from the intensity and the coherence properties of the emitted light.

After a first generation of experiments where the mere observation of a polariton BEC was a central result, a new generation of experimental work has recently begun where features typical of a non-equilibrium BEC are addressed. An important contribution in this development has been played by the Trento BEC group in collaboration with several other groups. On one hand, we have pushed forward our theoretical investigations to characterize the main features of the non-equilibrium Bose gas of polaritons such as the dispersion of the elementary excitations, the superfluidity properties, the Josephson dynamics in two-well geometries. On the other hand, we have developed a strong connection with a few experimental groups: thanks to this interplay between theory and experiment, an explanation for puzzling observations for the condensate shape has been proposed, and new observations on vortices in polariton condensates and on critical slowing down have been performed.
Figure 1: Scheme of microcavity system used for polariton BEC experiments. Figure and caption taken from A. Baas *et al.* [1]. (a) A microcavity is a planar Fabry-Perot resonator with two Bragg mirrors at resonance with excitons in quantum wells (QW). The exciton is an optically active dipole that results from the Coulomb interaction between an electron in the conduction band and a hole in the valence band. In microcavities operating in the strong coupling regime of the light-matter interaction, 2D excitons and 2D optical modes give rise to new eigenmodes, called microcavity polaritons. (b) Energy levels as a function of the in-plane wavevector $k_\parallel$ in a CdTe-based microcavity. Interaction between exciton and photon modes, with parabolic dispersions (dashed curves), gives rise to lower and upper polariton branches (solid curves) with dispersions featuring an anticrossing typical of the strong coupling regime. The excitation laser is at high energy and excites free carrier states of the quantum well. Relaxation towards the exciton level and the bottom of the lower polariton branch occurs by acoustic and optical phonon interaction and polariton scattering. The radiative recombination of polaritons results in the emission of photons that can be used to probe their properties. Photons emitted at angle $\theta$ correspond to polaritons of energy $E$ and in-plane wavevector $k_\parallel = (E/\hbar c) \sin \theta$. 
Elementary excitations of a polariton condensate

A central role in the theory of many-body systems is played by the elementary excitations of the system around the equilibrium state. Spatially uniform Bose-condensed gases at equilibrium are characterized by a sound-like branch with a linear dispersion $\omega(k) \approx c_s k$ for small $k$ that then transforms into a single-particle branch $\omega(k) \approx \hbar k^2 / 2m$ at higher $k$.

A few works of the Cambridge [2] and the Trento [3] groups had suggested that the non-equilibrium condition can dramatically affect the low-$k$ behaviour of the dispersion. To shine more light on this quite surprising fact, we have generalized the Gross-Pitaevskii equation to the case of non-equilibrium condensates including terms that describe non-resonant pumping and dissipation [4]. This simple model is inspired from the semiclassical theory of laser and is intended to be a “generic model” of polariton condensation where the microscopic details of the pumping and dissipation are summarized by a few parameters.

![Figure 2: Real and imaginary part of the excitation spectrum of a homogeneous polariton condensate as a function of the wavevector $k$. Dashed lines: standard equilibrium Bogoliubov dispersion.](image)

From the generalized Gross-Pitaevskii equation, the elementary excitations are immediately obtained by linearization around the spatially uniform stationary state. Ex-
amples are shown in Fig.2. The soft mode corresponding to the spontaneously broken $U(1)$ symmetry is still present (the $+$ branch in the figure) as predicted by the Goldstone theorem, but has a diffusive rather than propagating nature, i.e. $\omega(k) = -i\alpha k^2$. Physically, this means that phase modulations do not propagate as sound waves but simply relax down towards the uniform phase state on a time scale that grows with the wavelength of the perturbation. On the other hand, long-wavelength density modulations relax down on a much faster rate (the $-$ branch in the figure).

Josephson dynamics of polariton condensates in two-well geometries

Inspired by recent advances in the creation of polariton traps of widely tunable size and depth [5], we have investigated the Josephson dynamics of polariton condensates in two-well geometries under different pumping regimes.

![Figure 3: Time evolution of the population $N_1$ after an excitation pulse. Upper (a) panel: Josephson oscillations ($J = \gamma/2$). Lower (b) panel: overdamped relaxation ($J = \gamma/20$). Figure taken from [4].](image)

The case of condensation under a non-resonant pumping was considered in [4] and is here illustrated in Fig.3: depending on the ratio between the Josephson coupling $J$ and the damping rate $\Gamma$, Josephson oscillation may be replaced by an overdamped relaxation closely related to the diffusive part of the phonon dispersion shown in Fig.2.

The physics is a bit more complex when a resonant pump is considered: depending on the pump parameters, a variety of different behaviours can be explored [6], ranging from Josephson oscillations, to self-trapping, to self-pulsing. Generalization of this latter configuration to the strongly interacting case is promising in view of quantum optical applications along the lines of [7] and is presently under investigation.
The shape of finite-size polariton condensates

Figure 4: Numerical results of generalized GPE simulations in the absence of disorder for respectively a large $\sigma_p = 20 \mu m$ (a-d) and a small $\sigma_p = 2 \mu m$ (e-h) circular excitation pump spot. Panels (a,e) give the $(k,E)$ emission pattern, (b,f) the polariton distribution in momentum space, (c,g) the polariton distribution in real space and (d,h) the local wave vector $k_c(r)$. Figure taken from [8].

At equilibrium, the shape of a trapped condensate can be obtained from the lowest energy solution of the time-independent Gross-Pitaevskii equation: Unless the system is set into rotation, the phase of the condensate is uniform throughout the cloud and no net flow is present. In typical harmonic traps, the real-space density has a smooth Thomas-Fermi profile centered at the potential minimum, and the momentum distribution is strongly peaked around $k=0$. The only broadening mechanism is the one due to the Heisenberg position-momentum uncertainty relation.

The situation appears to be completely different in polariton condensates. Even in the absence of trapping, the finite size of the pump spot is responsible for a spatial inhomogeneity of the polariton condensate, and experimental evidence of a dramatic qualitative dependence of the condensate shape on the pump spot size was obtained as early as 2005 by the Grenoble group [9]: the momentum distribution inferred from
the far-field emission pattern dramatically changes from a single-peak shape in the case of a wide pump spot to a ring-like shape in the case of a narrow spot. In spite of the complex pattern, the emission at all different $k$ values remained fully coherent, which suggests that one is dealing with a single and not fragmented condensate.

The theoretical interpretation [8] of these experimental observations stems from the remark that at non-equilibrium no free energy minimization criterion is no longer present that forces BEC to occur in the $k = 0$ state. The condensate wave function has then to be determined by the dynamics, which leaves open a quite wide range of possibilities: as time-reversal is no longer a symmetry of the problem, there is in fact no reason for the condensate wavefunction to have a spatially uniform phase, so that vortices can spontaneously and deterministically appear in the condensate (see the next section). Similar conclusions were almost simultaneously drawn by the Cambridge group [10].

Even though polaritons are scattered into the condensate mostly at the center of the spot where the pump intensity is the strongest, polariton-polariton repulsion then tends to accelerate them in the outward direction. Examples of numerical calculations based on the non-equilibrium polariton Gross-Pitaevskii equation are shown in Fig.4: as expected, the shape of the condensate on the ratio $\eta$ between the pump spot size and the distance that polaritons are able to fly before recombining. If $\eta \gg 1$, slowly moving polaritons are the majority, and the momentum distribution remains peaked around $k = 0$. On the other hand, if $\eta \ll 1$, fast polaritons that ballistically propagate outside the cloud dominate, which provides the characteristic ring shape of the momentum distribution.

**Vortices in polariton condensates**

The shape of the condensate wavefunction can be even more complex in the presence of disorder: the resulting potential can in fact significantly affect the flow pattern discussed in the previous section and give rise to new peculiar patterns. An experimental investigation of this physics [11] has been performed at EPFL with the theoretical support from Trento.

The central result of the experiment (see Fig.5) is the observation of vortices in the polariton condensate with a quantized phase winding around the vortex core: the phase profile of the condensate has been obtained by interferometric techniques which superimpose the light emitted from the condensate to an unperturbed plane-wave beam. The observations can be interpreted in terms of the flow pattern that is deterministically created in the condensate by the disorder potential. Whenever the condensate phase...
Figure 5: Interferogram and extracted phase. (a) Interferogram with vortex: in the red circle one can see the forklike dislocation. b, Interferogram carrying the same information but this time the vortex is overlapped with a different region of the condensate and for different fringe orientation. The vortex appears at the same real space coordinates as before. c, Real space phase profile calculated from interferogram of (a). The red circle encloses the vortex (same real space area as on (a),(b)). d, Phase as a function of the azimuthal angle for a range of different radii as shown in the indent of figure (d) (zoom of (c)). Note that the data are repeated before and after the azimuthal angles 0 and 2\(\pi\) to better illustrate the 2\(\pi\) shift. Figure taken from [11].
in the different regions can not be continuously matched, vortex singularities have to appear.

This picture has been confirmed by numerical simulations based on the polariton Gross-Pitaevskii equation and is further validated by the experimental observation that vortices appear at deterministic positions with a given charge, which rules out thermal nucleation mechanisms such as the ones recently studied in the context of two-dimensional atomic gases [12].


QUANTUM OPTICS AND QUANTUM FIELDS

One of the most fascinating aspects of quantum fields as compared to classical ones is the possibility of creating real and propagating excitations of the field by simply modulating in space and/or time the background over which the quantum field is propagating.

The simplest example of this phenomenology consists of the radiation emitted by a planar mirror which is moving through the electromagnetic vacuum with a non-uniformly acceleration: when reflected by the moving mirror, the zero-point fluctuations of the electromagnetic field are transformed into propagating radiation, i.e. real photons that can be detected by standard photodetectors. As this process is a consequence of a time-dependent boundary condition, it often goes under the name of dynamical Casimir effect [1]. As a reaction, the quantum field exerces a friction force on the mirror that opposes its motion: electromagnetic radiation is emitted at the expenses of the kinetic energy of the mirror [2].

Although the time-dependence of the background plays a central role in the dynamical Casimir effect, a related zero-point emission effect has been predicted for backgrounds that are modulated in space but remain stationary in time. The most celebrated example of such effect is the so-called Hawking radiation from astrophysical black holes, i.e. the steady-state emission of a thermal radiation at a temperature inversely proportional to the mass of the black hole [3]. As the Hawking mechanism is a direct consequence of the space-time geometry, it was pointed out by Unruh [4] that a similar emission process should take place in any system developing a horizon for some wavy perturbation. During the last years, many systems have been investigated to this purpose, ranging from liquid Helium to ultracold atoms to time-dependent dielectrics [For a review, see e.g. Artificial Black Holes, edited by M. Novello, M. Visser, and G. Volovik (World Scientific, River Edge, 2002).].

Even if a variety of systems have been considered for experimental study of these effects, the weak intensity of the zero-point radiation and the presence of spurious effects has so far prevented a direct experimental confirmation of the theoretical predictions. The Trento activity in this field has been carried out along two distinct directions: on one hand, we have searched for novel systems where the dynamical Casimir effect may be observed in the next future. On the other hand, we have performed microscopic numerical experiments on the Hawking emission from acoustic black holes in atomic Bose-Einstein condensates.
Dynamical Casimir effect

The investigations on the dynamical Casimir effect has been performed in strict collaboration with Simone De Liberato and Cristiano Ciuti at the Paris 7 University; two distinct paths have been explored, involving respectively solid state and atomic systems.

Solid-state microcavities

Solid-state microcavities with embedded doped quantum wells have been recently realized in the lab and shown to display a unique ultra-strong coupling between the electronic transition in the quantum well and the photonic mode of the cavity. This means that the ground state of the system consists a squeezed state containing a significant amount of virtual photon and matter excitations, but a time-dependence of the cavity parameters is required in order to release and convert them into propagating radiation. Recent experiments on the wide tunability of the cavity parameters on a fast time scale via a gate bias [5] make the present system a very promising one for the observation of quantum vacuum radiation.

First theoretical studies for an isolated cavity [6] have supported this guess, which has then been firmly validated by the full input-output theory developed in [7]. For realistic parameters, the photon emission is predicted to largely exceed the blackbody radiation; for strong and resonant modulation a parametric oscillation regime should also be achievable.

Ultracold atoms in optical lattices

The optical properties of ultracold atom systems are in a sense complementary to the ones of solid-state devices: the significantly weaker light-matter coupling is compensated by linewidths which are order of magnitude narrower. The order of the atomic ordering can be made almost perfect using Mott insulator states where a given number of atoms is present and strongly localized at each site of an optical lattice. Furthermore, the optical properties of the atoms can be controlled by coherently dressing them with a laser field in a three-level Lambda configuration.

The main outcome of our investigations along this line are summarized in the long paper [8]: after a general review of the ultraslow-light propagation properties in these systems, we have studied the dynamical Casimir emission that occurs when the dressing field is modulated on a fast time scale: the resulting modulation of the optical properties of the medium is in fact able to parametrically amplify the zero-point fluctuations of the
ground state polariton field and convert them into observable radiation. Using a fully microscopic approach based on the Hopfield-Fano model [9], a significant production of pairs is predicted to occur for realistic parameters and geometries.

**Hawking radiation**

Our activity on Hawking radiation has been focussed on the quantum dynamics of Bogoliubov phonons propagating on top of a moving atomic Bose-Einstein condensate. Atomic BECs are among the cleanest system where quantum physics can be investigated: the temperature can in fact be made so low that the quantum dynamics is weakly affected by spurious thermal effects. Furthermore, in contrast to other quantum coherent condensed-matter systems such as superfluid liquid Helium, quantitative theories able to describe the collective dynamics from a microscopic standpoint are available. All the investigations on the Hawking radiation have been carried out in close collaboration with R. Balbinot and S. Fagnocchi in Bologna and A. Fabbri in Valencia, who have provided the expertise in quantum field theory on curved space-times and gravitational physics.

In a first paper [11] we have made use of the mathematical analogy between Bogoliubov phonons in non-uniformly flowing condensates and a quantum field on a curved space time [12] to evaluate the correlation function of density fluctuations on either side of the horizon. In particular, we have anticipated that a very characteristic pattern appears as a consequence of the Hawking effect: Since the Hawking effect consists of pairs of correlated phonons being emitted in opposite directions from the horizon, the quantum correlations within a pair of Hawking partners propagate across the condensate and result into long-range density correlations between distant points on opposite sides of the horizon.

While waiting for a real experiment to be performed, these theoretical predictions have been verified by an extensive campaign of numerical simulations where the dynamics of the condensate is simulated in a microscopic way by means of a truncated Wigner approach [10]. Since our numerical calculations never rely on the gravitational analogy, they provide an independent evidence of the existence of the Hawking effect in a realistic condensed-matter system and can be considered as a sort of numerical experiment. Moreover, they provide further support to the promise of experimental detection of the Hawking radiation from the density correlations rather than from the phonon flux: qualitative features to univocally distinguish the Hawking signal from fluctuations of different nature are in fact visible in the simulated density-density correlation images, an example of which is reproduced in Fig.1.
Figure 1: Density plot of the rescaled density correlation well after the switch-on of the horizon in the Bose-Einstein condensate. The dashed lines and the (i), (ii), (iii), (iv) labels identify the main features that are visible in the plot: (i) is the standard anti-bunching due to repulsive atom-atom interactions. (ii) follows from the emission of phonon pair by dynamical Casimir effect inside the black hole. (iii) is the correlation between Hawking partners. (iv) originates from the back-scattering of the emitted Hawking phonon into the black hole. Figure taken from [10].


The Casimir-Lifshitz force is an interaction of electromagnetic origin acting between neutral dispersive bodies without permanent polarization. Casimir predicted the existence of such a force between two ideal reflecting mirrors at zero temperature [1]. The interaction between an atom and a mirror takes instead the name of the Casimir-Polder force. Casimir’s idea was readily developed by Lifshitz who formulated a more general theory of the interaction including materials having realistic dielectric properties and finite temperature effects [2]. Lifshitz theory still represents the most advanced approach to the problem and is extensively employed to deal with dispersive forces in different fields of science and technology.

Lifshitz theory is formulated for systems at thermal equilibrium and the pure quantum effect at $T = 0$ is clearly separated from the thermal contribution to the force. The former gives the dominant contribution at small separation ($< 1 \mu m$ at room temperature) between the bodies and was confirmed experimentally with good accuracy for the surface-surface and surface-atom configurations. On the contrary the thermal effect, dominating at large distances where the total force is strongly reduced, remained for many decades just an elusive and intriguing prediction without any experimental confirmation. Only in 2007 the thermal effects of the Casimir-Lifshitz force were measured for the first time at JILA by the Eric Cornell group in experiments where ultra-cold trapped atoms were placed close to a dielectric substrate [3]. In these measurements a determinant role was played by the particular thermal configuration of the system, which was not at equilibrium and hence could not be described by the original Lifshitz theory. The use of non-equilibrium configurations results in a significant enhancement of the thermal effect as first pointed out by the Trento team [4].

Surface-atom force out of thermal equilibrium

In [3] the thermal effects of the surface-atom interaction was measured in a configuration out of thermal equilibrium. This work is the result of a successful collaboration between the JILA and Trento teams where the predictions of theory, previously developed in [4] were experimentally confirmed with good accuracy.

In our theoretical paper [4] we studied the surface-atom force out of thermal equilibrium, characterizing configurations where the temperature $T_S$ of the substrate and the temperature $T_E$ of the surrounding walls located at large distances (hereafter called environment temperature) do not coincide (see Fig. 1). In typical experiments with
ultra cold atomic gases the environment temperature is determined by the chamber containing the substrate and the trapped atoms.

In our analysis the atom has been treated as being at zero temperature in the sense that the surrounding radiation is not able to populate their excited states which are assumed to be located at energies much higher than the thermal energy, a condition very well satisfied at ordinary temperatures. Actually the first optical resonance of Rb atoms corresponds to 1.8 $10^4$ K. In this non-absorbing situation one can neglect the contributions of the atom to the radiation and the thermal component of the surface-atom force turns out to be proportional to the energy density of the electromagnetic field (AC Stark Effect): $F \propto \alpha_0 \partial_z \langle E(z, t)^2 \rangle$, where $\alpha_0$ is the static atomic polarizability.

To understand the peculiarity of the non equilibrium configuration we have focused on the large distance asymptotic behavior of the interaction which takes the form

$$F_{\text{net}}(T_S, T_E, z) = \frac{\pi \alpha_0 k_B^2 (T_S^2 - T_E^2)}{6 \varepsilon_{10}^3 c^3 \hbar} \frac{\varepsilon_{10} + 1}{\sqrt{\varepsilon_{10} - 1}},$$

holding at low temperature and at distances larger than $\lambda_T / \sqrt{\varepsilon_{10} - 1}$ where $\lambda_T \equiv \hbar c / k_B T$ is the thermal photon wavelength calculated at the relevant temperatures $T_S$ and $T_E$, and $\varepsilon_{10}$ is the static dielectric functions of the substrate. Differently from the attractive force characterizing the thermal equilibrium, Eq.(1) shows that out of equilibrium the force can be attractive or repulsive depending on whether the substrate temperature is higher or smaller than the environment one. Furthermore Eq. (1)
exhibits a strong $\sim T^2$ temperature dependence and is characterized by a $1/z^3$ decay at large distances. All this gives rise to a strong combined effect that increase in a relevant way the force with respect to the equilibrium case, where $F^{\text{eq}}(T, z) \propto T/z^4$.

Ref. [3] reports the first measurement of the temperature dependence of the Casimir-Polder force performed by the JILA group. Here the Casimir-Lifshitz force was measured at large distances ($\sim 10\mu$m) and in a configuration out of thermal equilibrium. This measurement was obtained by positioning a nearly pure $^{87}$Rb Bose-Einstein condensate (BEC) a few microns from a dielectric substrate and exciting its dipole oscillation. Changes in the collective oscillation frequency of the magnetically trapped atoms result from spatial variations in the surface-atom force. In the experiment, the dielectric substrate is heated up to 605 K, while the surrounding environment is kept near room temperature (310 K). The effect of the Casimir-Polder force was measured to be nearly 3 times larger for a 605 K substrate than for a room-temperature substrate, showing a clear temperature dependence in agreement with theory.

Figure 2(a) shows the experimental results for $\gamma_x$. The blue squares show the measured effect of the room-temperature Casimir-Polder force [$T_S = 310(5)$ K] on the trap frequency. The increase in the strength of the CP force due to thermal corrections becomes evident when the substrate is heated to 479(20) K (green circles) and even more pronounced at 605(28) K (red triangles). These measurements were all done maintaining a room temperature environment for which the pyrex vacuum chamber walls were measured to be $T_E = 310(5)$ K.

The value of $\gamma_x$ averaged over the positions 7.0, 7.5, and 8.0 $\mu$m has been measured for each substrate temperature. These values, plotted in Fig. 2(b), clearly show a significant increase in the strength of the Casimir-Polder force for hotter substrate temperatures; they also distinguish the nonequilibrium theory (solid) curve from the equilibrium (dash-dotted) curve, for which a much smaller force increase is predicted.

**Surface-Surface forces out of thermal equilibrium**

In a series of recent works [5, 6] we have studied the Casimir-Lifshitz force acting between two parallel bodies locally at equilibrium but with different temperatures ($T_1$ and $T_2$ respectively) and separated by a distance $l$ (see figure 3), the whole system being in a stationary thermodynamic configuration.

We find that the final result for the total pressure can be expressed as

$$P^{\text{neq}}(T_1, T_2, l) = P_0(l) + P^{\text{neq}}_{\text{th}}(T_1, T_2, l),$$

(2)
Figure 2: (a) Fractional change $\gamma$ in the dipole frequency due to the Casimir-Polder force. Pictured are three sets of data and accompanying theoretical curves with no adjustable parameters for various substrate temperatures. The blue squares represent data taken with a 310 K substrate; green circles, a 479 K substrate; and red triangles, a 605 K substrate. The environment temperature is maintained at 310 K. The error bars represent the total uncertainty (statistical and systematic) of the measurement. (b) Average values of $\gamma_{x}$ from (a) (for trap center to surface positions 7.0, 7.5, and 8.0 $\mu$m) plotted versus the substrate temperature, demonstrating a clear increase in strength of the CP force for elevated temperatures. The solid curve represents the theoretical prediction for the out of equilibrium configuration, while the dash-dot curve represents the case of equal temperatures.
Figure 3: Schematic figure of the surface-surface system out of thermal equilibrium. Here the two bodies occupy infinite half-spaces.

Figure 4: Interaction energy between two identical impurities as a function of the relative distance in a one-dimensional Fermi liquid. Dashed line: non-interacting fermions. Full line: interacting fermions (see [8] for details).
where \( P_0(l) \) is the \( T = 0 \) pressure, and the thermal pressure is the sum of two terms:

\[
P_{\text{th}}^{\text{neq}}(T_1, T_2, l) = P_{\text{th}}^{\text{neq}}(T_1, 0, l) + P_{\text{th}}^{\text{neq}}(0, T_2, l),
\]

(3)
each of them corresponding to a configuration where the thermal fluctuations are produced only in one of the two bodies.

We calculated the thermal pressure \( P_{\text{th}}^{\text{neq}}(T_1, T_2, l) \) at all distances as a function of the temperatures and the dielectric functions of the two bodies (\( \varepsilon_1(\omega) \) and \( \varepsilon_2(\omega) \) respectively). We have focused in particular on the case when one of the two body is a rarefied gas in order to recover the surface-atom force starting from the surface-surface configuration. To solve this problem the expansion of the pressure \( P_{\text{th}}^{\text{neq}}(T, 0, l) \) should be performed through two limiting procedures: the large distance \( l \to \infty \) and the diluteness \( (\varepsilon_2 - 1) \to 0 \) conditions. The order in which we perform such expansions is crucial, providing two asymptotic behaviors holding both at large distances but in different regions.

The first limiting procedure, where the expansion for a dilute body (body 2) is performed only after the large distance limit, gives the result

\[
P_{A1}^{\text{neq}}(T, 0, l) = \frac{k_B T}{l^3} C \sqrt{\frac{\varepsilon_{10} + 1}{\varepsilon_{10} - 1}} \sqrt{\frac{\varepsilon_{20} - 1}{\varepsilon_{20} - 1}},
\]

(4)
where \( C \approx 3.83 \cdot 10^{-2} \) is a numerical factor, independent of the dielectric functions, and \( \varepsilon_{10} \) and \( \varepsilon_{20} \) are the dielectric functions at zero frequency \( \omega = 0 \). This law is valid in the region \( \ell \gg \frac{\lambda_T}{\sqrt{\varepsilon_{20} - 1}} \). Since for a dilute gas \( (\varepsilon_2 - 1) = 4\pi n\alpha \), where \( n \) is the density of the material 2 and \( \alpha \) is the dipole polarizability of its constituents (for example atoms), the resulting force is not additive with respect to the constituents of the system.

A second asymptotic behavior in the surface-rarefied body pressure holds in a different range of distances. In order to obtain such a result one should perform an opposite limiting procedure, carrying out first the diluteness limit \( (\varepsilon_2 - 1) = 4\pi n\alpha_2 \to 0 \), and then the limit of large distances \( l \to \infty \).

Such expansions provide the total asymptotic pressure

\[
P_{A2}^{\text{neq}}(T, 0, l) = \frac{(k_B T)^2}{24 l^2} \frac{\varepsilon_{10} + 1}{ch} \sqrt{\frac{\varepsilon_{10} + 1}{\varepsilon_{20} - 1}} (\varepsilon_{20} - 1),
\]

(5)
holding in the interval \( \lambda_T \ll l \ll \frac{\lambda_T}{\sqrt{\varepsilon_{20} - 1}} \) and satisfies the additivity property. This latter result agrees with the surface atom force discussed in the previous section.
From the previous results it emerges that if the dielectric 2 is very dilute but still occupies a thick region of space, there is a first region where the pressure is additive and coincides with (5). At larger distances the pressure is given by (4) and is no longer additive. Notice that the interval $\lambda T \ll l \ll \lambda T / \sqrt{\varepsilon_2 - 1}$ practically disappears for dense dielectrics.

**Casimir forces between impurities in 1D Fermi liquids**

The Casimir effect is not restricted only to excitations of the electromagnetic field. More generally, objects embedded in a medium at zero temperature will feel a force due the quantum fluctuations of the excitations of the medium. Such a force can be in general very different from the one originating from the fluctuations of the electromagnetic field. In this context we carried out [8] a detailed analysis of the Casimir force between defects in a repulsive one-dimensional Fermi liquid. We have shown (see Fig.4) that the strength and sign of this Casimir interaction depend in a sensitive way on the distance between the impurities. The resulting oscillations of the force have a Friedel nature and are amplified by the interactions present in the medium.


MATTER-WAVES INTERFEROMETRY

The aim of an interferometric device is to measure a phase shift between two fields travelling down separated paths. The shift can be originated by an external force or by a length difference between the two paths, thus phase estimation through an interferometer gives information about the external perturbation acting on our system. Indeed, interferometers take a central role in the development of many technologies based on sensing devices. Clearly, research in such a field would strive for the best precision attainable with a given apparatus thus the uncertainty with which the interferometric phase shift can be determined becomes crucial. This issue is tightly connected to information theory and single parameter estimation. The latter pose as a fundamental question how the uncertainty of a given device scales with the “energy” required for the measurement. In optical or matter-wave interferometry this “energy” corresponds to the number of particles entering the apparatus. A generic linear interferometer with classical input states characterized by a Poissonian distribution of particles has a sensitivity which scales as $1/\sqrt{N}$, where $N$ is the total number of particles. This is referred to as shot-noise limit or standard quantum limit (SQL). The number of particles cannot be increased ad infinitum because the back-reaction of the mirrors under an increasing light pressure limits the sensitivity in an optical interferometer [1]. Similar problems occur in atom interferometers where mirrors and beam splitters are implemented by standing light waves [2]. In addition, there are interaction induced shifts at higher densities. Hence, there is an optimal number of particles that can be used in a particular setup. Using this number, the sensitivity can still be improved by a factor $1/\sqrt{N}$ by using correlated states, which can provide a huge gain in sensitivity. In particular, entangled input states would allow for a scale $1/N$, referred to as the Heisenberg limit (HL). Progress in this direction is not only important for interferometry but the results apply to a wide range of quantum technological applications such as quantum frequency standards, lithography, positioning, and imaging (see [3], and references within). There has been a substantial advance in the last years on the experimental side, and recently phase resolution beyond the SQL has been achieved in proof-of-principle experiments with entangled states of up to four photons [4], of up to six ions [5], and with squeezed light injected into Michelson interferometers as used in gravitational wave detectors [6]. It is generally believed that plays a crucial role in new quantum technologies such as quantum communication, quantum simulation, and quantum computation [7]. Moreover, as introduced above, entanglement is also a key resource to beat the SQL. Thus there seem to be a relation between phase estimation
Matter-waves interferometry

Entanglement measure and quantum interferometry

General bounds on the sensitivity of an interferometer can be obtained from the Fisher information $F$. This is a function of the conditional probabilities $p(\eta|\theta)$ that a measurement outcome $\eta$ is obtained if the true value of the phase shift was $\theta$. The conditional probabilities can be calculated analytically if the input state, the interferometer sequence, and the measurement observable are known. The Fisher information provides the ultimate limit to the phase estimation uncertainty, the Cramer-Rao bound $\Delta \theta \geq 1/\sqrt{F}$ [8]. There is a clear physical picture behind this: an input $|\psi(0)\rangle$ is transformed into a state $|\psi(\theta)\rangle$ under the action of the Hamiltonian $\hat{H}$ (the generator of the phase shift $\theta$). The Fisher information is related to the rate at which $|\psi(\theta)\rangle$ becomes distinguishable from $|\psi(0)\rangle$. Hence, the larger is the Fisher information, the better the final state can be distinguished from the input state, and the better can the phase be estimated. The maximal Fisher information that can be achieved if it is optimized over all possible choices of the output observable is called the quantum Fisher information $F_Q$. This yields the quantum Cramer-Rao bound, the fundamental limit on the phase precision that can be achieved according to quantum mechanics. The quantum Fisher information can be calculated analytically from the input state and the interaction Hamiltonian $\hat{H}$ [9]. The Cramer-Rao bound leads to the condition $F > N$ for sub shot-noise sensitivity. For linear interferometers, the rigorous upper bound for the Fisher information is $F \leq N^2$, corresponding to Heisenberg limit. In our work [10] we develop a general framework to study the interplay between entanglement and phase estimation in metrology and quantum sensors. As stressed above, a quantum state $\hat{\rho}_{inp}$ must necessarily be entangled in order to be useful for estimating a phase shift $\theta$ with a sensitivity $\Delta \theta$ beyond the shot-noise, which is the maximum limit attainable with separable states. Nevertheless not all entangled states can perform better than separable states. Here we introduce a new criterion, on a generic $\hat{\rho}_{inp}$, which is sufficient to recognize multi-particle entanglement and is necessary and sufficient for sub shot-noise phase estimation sensitivity. A state of $N$ particles in two modes ($N$ qubits) is separable (non-entangled) when it can be written as

$$\hat{\rho}_{sep} = \sum_k p_k \hat{\rho}_k^{(1)} \otimes \hat{\rho}_k^{(2)} \otimes \ldots \otimes \hat{\rho}_k^{(N)},$$  \hspace{1cm} (1)
where \( p_k > 0 \), \( \sum_k p_k = 1 \) and \( \hat{\rho}_k^{(i)} \) is the density matrix for the \( i^{th} \) particle. How to recognize entangled states? Let us introduce the “fictitious” angular momentum operator, \( \hat{J} = \sum_{l=1}^{N} \hat{\sigma}^{(l)} \) where \( \hat{\sigma}^{(l)} \) is a Pauli matrix operating on the \( l^{th} \) particle. According to the current literature, if a state \( \hat{\rho}_{inp} \) satisfies the inequality 
\[
\chi^2 \equiv \frac{N(\Delta \hat{J} \cdot \vec{n}_1)^2}{(\langle \hat{J} \cdot \vec{n}_2 \rangle^2 + \langle \hat{J} \cdot \vec{n}_3 \rangle^2 < 1)} \leq 1
\]
then is particle-entangled [23] and spin squeezed along the direction \( \vec{n}_1 \), being \( \vec{n}_1, \vec{n}_2 \) and \( \vec{n}_3 \) three mutually orthogonal unit vectors and \( \hat{J}_{\vec{n}_i} = \hat{J} \cdot \vec{n}_i \). Here we introduce a different sufficient condition for particle-entanglement:
\[
\chi^2 = \frac{N}{F_Q[\hat{\rho}_{inp}, \hat{J}_{\vec{n}_2}]} < 1, \tag{2}
\]
where \( F_Q[\hat{\rho}_{inp}, \hat{J}_{\vec{n}_2}] = 4(\Delta \hat{R})^2 \). The Hermitian operator \( \hat{R} \) is the solution of the equation \( \{ \hat{R}, \hat{\rho}_{inp} \} = i[\hat{J}_{\vec{n}_2}, \hat{\rho}_{inp}] \). It is possible to demonstrate that \( \chi^2 \leq \xi^2 \). Therefore, Eq.(2) recognizes a class of states which are entangled, \( \chi^2 < 1 \) and not spin-squeezed, \( \xi^2 \geq 1 \). Generally speaking, an interferometer is quantum mechanically described as a collective, linear, rotation of the input state by an angle \( \theta \): \( \hat{\rho}_{out} = e^{i\theta \hat{J}_{\vec{n}_2}} \hat{\rho}_{inp} e^{-i\theta \hat{J}_{\vec{n}_2}} \), where the generator of the rotation is the projection of the angular momentum operator \( \hat{J} \) over an arbitrary direction \( \vec{n}_2 \). For instance, in Mach-Zehnder (MZ) interferometry, the rotation is about the \( \vec{n}_2 = \vec{y} \) axis. Since for an arbitrary interferometer and phase estimation strategy, the phase sensitivity is limited by the Quantum Cramer-Rao, which only depends on the specific choice of the input state,
\[
\Delta \theta_{QCR} = \frac{1}{\sqrt{F_Q[\hat{\rho}_{inp}, \hat{J}_{\vec{n}_2}]}} = \frac{\chi}{\sqrt{N}} \tag{3}
\]
a comparison with Eq.(3) reveals that Eq.(2) is not only a sufficient condition for particle-entanglement, as already discussed, but also a necessary and sufficient condition for sub shot-noise phase estimation. This is a main result of this work: \( \chi < 1 \) provides the class of entangled states which are useful for sub shot-noise sensitivity. In other words, chosen \( \hat{\rho}_{inp} \) and \( \hat{J}_{\vec{n}_2} \), if the corresponding value of the \( F_Q \) is such that \( \chi < 1 \), then the state is entangled and, if used as input of an interferometer realizing the unitary transformation \( e^{i\theta \hat{J}_{\vec{n}_2}} \), it provides a phase estimation sensitivity higher than any interferometer using classical (separable) states. On the other hand, the class of entangled states for which \( \chi > 1 \) cannot provide a sensitivity higher than the classical shot-noise.

We also discuss the connection between non-linear dynamics, entanglement and spin-squeezing. We consider a coherent spin state, \( |j, j\rangle_{\vec{n}_1} = \sum_{\mu=-j}^{+j} \hat{a}^{\dagger \mu} \sqrt{\binom{2j}{j+\mu}} |j, \mu\rangle_{\vec{n}_3} \), with \( j = N/2 \). This state is separable (\( \chi^2 = 1 \)) and we quest the possibility to strongly
entangle the particles by the non-linear evolution $e^{-i\tau \hat{J}_3^2}$.

A comparison between $\xi(\tau)$ and $\chi(\tau)$ is presented in figure (1) for $N \gg 1$. Notice that, $\xi^2 \geq 1$, while $\chi^2 \leq 1$ for all values of $\tau$: the state $|\psi(\tau)\rangle$ is not spin-squeezed along the $\vec{n}_3$ direction but still (usefully) entangled. There are three interesting time scales in the dynamical evolution of $\chi^2$: for $0 < \tau < 1/\sqrt{N}$, $\chi^2$ decreases assuming values between 1 and $2/N$; for $1/\sqrt{N} \leq \tau \leq \pi/2 - 1/\sqrt{N}$, $\chi^2$ reaches the plateau $\chi^2 = 2/N$. The dynamics is periodic with period $T = \pi/2$ for even values of $N$ and $T = \pi$ for odd $N$ (in which case $\chi^2 = 1/N$ at $\tau = \pi/2$).

Bayesian phase estimation with Mach-Zehnder interferometry

In the previous section we have discussed the connection between entanglement and the quantum Fisher information $F_Q$, that is, the maximal Fisher information that can be achieved if it is optimized over all possible choices of the output observable. Now we go back and restrict our problem to a particular choice of the output observable to illustrate the problem of phase estimation strategies. Indeed, once the physical setup and the measurement operation are chosen, the phase shift has to be estimated from the measurement results. An estimation strategy which can saturate the Cramer-Rao bound is the following: the conditional probability distribution $p(\eta|\theta)$ (which can be estimated from the measurement results) can be inverted using Bayes rule $p(\eta|\theta) = p(\theta|\eta)p(\theta)/p(\eta)$. Here, $p(\theta)$ is the knowledge about $\theta$ before the estimation and $p(\eta)$ is
Figure 2: (a) Schematic of a Mach-Zehnder interferometer. A phase sensitive measurement is provided by the detection of the number of particles $N_c$ and $N_d$ at the two output ports. (b) Pulse height distribution for a visible light photon counter (VLPC) used in the experiment. The vertical lines show the decision thresholds.

A normalization constant. The phase shift can be estimated as the value $\theta_{\text{est}}$ leading to the maximal value of $p(\theta|\eta)$, with $\Delta \theta$ given by the 68% confidence interval around $\theta_{\text{est}}$. In this formalism, also finite efficiencies of the detectors can be directly implemented into the probability distribution [12, 13]. This approach has advantages with respect to the frequently used maximum likelihood method or other semi-classical approaches [14]. In practice, the optimal measurement that maximizes $F$ depends on the physical setup. In the following we will focus on a paradigmatic example: the Mach-Zehnder interferometer.

The Mach-Zehnder (MZ) interferometer [15, 16] is a truly ubiquitous device that has been implemented using photons, electrons and atoms. Its applications range from micro- to macro-scales, including models of aerodynamics structures, near-field scanning microscopy and the measurement of gravity accelerations. In the standard configuration of the MZ interferometer, a coherent optical state with an average number of photons $\bar{n} = |\alpha|^2$ enters input port $a$ and the vacuum enters input port $b$, as illustrated in figure (2). The goal is to estimate the value of the phase shift $\theta$ after measuring a certain number of photons $N_c$ and $N_d$ at output ports $c$ and $d$. In the experiment discussed in our work [13], the above photon counting is made possible by two number-resolving photodetectors. Here we discuss the possibility to achieve the standard quantum limit, which in this case corresponds to the Cramer-Rao lower bound, by means of Bayesian phase estimation. A common phase inference protocol for the scheme of Fig.(2) estimates the true value of the phase shift $\theta$ as $\Theta_{\text{est1}} = \arccos \left( \frac{M_p}{\bar{n}} \right)$, where $M_p = \sum_{k=1}^{p}(N_c^{(k)} - N_d^{(k)})/p$ is the photon number difference detected at the output ports, averaged over $p$ independent measurements. The phase uncertainty of estimator
\( \Theta_{\text{est}} \) is \( \Delta \Theta_1 = 1 / \sqrt{\bar{n} \sin \theta} \), which follows from a linear error propagation theory. It predicts an optimal working point at a phase shift \( \theta = \pi / 2 \), where the average photon number difference varies most quickly with phase. As \( \theta \) approaches 0 or \( \pi \), the confidence of the measurement becomes very low and eventually vanishes. On the other hand, the CRLB can be calculated analytically. The coherent \( \otimes \) vacuum input state, gives \( F(\theta) = \bar{n} \). Therefore, the CRLB is

\[
\Delta \Theta_{\text{CRLB}} = \frac{1}{\sqrt{\bar{n} n}}
\]

which, in contrast with \( \Delta \Theta_1 \), is independent of the true value of the phase shift. The only assumption here is that the observable measured at the output ports is the number of particles. In our Bayesian protocol the goal is to determine \( P(\phi|N_c, N_d) \), the probability that the phase equals \( \phi \) given the measured \( N_c \) and \( N_d \). Assuming no prior knowledge of the phase shift in \([0, \pi]\), \( P(\phi) = 1/\pi \). In the ideal case, the Bayesian phase probability distribution can be calculated analytically for any value of \( N_c \) and \( N_d \),

\[
P(\phi|N_c, N_d) = C \left( \cos \frac{\phi}{2} \right)^{2N_c} \left( \sin \frac{\phi}{2} \right)^{2N_d},
\]

where \( C = \frac{\Gamma(1/2+N_c)\Gamma(1/2+N_d)}{\Gamma(1+N_c+N_d)} \) is a normalization constant. In practice, one must measure \( P(N_c, N_d|\phi) \) and, from this, determine \( P(\phi|N_c, N_d) \). This distribution provides both the phase estimate and the estimate uncertainty. For instance, with a single measurement, \( p = 1 \), it predicts an uncertainty that scales as \( \Delta \Theta \approx \frac{1}{\sqrt{n}} \). Since Eq.(5) does not depend on \( \bar{n} \), the estimation is insensitive to fluctuations of the input laser intensity. Most importantly, its uncertainty, in the limit \( p \gg 1 \), is \( \Delta \Theta = \frac{1}{\sqrt{p n}} \) which coincides with the CRLB. To implement the proposed protocol we have realized a polarization Mach-Zehnder interferometer with photon-number-resolving coincidence detection. After the calibration, we can proceed with the Bayesian phase estimation experiment. For a certain value of the phase shift, \( \theta \), we input one laser pulse and detect the number of photons \( N_c \) and \( N_d \). We repeat this procedure \( p \) times obtaining a sequence of independent results \( \{N_c^{(i)}, N_d^{(i)}\}_{i=1}^p \). The \( p \) photon-number measurements comprise a single phase estimation. The overall phase probability is given by the product of the distributions associated with each experimental result: \( P_{\text{exp}}(\phi|\{N_c^{(i)}, N_d^{(i)}\}_{i=1}^p) \propto \prod_{i=1}^p P_{\text{exp}}(\phi|N_c^{(i)}, N_d^{(i)}). \) The phase estimator \( \Theta_{\text{est}} \) is given by the mean value of the distribution, \( \Theta_{\text{est}} = \int_0^{\pi} d\phi \Phi_{\text{fit}}(\phi|\{N_1^{(i)}, N_2^{(i)}\}_{i=1}^p) \), and the phase uncertainty \( \Delta \Theta \) is the 68.27% confidence interval around \( \Theta_{\text{est}} \), where \( \Phi_{\text{fit}}(\phi|\{N_1^{(i)}, N_2^{(i)}\}_{i=1}^p) \) is a fit of the experimental distribution \( P_{\text{exp}} \).
Figure 3: (color online). Phase sensitivity as a function of the true value of the phase shift. Circles are averages of 20 replica of $\sqrt{p} \Delta \Theta$, obtained from Bayesian distributions $P_{\text{exp}}$, with $p = 10000$. The error bars give the fluctuations. The solid black line is the theoretical prediction Eq.(4). The dashed blue line is the CRLB calculated with the experimental distributions. Squares are the uncertainty obtained with a generalization of the estimator $\Theta_{\text{est1}}$ taking into account the experimental imperfections, while the dotted red line is the phase sensitivity predicted by $\Delta \Theta_1$. Inset: difference between the true value of the phase shift $\theta$ and the mean value, $\langle \Theta_{\text{est}} \rangle$, of 20 replica of $p = 10000$ independent measurements. The vertical bars are mean square fluctuations $\sigma_{\text{est}}^2 = \langle (\Theta_{\text{est}} - \langle \Theta_{\text{est}} \rangle)^2 \rangle$. The $\sigma_{\text{est}} \gg (\theta - \Theta_{\text{est}})$ provides evidence that our result is unbiased.
The main result of this work is presented in Fig.(3). We show the phase sensitivity for different values of the phase shift $\theta$, calculated after $p = 10000$ photon-number measurements. The circles are averages, $\Delta \Theta$, of 20 independent phase measurements, and the bars give the corresponding mean square fluctuations. The dashed blue line is the CRLB calculated with the experimental probability distributions, $\Delta \theta_{\text{exp}} = 1/\sqrt{p F_{\text{exp}}(\theta)}$, where $F_{\text{exp}}(\theta) = \sum_{N_1,N_2} \frac{1}{N_1!N_2!} (\partial P_{\text{exp}}(N_1,N_2|\theta)/\partial \theta)^2$. For $0.1 \lesssim \theta/\pi \lesssim 0.9$, it follows the theoretical prediction (solid black line), Eq.(4). The experimental results (circles) do saturate the CRLB.

Interferometry at the Heisenberg limit with non-classical states

In the experimental work discussed above, a protocol able to saturate the CRLB has been discussed for the case of a MZ fed with classical input states: a coherent state and the vacuum state. The use of such states imposed a lower bound to the sensitivity (CRLB) which coincided with the shot-noise limit or standard quantum limit, consistently with the prediction discussed above that separable states (non entangled states) should not provide sub-shot noise sensitivity. Thus, in order to beat the SQL, we must make use of non-classical inputs for our MZ interferometer. This quest was initiated by Caves in 1981 [17], who considered a lossless MZ fed by coherent $\otimes$ squeezed-vacuum light. The coherent state can be written as $|\alpha\rangle_a \equiv \sum_{m=0}^{+\infty} C_m |m\rangle_a$, with $\alpha \equiv e^{i\theta_c} |\alpha|$, and $C_m \equiv \alpha^m e^{-|\alpha|^2/2} \sqrt{m!}$. The squeezed-vacuum is $|\zeta\rangle_b \equiv \sum_{m=0}^{+\infty} S_m |m\rangle_b$, with $\zeta \equiv re^{i\theta_s}$ and $S_m \equiv (e^{i\theta_s \tanh r})^m / 2^{m^2} \sqrt{m!} \cosh r \text{H}_m(0)$, $\text{H}_m(x)$ being the Hermite polynomials. In our work [18] we show that the ultimate phase sensitivity of a Mach-Zehnder fed by coherent $\otimes$ squeezed-vacuum light is

$$\Delta \theta = \frac{1}{\sqrt{p}} \frac{1}{\sqrt{|\alpha|^2 e^{2r} + \sinh^2 r}} \quad (0 \leq \theta \leq \pi).$$

(6)

The phase sensitivity Eq.(6) is i) independent from the true value of the phase shift over the whole interval $0 \leq \theta \leq \pi$ and ii) it reaches, at the optimal point $|\alpha|^2 = \sinh^2 r$, the Heisenberg limit:

$$\Delta \theta = \frac{1}{\sqrt{p \bar{n}}}, \quad (0 \leq \theta \leq \pi),$$

(7)

asymptotically in the average number of photons $\bar{n} = |\alpha|^2 + \sinh^2 r$ and with a number of independent measurements $p \gtrsim 30$. Then, with a protocol similar to the one discussed in the previous section, we demonstrate the the CRLB is saturated by a Bayesian
phases inference approach. A natural question arises: What is the physics underlying the increase in phase sensitivity using squeezed vacuum light? We can understand the saturation at the Heisenberg limit by quantum interference effects created by the beam splitter. Indeed, the latter creates a relative number of particles distribution characterized by a mean-square fluctuation of the order of $N$, which indicates that the quantum state after the beam splitter contains a large “NOON” component $|NOON\rangle \sim (|N,0\rangle + |0,N\rangle)$. Such a distribution is typical of states attaining the Heisenberg limit $\Delta \theta \sim 1/N$. An example of how “NOON”-like states can be created and employed for phase estimation with BECs will be discussed below.

**Interferometry with BEC.**

Interferometry with BECs might offer additional advantages with respect to atom interferometry. The reduced spreading in the momentum distribution is advantageous for the operation of interferometers with standing light waves functioning as beam splitters. Moreover, the low velocities enable the realization of interferometers enclosing a larger area. Very promising experiments using BECs in double well potentials for interferometry have been performed recently [19]. Finally, the experimental possibility to tune the strength of the atom-atom interaction and dynamically adjust it near so-called Feshbach resonances has opened many new intriguing scenarios. In particular it can be tuned to zero to allow for the realization of non-interacting BECs [20]. In particular, the LENS group in Florence has exploited this feature to realize an interferometer with BEC trapped inside an optical lattice [21]. Here Bloch oscillations were forced by gravity and imaged at different times after releasing the trap. Due to the micrometric size of the atomic sample (related to the BEC small spreading in momentum distribution) and the enhanced coherence times (allowed by the tuning of the inter-particle interactions almost to zero) this sensor seems an ideal tool for measuring forces with high sensitivity.

**Creation of entangled states with BEC.**

The above experiment required to have almost non-interacting atoms during the interferometric sequence in order to avoid phase diffusion [22], but interactions are not always enemies to interferometry. Indeed, the strong inter-particle interactions which are naturally present in a BEC can be used to create highly entangled states of a large
Figure 4: (color online). a),b) Montecarlo phase distribution (dots, with the phase values distributed over 20 bins) and the SU(2) phase distribution $P(\theta)$ (solid line), for the state Eq.(8) with $s = 1.5$.

number of atoms useful for interferometry below the SQL [23]. Nonlinearity is crucial for the creation of a superposition of macroscopically distinguishable states, often referred to in the literature as “Schrödinger’s cat”. On the atomic BEC side, several thermodynamical and dynamical schemes for creating a superposition of two BEC states differing by a macroscopically large number of particles have been proposed in the literature (see [24], and references therein). These so called “NOON states” maximize particle entanglement. In our work [24] we discuss an experimentally feasible protocol for the creation and detection of a macroscopic superposition of states having different relative phases with a Bose-Einstein condensate trapped in a periodic potential. These entangled states [25] are generated by the nonlinear unitary evolution governed by the decoupled $N_S$-mode Bose-Hubbard Hamiltonian: $\hat{H}_{NS} = \sum_{j=1}^{N_S} \epsilon_j \hat{c}_j^\dagger \hat{c}_j + \frac{U}{2} \hat{c}_j^\dagger \hat{c}_j^\dagger \hat{c}_j \hat{c}_j$ and are closely related to NOON states. Here $\hat{c}_j$ ($\hat{c}_j^\dagger$) annihilate (create) a particle in the $j$th condensate, $\epsilon_j$ is the energy offset due, for instance, to an external potential superimposed to the optical lattice and $U_j$ is the single-condensate interaction energy.

For a symmetric double well potential ($\epsilon_1 = \epsilon_2$, $U_1 = U_2 \equiv U$), with the state of $N_T$ particles $|\psi(\theta_0)\rangle = \sum_{n=0}^{N_T} C_n e^{-i n \theta_0} |N_T - n, n\rangle_z$ as initial condition (where $|N_T - n, n\rangle$ is a number Fock state ), the state $|\Psi(t_{\pi/2})\rangle = e^{-i \hat{H} t_{\pi/2}/\hbar} |\psi(\theta_0)\rangle$, obtained after a time $t_{\pi/2} \equiv \frac{\hbar \pi}{2U}$, can be written as

$$|\Psi(t_{\pi/2})\rangle = \frac{e^{-i \frac{\pi}{2}}}{\sqrt{2}} |\psi(\theta_0 + \frac{\pi}{2}, \xi)\rangle + \frac{e^{+i \frac{\pi}{2}}}{\sqrt{2}} |\psi(\theta_0 - \frac{\pi}{2}, \xi)\rangle,$$

(8)

where $\xi = 0$ ($\xi = 1$) for $N_T$ even (odd). It is instructive to project the state Eq.(8) over the SU(2) basis states $|N_T, \theta\rangle = \frac{1}{\sqrt{2\pi \sqrt{N_T + 1}}} \sum_{n=0}^{N_T} e^{-i n \theta} |N_T - n, n\rangle_z$. If the initial $|\psi(\theta_0)\rangle$ has a relative phase distribution localized about $\theta_0$, the phase distribution of the
evolved state, $P(\theta) = |\langle NT, \theta | \Psi(t_\pi/2) \rangle|^2$, is characterized by 2 peaks, see Fig. 4, which is the consequence of being $|\Psi(t_\pi/2)\rangle$ a superposition of 2 states having different relative phases. With trapped BEC, a realistic scheme for the creation of a superposition of two states having different relative phases involves the sudden splitting of a single condensate. The BEC is left in a state slightly squeezed in the relative number of particles, $|\psi_0\rangle \equiv |\psi(\theta_0 = 0)\rangle \sim \sum_{n=0}^{NT} e^{-(n-N_T/2)^2/4\sigma_s^2} |N_T - n, n\rangle_z$, which provides the initial condition of the decoupled non-linear evolution. The width of the relative number distribution is $\sigma_s = \sqrt{NT/(2s)}$ and $s$ is the squeezing parameter. After a time $t_\pi/2$, this state evolves in the superposition Eq.(8). We release the confining potential and let the condensate ballistically expand and overlap, giving rise to an interference pattern from which we extract a single value of the relative phase. We show, by simulating the formation of several single-shot interference density profiles with a many-body Montecarlo technique, that the phase distribution obtained upon several interference experiments is reasonably well described by the SU(2) probability $P(\theta)$, when the initial state is number-squeezed $s > 1$. In FIG. 4(a),(b) we show the results of 400 independent phase estimations (dots) for $NT = 10$ (a) and $NT = 11$ (b). In FIG. 4(a),(b) we can distinguish two peaks separated by $\pi$, each corresponding to a different phase component of Eq.(8). The phase shift due to a change in the parity of $NT$ can be clearly seen as a shift of $\pi/2$ among the distributions of (a) and (b).

As expected, the width of the peaks $w \sim s/\sqrt{NT}$ increases with the relative number squeezing of the initial state. In typical experimental conditions there is no control on the parity of the total number of atoms. In this case, the system is described by a classical mixture of superposition states, half corresponding to odd $NT$ and half to even $NT$. Notice that the occurrence of two peaks (or four if the parity of $NT$ is not controlled) in the phase distribution constitutes, by itself, a signature of the presence of a quantum superposition, rather than a statistical (non coherent) mixture with the same components. This because the state Eq.(8) is created and lives in a very narrow temporal window, $\sim t_\pi/2 \pm \hbar/U NT$.

We have also demonstrated that a clear signature of the creation of these macroscopic superpositions also appears in the interference of an array of BECs, even in a single-shot density profile. We finally account for the problem of decoherence due to one, two, and three-body losses. For typical experimental trapping parameters and single well occupations up to a few hundred atoms, the decoherence time can be about 500 ms, longer than the typical formation time of the macroscopic superposition of phase states which can therefore be experimentally created and detected within current technology.
Cat states and sub-shot noise interferometry.

Quite recently, several efforts have been directed (and succeeded with 6 trapped ions [5]) to the experimental realization of “NOON” states:

$$|\Psi_N\rangle = \frac{1}{\sqrt{2}}(|N,0\rangle + e^{i\phi}|0,N\rangle).$$  \hspace{1cm} (9)

The state $|N,0\rangle$ contains $N$ particles in mode $a$ and 0 particles in mode $b$ (vice versa the state $|0,N\rangle$), while $\phi$ is an arbitrary phase. It is widely believed that interferometry with the state Eq.(9) can estimate unknown phase shifts with sensitivity at the Heisenberg limit. This claim is often accompanied by a simple example. The phase shift, induced by an external classical perturbing field, is created by the unitary operator $\hat{U}_\theta = e^{-i\hat{J}_z\theta}$, where the generator of the unknown phase translation $\theta$ is the two-mode relative number of particles operator, $\hat{J}_z = (\hat{N}_a - \hat{N}_b)/2$. The projection of the new state $|\Psi_N(\theta)\rangle = \hat{U}_\theta|\Psi_N\rangle = (|N,0\rangle + e^{i(\theta N + \phi)}|0,N\rangle)/\sqrt{2}$ over the initial one gives

$$|\langle \Psi_N|\Psi_N(\theta)\rangle|^2 = \cos^2(N\theta/2).$$  \hspace{1cm} (10)

Orthogonality, $\langle \Psi_N|\Psi_N(\theta)\rangle = 0$, is first reached at $\theta = \pm \pi/N$, which would suggest that the smallest measurable phase shift is of the order of $1/N$ as well. There is a problem, though: in interferometry the incremental phase shift, albeit supposedly small, is unknown and the phase estimation based on the projective measurement Eq.(10) is ambiguous. Indeed, $\langle \Psi_N|\Psi_N(\theta)\rangle = 0$ when $\theta = \pm (2n + 1)\pi/N$, with $n = 0, 1, 2, \ldots, N - 1$. Orthogonality alone is not sufficient to determine $n$, with unpleasant consequences when trying to estimate the unknown value of $\theta$ with an arbitrary large number of particles and complete prior ignorance. The $2\pi/N$ oscillation period of Eq.(10) is typical in quantum enhanced technology with state Eq.(9). In this our work [12] we propose i) a measurement strategy for the unambiguous estimation of phase shifts with uncertainty $\sim 1/N_T$ by using the state Eq.(9), within ii) a rigorous Bayesian analysis of the measurement results which can be implemented experimentally incorporating decoherence and classical noise and iii) maximum prior ignorance about the phase shift: $-\pi \leq \theta \leq \pi$. The protocol requires $p$ independent interferometric measurements performed with “NOON” states having a different number of particles, $N = 1, 2, 4, \ldots, 2^{p-1}$.

The basic idea underlying this choice of different $N$ for different measurements, which is crucial for the unambiguous phase estimation, is to kill all the oscillations of Eq.(10) but one, in order to cancel the periodicity and therefore the ambiguity in the choice of the estimator. The sensitivity is calculated as a function of the total number of
particles used in the process, $N_T = 2^n - 1$. From the experimental point of view, the demonstration of the Heisenberg limit Eq.(7) requires the creation of Schrödinger cat states with a minimum of $N = 8$ particles, which is within the reach of the present state-of-the-art.


Neutral atoms in optical lattices and atom chips are unique for quantum information purposes, as they are one of the few physical systems, in which both an outstanding degree of single particle control exists, while simultaneously large scale qubit systems can be realized. Thus, neutral atoms are one of the most promising candidates for realizing quantum information processing devices.

The experimental demands on the next phase of Quantum Information Processing and Communication (QIPC) research will have a larger focus on integration of components and their reliability as the field moves from research oriented problems to applied and even commercial quantum technologies. Therefore an even closer interplay between theory and experiment will be needed in order to tame and overcome the broad variety of sources of errors that hinder present developments of large-scale quantum information applications: noise, inhomogeneous broadening, parameter fluctuations, device imperfections and coupling to a thermal environment. The work of the Quantum Information group at the BEC center has focused mainly along this direction.

Optimal control

In [1] we have applied optimal control techniques to model and optimize the manipulation of the external quantum state (center-of-mass motion) of atoms trapped in adjustable optical potentials. In particular we have considered in detail the cases of both non interacting and interacting atoms moving between neighboring sites in a lattice of a double-well optical potentials, since such a lattice can perform interaction-mediated entanglement of atom pairs and can realize two-qubit quantum gates.

Our numerical analysis shows that the optimized control sequences for the optical potential allow transport faster and with significantly larger fidelity than is possible with processes based on adiabatic transport. In particular, the fidelity of the transport process for $T = 0.15$ ms improves from $F_{\text{int}} = 0.22$, using simple adiabatic switching, to $F_{\text{int}} = 0.97$, using optimal control theory, with better results expected for longer control times. When including the effect of atom-atom interactions on the transport process we found that the optimal control parameter sequences found in the non-interacting case still work: a better fidelity can be achieved in a time shorter by more than a factor of three, which represents a relevant improvement in terms of scalability of the number of gates that can be performed before the system decoheres due to the coupling to its environment. This technique can be easily adapted to other similar transport processes and also extended to atoms in different magnetic states, which can allow the
implementation of a fast, high-fidelity quantum gate in a real optical lattice setup with the qubits encoded in the atomic internal states.

In [2] we employed control theory to design high-fidelity two-bit gates for Josephson charge qubits realized by an appropriate choice of pulses in the gate potentials and in the presence of both leakage and noise. The optimized protocol allows to obtain very high fidelities, and, more important, it has proven quite robust in the disruptive presence of $1/f$ noise. The improvement in the gate performances observed (with errors $\sim 10^{-3} \div 10^{-4}$ in realistic cases) allows to cross the fault tolerance threshold, opening thus an avenue to the realization of high-fidelity computations with Josephson nanocircuits.

**Atom-ion interactions**

Motivated by recently opened experimental possibilities within combined systems – currently being built in several groups worldwide – where magneto-optical traps or
optical lattices for neutral atoms coexist with electromagnetic traps for ions, in [3] we analyzed in detail the interaction between a single atom and a single ion guided by external trapping potentials.

The ion charge induces in the atom an electric dipole moment, which attracts it with an $r^{-4}$ dependence at large distances, while the short-range part of the interaction is described in the framework of quantum-defect theory, by introducing some short-range parameters, which can be related to the $s$-wave scattering length (the application of the contact pseudopotential is forbidden since the characteristic range of the atom-ion interaction is comparable or larger than the characteristic size of the trapping potential). A remarkable feature identified in such system is the presence of resonances between molecular-ion bound states and motional excitations within the trap. These trap-induced resonances (which are similar in nature to Feshbach resonances driven by external fields), make such systems very interesting from the point of view of quantum information processing, since one could utilize controlled atom-ion interactions to effect coherent transfer of qubits, thereby creating interfaces between atoms and ions. By considering quasi-one-dimensional systems and investigating the effects of coupling between the center of mass and relative motion which occurs for different trapping frequencies of the atom and ion traps, we have shown how the two types of resonances can be employed for quantum state control and spectroscopy of atom-ion molecules.
Entanglement and decoherence in many atom systems

In [4] we studied decoherence induced on a two-level system coupled to a one-dimensional quantum spin chain. In particular, we considered the cases where the dynamics of the chain is determined by the Ising, $XY$, or Heisenberg exchange Hamiltonian, since this model of quantum baths can be of fundamental importance for the understanding of decoherence in open quantum systems (it can, e.g., be experimentally engineered by using atoms in optical lattices).

We provided results that go beyond the case of a central spin coupled uniformly to all the spins of the bath, in particular showing what happens when the bath enters different phases, or becomes critical; we also studied the dependence of the coherence loss on the number of bath spins to which the system is coupled and we described a coupling-independent regime in which decoherence exhibits universal features, irrespective of the system-environment coupling strength. Finally, we establish a relation between decoherence and entanglement inside the bath. For the Ising and the $XY$ models we are able to give an exact expression for the decay of coherences, while for the Heisenberg bath we resort to the numerical time-dependent density matrix renormalization group.
In [5] we have considered a class of random critical spin chains, in which the couplings $J$ and the local transverse magnetic fields $h$ are random but share a given amount of correlations characterized by the coefficient $\alpha \in [0, 1]$. In particular we concentrated on two models: the random Ising model with dimension $d = 2$ and the quantum Potts model with $d = 3$. These random correlated models are particular interesting since could be realized experimentally in engineered quantum systems, e.g. in optical lattices, ion traps or arrays of Josephson junctions. Our analysis shown that whenever $J$ and $h$ are correlated the prefactor of the Von Neumann entropy is always greater than the prefactor for the uncorrelated case. Moreover, when the correlations are above a certain threshold $\alpha^*$ the prefactor of the random correlated model is larger than the prefactor for the homogenous chain. Thus, contrary to the standard lore that this could only be achieved in higher dimensional systems ($d > 41$) we demonstrated how to increase entanglement by putting static disorder in systems with small local dimension showing that there exists a class of disordered spin one-half models where the ground state entanglement is larger than in the correspondent translational-invariant ones. Since the correlated random models studied lay outside the random singlet-like phase for which a generalized c-theorem holds, we proved that it is thus ambiguous the association of a renormalized central charge to general non-homogeneous systems.

### Strategic planning and networking at European level

In the context of the coordination action QUROPE – Quantum Information Processing and Communication in Europe the position document [6] “Quantum Information Processing and Communication: Strategic report on current status, visions and goals for research in Europe, has been constantly updated under the joint coordination of the Trento group and of the Institute for Quantum Optics and Quantum Information of the Austrian Academy of Sciences in Innsbruck. This document has been elaborated, following a former suggestion by FET (the Future and Emerging Technologies Unit of the Directorate General Information Society and Media of the European Commission), by a committee of QIPC scientists, and it has been instrumental to provide input towards the European Commission for the preparation of the Seventh Framework Program. Besides being a document addressed to policy makers and funding agencies (both at the European and national level), the document contains a detailed scientific assessment of the state-of-the-art, main research goals, challenges, strengths, weaknesses, visions and perspectives of all the most relevant QIPC sub-fields.
Figure 4: Prefactor $\kappa^2_\alpha$ of the entropy versus the correlation coefficient $\alpha$ of the $J$ and $h$ couplings for different chain lengths ($L = 50$, black circles; $L = 70$, red squares; $L = 100$, blue diamonds) and $N = 10^4$ configurations. The solid and dashed line gives $\kappa^2_\alpha = 0 = c_2 \ln 2 \approx 0.35$ and $\kappa^h_2 = c_2 = 0.5$ respectively. The inset shows the finite-size scaling of the prefactor with best-fit lines (solid, $\alpha = 0$; dotted, $\alpha = 0.95$; dashed-dotted, $\alpha = 1$). Extrapolation to the thermodynamic limit $1/L \to 0$ gives $\kappa^2_{\alpha = 0} = 0.30 \pm 0.04$ and $\kappa^2_{\alpha = 1} = 0.60 \pm 0.01$.


[6] The document can be downloaded at the following address
http://www.qurope.net/Q_Reports/q_reports.php?type=reports&trig_id=1

Projects and scientific collaborations

Projects

During the period June 2006 - May 2008 the members of the BEC Center have been involved in national projects supported by the Italian Ministry of Research, by INFM-CNR and the European Union. Among them:

- 2005-2007 MIUR PRIN on "Ultracold Fermi gases and optical lattices"
- 2006-2008 Marie-Curie Intra-European Fellowship TUCAL (Theory of Ultra-Cold Atoms in Lattices)
- 2005-2008 Marie-Curie Outgoing International Fellowship QOQIP (Quantum Optics for Quantum Information Processing)
- 2005-2009 IST-FET Integrated Project SCALA (Scalable quantum computing with Atoms and Light)
- 2005-2007 Coordination Action ERA-Pilot QIST (Quantum Information Sciences and Technologies)
- 2006-2009 Coordination Action QUROPE (Quantum Information Processing and Communication in Europe)
- 2007-2009 Research Training Network EuroQUAM (Fermionic Mixtures of Ultracold Atoms)

The BEC Center is also supported by the Provincia Autonoma di Trento (PAT) on the basis of an official agreement with INFM-CNR.

Main scientific collaborations

The scientific activity carried out at the BEC Center is the result of numerous national and international collaborations. Some of the most significant ones are briefly described below:

- **Florence, European Laboratory for Nonlinear Spectroscopy.** The Trento team has a long and fruitful experience of collaboration with the experimental group of Massimo Inguscio at LENS. Regular meetings are organized between the two groups on topics of common interest. Michele Modugno works at LENS, but he has been also a member of the BEC center, working on theoretical problems in collaboration with both groups in Florence and Trento. Recent projects include the behavior of condensates in optical lattices and in random potentials.
Projects and scientific collaborations


• Paris, Ecole Normale Superieure. The collaboration with Yvan Castin is actively going on since 1999. In the last years, the collaboration has concerned the development and the application of semi-classical Monte Carlo schemes for the study of the density and the pair distribution function of thermal vortices in two-dimensional Bose gases at thermal equilibrium. The BEC center also collaborates with Roland Combescot; recent joint activities has focused on the study of elementary excitations in superfluid Fermi gases and on the normal state of highly polarized configurations.


• Paris, University Paris 7. In the last years the long-standing collaboration with Cristiano Ciuti has followed two main axis. On one hand we have gone on with our study of polariton condensates, in particular of their shape and superfluidity properties. On the other hand we have investigated the physics of the electromagnetic vacuum in (i) solid-state systems that display an ultra-strong coupling of light to an material excitation and (ii) in optically dressed ultracold atomic gases trapped in optical lattices. The promise of both systems in view of an experimental observation of the dynamical Casimir effect has been established.
Projects and scientific collaborations


- Paris, École Polytechnique, Palaiseau. The collaboration with T.-L. Dao, C. Kollath, and A. Georges has started from a common interest on new diagnostic techniques that may be sensitive to the peculiar correlations that appear in strongly interacting Fermi gases. Stimulated Raman spectroscopy has been shown to provide information on the first order Green’s function in energy-momentum space, while light-stopping techniques appear as very promising in view of a measurement in real-time.


- Lausanne, EPFL. In the last years, the Trento activity on polariton condensates has taken great advantage from the collaboration with a few groups at EPFL. On the theoretical side, M. Wouters, a former member of the BEC Center, has recently joined V. Savona’s group and D. Sarchi, former PhD student at EPFL, will join the BEC Center starting from early 2009. These exchanges have led to joint works on several aspects of the many-body and non-equilibrium physics of polariton condensates, in particular the condensate shape, the elementary excitations, and the Josephson dynamics in two-well geometries. From the experimental side, the Trento expertise in many-body physics has supported B. Deveaud’s group in the theoretical interpretation of their observations of vortices in polariton condensates.

Projects and scientific collaborations


- **Innsbruck**, Austria. Since the very beginning, the BEC center has been continuously in touch with the colleagues in Innsbruck, namely with the P.Zoller and co-workers at the Institute for Theoretical Physics and with the group of R.Grimm at the Institute for Experimental Physics at the University of Innsbruck and at the Institute for Quantum Optics and Quantum Information (IQOQI) of the Austrian Academy of Sciences. Regular meetings are organized among these groups both in Trento and Innsbruck. Several experiments in Innsbruck have been stimulated by and/or interpreted with theories developed in Trento.

- **Zürich, ETH**. A collaboration with A. Imamoglu at ETH has recently started about the possibility of realizing a Tonks-Girardeau gas of strongly interacting polaritons and the new properties that one is to expect as a consequence of the non-equilibrium nature of the polariton gas. To this purpose, I. Carusotto will spend the winter semester of the 2008-09 academic year at ETH as a visiting professor.

- **Universität Ulm**, Germany. In the last years, the Trento activity on quantum information has taken great advantage from the collaboration with T. Calarco, a former member of the BEC Center, who got a professorship in Ulm. The activity is mainly based on proposals of realistic quantum gates and entanglement on atom chip.

- **Munich, TUM and LMU** Germany. In the past years we have had a fruitful collaboration with W. Zwerger at Technical University of Munich (TUM) on the study of cold gases in reduced geometries. Very recently we started a new collaboration with Zwerger and the experimental group led by Philipp Treutlein.
at the Ludwig-Maximilians-Universität (LMU) on the measurement of the so-called dissipative Casimir effect on atom chips. Such an effect would reveal in the collective oscillations of atomic clouds. Alessio Recati will spend several months in the group of Zwerger during the academic year 2008-09.


- **Università di Bologna & Universidad de Valencia.** A project on the physics of Hawking radiation from acoustic black holes in atomic BECs has started in early 2006 in collaboration with R. Balbinot in Bologna and his coworker A. Fabbri in Valencia: their expertise in the physics of black holes and quantum fields in curved space-time has been combined with the Trento one on many-body physics to conceive and then perform a first numerical observation of Hawking radiation from acoustic black holes in atomic Bose-Einstein condensates.


- **Amherst, Univ. Massachusetts.** Since the sabbatical leave spent with our group by Prof. Nikolay Prokof’ev in 2006, we have started a fruitful collaboration with him and his group at University of Massachusetts and also with the group of Prof. Matthias Troyer at ETH in Zurich, visited by Nikolay during the year 2007. As a result of this collaboration we have published a Letter on the critical temperature of interacting Bose gases in three and two dimensions and we are presently collaborating on a project concerning the quantitative comparison between mean-field theories and “exact” quantum Monte simulations of bosons at finite temperature.


- **Boulder, JILA and University of Colorado.** The Trento team has fruitful collaborations with the groups at JILA since a long time. In the most recent joint work, the group of Trento developed a theory for the temperature dependence of the Casimir-Polder forces, which was experimentally tested by the group of Eric Cornell.
• *Barcelona, UPC.* The collaboration between the Trento BEC group and the quantum Monte Carlo group at the Universitat Politecnica de Catalunya (UPC) has been a constant thread over the last years. Members of our group have often visited UPC for short periods and Grigory Astrakharchik, a former research associate in Trento and now spending a long post-doc at UPC, comes regularly to Trento collaborating with our group on a project concerning quantum Monte Carlo simulations of ultracold fermions with resonant interactions.


• *Barcelona, IFCO.* The long lasting collaboration with Maciej Lewenstein (till 2005 in Hannover) has been continued after his move to ICFO - The Institute for Photonic Sciences, Barcelona. Chiara Menotti has spent in his group two years in the framework of an individual Marie Curie Fellowship centered on the study of correlated atoms in optical lattices and in particular the quantum phases and metastable states of dipolar atoms. Moreover, a former Diploma student of the Trento group, Christian Trefzger, is presently doing his PhD at ICFO.


Projects and scientific collaborations

- **Columbus**, Ohio State University. A collaboration between Chiara Menotti and Nandini Trivedi has been carried out on the study of the excitations and spatial correlations in the Bose-Hubbard model. A RPA treatment has allowed to identify gapped excitations in the superfluid phase which arise when the Mott transition is approached, and finite range spatial correlations in the Mott phase at finite tunneling.


- **Los Alamos National Laboratory**. Augusto Smerzi has a stable collaboration with the Los Alamos National Laboratory, Los Alamos (New Mexico, USA), namely with the groups of Lee Collins and Wojciech Zurek. The investigated problems are mostly related with the superfluidity properties of BEC and entanglement phenomena.
Publications

The following list includes the papers published or submitted for publication starting from January 2006 having at least one of the authors with the Trento BEC affiliation.

2008


Superfluid pairing between fermions with unequal masses; M.A. Baranov, C. Lobo, G.V. Shlyapnikov, arXiv:0801.1815


Entanglement, Non-linear Dynamics and Heisenberg Limit with Bose-Einstein Condensates; L. Pezzè, A. Smerzi, arXiv:0711.4840

Dynamics of coherent polaritons in double-well systems; D. Sarchi, I. Carusotto, M. Wouters, V. Savona, arXiv:0711.4270


Spatial and spectral shape of inhomogeneous non-equilibrium exciton-polariton condensates; M. Wouters, I. Carusotto, C. Ciuti, cond-mat/0707.1016


Structural phase transitions in low-dimensional ion crystals; Shmuel Fishman, Gabriele De Chiara, Tommaso Calarco, Giovanna Morigi, arXiv:0710.1831

Anderson localization of a non-interacting Bose-Einstein condensate; G. Roati, C. D’Errico, L. Fallani, M. Fattori, C. Fort, M. Zaccanti, G. Modugno, M. Modugno,


*Ultra-cold dipolar gases*; Chiara Menotti, Maciej Lewenstein, Proceedings of the RPMBT14, Barcelona, 2007; Series on Advances in Quantum Many-Body Theory, World Scientific


Theory of ultracold Fermi gases; Stefano Giorgini, Lev P. Pitaevskii, S. Stringari, Rev. Mod. Phys. 80, 1215 (2008)

2007

Sub shot-noise interferometric phase sensitivity with beryllium ions Schrödinger cat states; L. Pezzè, A. Smerzi, Europhys. Lett. 78, 30004 (2007)


Axial breathing mode in rapidly rotating Bose-Einstein condensates and uncertainty of the rotation velocity; Gentaro Watanabe, Phys. Rev. A 76, 031601(R) (2007)


Semiclassical field method for the equilibrium Bose gas and application to thermal vortices in two dimensions; L. Giorgetti, I. Carusotto, Y. Castin, Phys. Rev. A 76, 013613 (2007)


Breathing modes of a fast rotating Fermi gas; Mauro Antezza, Marco Cozzini, S. Stringari, Phys. Rev. A 75, 053609 (2007)


Landau and dynamical instabilities of Bose-Einstein condensates with superfluid


Sub Shot-Noise interferometric phase sensitivity with Beryllium ions Schroedinger Cat States; L. Pezzè and A. Smerzi, Europhysics Letters 78, 30004 (2007)

Structure, superfluidity and quantum melting of hydrogen clusters; Fabio Mezzacapo, Massimo Boninsegni, Phys. Rev. A 75, 033201 (2007)


Attractive Fermi gases with unequal populations in high elongated traps; Giuliano Orso, Phys. Rev. Lett. 98, 070402 (2007)


Controlled collisions of a single atom and an ion guided by movable trapping potentials; Zbigniew Idziaszek, Tommaso Calarco, and Peter Zoller, Phys. Rev. A 76, 033409 (2007)


2006


Quantum fluctuations and collective oscillations of a Bose-Einstein condensate in


Travelling to exotic places with ultracold atoms; M. Lewenstein, A. Kubasiak, J. Larson, C. Menotti, G. Morigi, K. Osterloh, and A. Sanpera, cond-mat/0609587


Density Matrix Renormalization Group for Dummies; G. De Chiara, M. Rizzi, D. Rossini and S. Montangero, cond-mat/0603842

Non-Positive alpha-Rule for Continuous Phase Transition Lines; Nikolay Prokof’ev, Boris Svistunov, cond-mat/0603626

Dark solitons as quasiparticles in trapped condensates; V.A. Brazhnyi, V.V. Konotop, L.P. Pitaevskii, cond-mat/0603197


Kinetic energy of uniform Bose-Einstein condensate; E. E. Nikitin, L. P. Pitaevskii, cond-mat/0508684


Mott-insulator phase of coupled 1D atomic gases in a 2D optical lattice; D.M. Gangardt, P. Pedri, L. Santos, G.V. Shlyapnikov, Phys. Rev. Lett. 96, 040403 (2006)


Talks at workshops and conferences

Sandro Stringari: “Superfluidity of ultracold atomic Fermi gases”, Lausanne (Latsis Symposium, 28-30/1 2008)

Sandro Stringari: “Superfluidity of trapped atomic gases”, Chernogolovka (Landau Memorial Conference (22-25/6 2008)


Sandro Stringari: “Effects of spin polarization in atomic Fermi gases at unitarity”, Orsay (Symposium on Quantum fluids and strongly correlated systems, 15-16 September 2008)


Franco Dalfovo: “Solitons in ultracold atomic gases”, Tel Aviv (Italy-Israel Forum on Science and Technology, April 2008)


Lev Pitaevskii: “Thermal Lifshitz forces”, Frascati (2-nd Italo-Russian meeting, March 14, 2008)

Lev Pitaevskii: “Lecture "Unitary Fermi liquid", Moskow (Meeting of the Physical Department of the Russian Academy of Sciences, January 22, 2008)

Lev Pitaevskii: “Landau and theory of Van der Waals forces”, Moskow (Landau memorial conference, June 20, 2008)

Lev Pitaevskii: “Casimir-Lifshitz forces and entropy”, Paris (Casimir network meeting, November 29, 2008)

Stefano Giorgini: “Polarized Fermi gases”, Copenhagen (NORDITA, Workshop on Ultracold atoms and quark-gluon plasma, June 23-27, 2008)
Stefano Giorgini: “Phase diagram of a polarized Fermi gas at zero temperature”, Trondheim, Norway (Workshop on Laser Physics, June 30 - July 4, 2008)

Stefano Giorgini: “Polarized Fermi gases with resonant interactions”, Paris (Laboratoire de Physique Statistique, ENS, September 23 2008)


Augusto Smerzi: “Quantum Interferometry”, Los Alamos (LANL, USA, August 2008)

Augusto Smerzi: “Entanglement, nonlinearity and Heisenberg limit with Bose-Einstein condensates”, Hannover (Leibniz University, March 2008)


Chiara Menotti, ”Metastable states of dipolar bosons in optical lattices and other dipolar studies at ICFO”, Villetaneuse (“Dipolar quantum gases” one-day meeting, 25 June 2008)

Iacopo Carusotto: “Non-equilibrium superfluidity effects in exciton-polariton systems”, Madrid (Universidad Autó noma, 17 Gennaio 2008)


Talks at workshops and conferences


Iacopo Carusotto: “Numerical observation of Hawking radiation from acoustic black holes in atomic Bose-Einstein condensates”, Ginevra (Université de Genève, 14 Novembre 2008)


Alessio Recati: “Spin-polarized atomic Fermi gases at Unitarity”, Grenoble (Workshop on Theory of Quantum Gases and Quantum Coherence, June 2008)

Alessio Recati: “Polarized Fermi gases” and “Sonic black-hole in 1D Bose-Einstein condensates”, Munich (two invited talks at Physics Department of the Technische Universität, October 2008)

Edward Taylor: “Two-fluid hydrodynamic modes in a strongly interacting Fermi gas” Innsbruck (University of Innsbruck, November 3, 2008)

Edward Taylor: “Spin-polarized Fermi superfluids as Bose-Fermi mixtures”, Erice (Stripes08, July 28, 2008)

Edward Taylor: “Josephson’s other relation: an exact identity for the superfluid density in the BCS-BEC crossover”, Utrecht (Institute for Theoretical Physics, University of Utrecht, The Netherlands, May 23, 2008)

Talks at workshops and conferences

Franco Dalfovo: “Ultracold atoms: overview and perspectives”, IFIC - Instituto de Fisica Corpuscular, Universitat de Valencia (Spain), 25 April 2007

Sandro Stringari: “Superfluidity of ultracold Fermi gases”, Van der Waals-Zeeman Instituut, Amsterdam, 6 February 2007

Sandro Stringari: “Superfluidity of ultracold Fermi gases”, Department of Physics, Utrecht, 7 February 2007

Sandro Stringari: “Strongly interacting Fermi gases in traps”, Garching, Munich, 6 March 2007


Sandro Stringari: “Rotating superfluid Fermi gases”, Workshop on Superfluids under rotation, Jerusalem, 16-19 April 2007

Sandro Stringari: Series of lectures “Dynamics of quantum gases”, Institut Poincaré, Paris, 18-22 June 2007,


Sandro Stringari: Series of lectures “Collective oscillations of superfluid atomic Bose and Fermi gases”, School on Novel Quantum Phases and Non-equilibrium Phenomena in Cold Atomic Gases, ICTP Trieste, 27 August - 7 September 2007,

Sandro Stringari: “Verso lo Zero Assoluto”, Festival della Scienza, Genova, 28 October 2007


Augusto Smerzi: “Precision measurements with Bose-Einstein condensates”, Firenze (LENS, December 2007)

Augusto Smerzi: “Interferometry with trapped Bose-Einstein condensates”, Pisa (ENS, November 2007)

Augusto Smerzi: “Quantum Interferometry”, San Feliu de Guixols, Spain (Frontiers in Quantum Gases, September 15-20, 2007)


Stefano Giorgini: “Recent progress on the microscopic study of strongly interacting fermions with quantum Monte Carlo methods”, Paris (Institute Henri Poincaré Programme on Quantum Gases, July 13 2007)


Lev Pitaevskii: “Non-equilibrium surface-atom Casimir-Lifshitz forces and their measurement in cold atoms experiments”, Orsay, Paris, 8-9 March 2007


Iacopo Carusotto: “Non-equilibrium Bose-Einstein condensation phenomena in microcavity polariton systems”, Aarhus, Denmark, 11 October 2007

Iacopo Carusotto: “Non-equilibrium Bose-Einstein condensation phenomena in microcavity polariton systems”, ”Laser seminar”, ETH Zurich, 1 November 2007


Iacopo Carusotto: “Non-equilibrium Bose-Einstein condensation phenomena in microcavity polariton systems”, Trento-Innsbruck meeting, Innsbruck, 30 November 2007

Alessio Recati: “The physics of Cold Gases in 1D”, Houston, Texas (Physics Department at the RICE University, May 2007)


Alessio Recati: “Bose-Einstein Condensation”, Palermo (Physics Department of the Palermo University, December 2007)


Mauro Antezza: “Vortices and collective oscillations in superfluid atomic gases” and ”Surface-atom interaction out of thermal equilibrium: theory and recent experimental results”, Lectures at the PhD School, Department of Physics, University of Pavia, 9 March 2007


Gabriele De Chiara: “Optimal control of single atom motion in optical lattices”, QuantenInformationsVerarbeitung, Universitaet Ulm, 16 May 2007

Gabriele De Chiara: “Optimized atom transport for quantum gates in optical lattices”, Laser Cooling and Trapping Group, NIST Gaithersburg, USA, 2 August 2007


Sebastiano Pilati: “Phase separation in a polarized Fermi gas at zero temperature”, Joint Meeting Innsbruck-Trento on Ultracold Bose and Fermi Gases, Innsbruck, 30 November 2007

Luca Pezzè: Talk at the Conference on ”Problemi Attuali di Fisica Teorica”, Vietri sul Mare, 30 March - 4 April 2007,

Luca Pezzè: Talk at Institut d’Optique, Paliseau, France, June 23th 2007

Luca Pezzè: Talk at Summer School on Quantum Optics and Molecular Physics, Casper, WY, USA, 15-21 July 2007

Luca Pezzè: Talk at Los Alamos National Laboratory, Theoretical Division, Los Alamos, USA, 7 August 2007

Luca Pezzè: “Entanglement, Non-Linear Dynamics and Heisenberg Limit with Bose-Einstein Condensates”, Joint Meeting Innsbruck-Trento on Ultracold Bose and Fermi Gases, Innsbruck, 30 November 2007

Chiara Menotti: “Dynamics and thermodynamics of systems with long range interactions: theory and experiments”, Domus Pacis, Assisi, 4-8 July 2007
Ingrid Bausmerth: “Breaking resonant superfluidity at T=0 by rotation”, Joint Meeting Innsbruck-Trento on Ultracold Bose and Fermi Gases, Innsbruck, 30 November 2007


Sandro Stringari: “Test of the Casimir-Polder force with ultracold atomic gases”, Boulder (JILA Physics Department, 23 January 2006)

Sandro Stringari: “Test of the Casimir-Polder force with ultracold atomic gases”, Gaithersburg (NIST, 25 January)

Sandro Stringari: “Verso lo zero assoluto”, Parma (Scientiae Munus, 31 January 2006)

Sandro Stringari: “Superfluidity of ultracold quantum gases”, Barcelona (Facultat de fisica, 14 February 2006)

Sandro Stringari: “Molecular signature in the structure factor of an interacting Fermi gas”, Barcelona (Institute of Photonic Sciences) 15 February 2006

Sandro Stringari: “Fisica: Perche’?”, Udine (Universita’ Popolare, 23 February 2006)

Sandro Stringari: “Le due culture (dalla parte della fisica)”, Trento (Saperi e linguaggi a confronto, 2 March 2006)

Sandro Stringari: “Effects of spin polarization in a superfluid Fermi gas”, Bad Honnef (Workshop on Qubits and Macroscopic Quantum Coherence, 7-11 May 2006)

Sandro Stringari: “Superfluidity of ultracold quantum gases”, Milano (Dipartimento di Fisica, 16 May 2006)


Sandro Stringari: “Superfluidity of ultracold quantum gases”, Varenna (11th Int. Conf. on nuclear reaction mechanisms, 12-16 June 2006)


Sandro Stringari: “Dynamical and superfluid effects in Bose-Einstein condensates”, Vienna (School on Gross-Pitaevskii equations for superfluids and Bose-Einstein condensates, 18-22 September 2006)

Sandro Stringari: “Dynamical Behavior of Bosons and Fermions”, Beijing (Workshop on Current Development in Quantum Gases 2-8 December 2006)

Stefano Giorgini: “Fermi gas in the BCS-BEC crossover: a quantum Monte Carlo study”, Innsbruck (20th International Conference on Atomic Physics, ICAP 2006, 16-21 July)


Mauro Antezza: “New asymptotic behaviour of the Casimir surface-atom force out of thermal equilibrium”, Pavia (lectures at the Ph.D. school of the University of Pavia, 6 April 2006)


Mauro Antezza: “Thermal effects of the Casimir force for surface-atom and surface-surface configurations”, Bertinoro (QMFPA 2006 - Quantum Mechanics: from fundamental problems to applications, 4-7 December 2006)

Mauro Antezza: “Surface-atom force out of thermal equilibrium and its effect on cold atoms”, Paris (YAO06 - Young Atom Opticians, 14-18 February 2006)


Augusto Smerzi: talk at “Quantum Mechanics: from fundamental problems to applications”, Bertinoro, Italy, 2006


Augusto Smerzi: “Quantum Interferometry”, Baton Rouge (Louisiana, 2006)

Alessio Recati: “Polarized Superfluid Fermi gas at Unitarity”, Levico (Conferenza di Fisica Teorica e Struttura della Materia, September 2006)

Alessio Recati: “Polarized Superfluid Fermi gas at Unitarity”, Barcelona (Physics Department, Universitat Autonoma, November 2006)


Visitors

Long term visitors

Nikolai Prokof’ev (Univ. Massachusetts, Amherst, USA), Dec. 2005 - Aug. 2006

Massimo Boninsegni (Univ. Alberta, Edmonton, Canada), Sept. 2006 - Jan. 2007

Dörte Blume (Washington State Univ., Pullman, USA), July 2006 - Dec. 2006


Short term visitors

Sebastiano Anderloni (INFN, Trieste, Italy), May 26-28, 2008

Luca Pezzè (LCFIO, Palaiseau, France), May 13-20, 2008

Michiel Wouters (EPFL, Lausanne, Switzerland), May 4-11, 2008

Grigory E. Astrakharchik (UPC, Barcelona), April 17-24, 2008

Dario Gerace (ETHZ, Switzerland, and Univ. Pavia), April 13-18, 2008

William D. Phillips (NIST, USA), April 2, 2008

Tung-Lam Dao (Polytechnique, Paris), April 1-19, 2008

Thierry Giamarchi (DPMC-MaNEP, University of Geneve), March 4, 2008

Nicolai Prokof’ev (Univ. Massachusetts, Amherst), February 29, 2008

Markus Cirone (Universit di Palermo, Palermo), February 26-28, 2008


Kumar Tapan Das (Calcutta University, India), February 15-29, 2008

Nicolas Pavloff (LPTMS, Orsay, France), February 6-9, 2008

Paul Eastham (Imperial College, London), January 28 - February 1, 2008

Alain Aspect (LCFIO and Univ. Paris-Sud), November 21, 2007
Visitors

Michal Kolar (Palacky University), November 5-8, 2007
Anna Posazhennikova (Karlsruhe University), October 16-18, 2007
Boris A. Malomed (Tel Aviv University), October 1st, 2007
Petra Scudo (SISSA, Trieste), September 24-29, 2007
Carlos Sa de Melo (Univ. Maryland/NIST, USA), September 11, 2007
Serena Fagnocchi (Bologna University), August 1-3, 2007
Shunji Tsuchiya (Keio University, Tokyo, Japan), July 23-26, 2007
Jean-Noel Fuchs (LPS, University Paris Sud, France), July 9-11, 2007
Shyamal Biswas (Dept. of Theor. Phys., Jadavpur, India), July 6, 2007
Francesco Buonsante (Politecnico di Torino), May 21, 2007
Giuliano Benenti (Univ. dell’Insubria, Como), May 17, 2007
Sergio Gaudio (Univ. la Sapienza, Roma), May 14, 2007
Francesco Nesi (Inst. Theoretical Physics, Regensburg), April 26, 2007
Joe Carlson (Los Alamos, USA), March 26, 2007
Yusuke Nishida (Tokyo University, Japan), March 26, 2007
Chris Pethick (NORDITA, Copenhagen, Denmark), March 26, 2007
Biagio Pieri (Camerino, Macerata), March 26-28, 2007
Randall Hulet (Rice University), March 21, 2007
Ippei Danshita (NIST, Gaithersburg, USA), March 14, 2007
Davide Sarchi (EPFL, Lausanne, Swiss), February 26 - March 9, 2007
Oliver Morsch (Univ. Pisa), February 19, 2007
Markus Oberthaler (Univ. Heidelberg, Germany), February 13, 2007
Guido Pupillo (ITP/QIQ, Innsbruck, Austria), February 9, 2007
Grigory E. Astrakharchik (UPC, Barcelona, Spain), Jan 22 - Feb 27, 2007

Maria Luisa Chiofalo (SNS, Pisa), January 22-25, 2007

Roland Combescot (LPS, ENS, Paris), January 11-12, 2007

Simone Montagero (SNS, Pisa), January 8-10, 2007

Andrea Trombettoni (SISSA, Trieste), December 18-21, 2006

Frederic Chevy (ENS, Paris, France), December 18-19, 2006

Roberto Onofrio (Dartmouth College and Univ. Padova), Dec 18, 2006

Grimm’s group (Inst. fuer Exp. Physik, Innsbruck), December 15, 2006

M.Inguscio, G. Roati, G. Modugno (LENS, Firenze), November 13-14, 2006

Flavio Toigo and Luca Salasnich (Univ. Padova), November 13, 2006

Eric Cornell (JILA/NIST, Boulder, USA), November 12-14, 2006

Ryan Kalas (Washington State University), November 6-9, 2006

Vittorio Giovanetti (SNS, Pisa), October 30-31, 2006

Serena Fagnocchi (Univ. Bologna), October 26-27, 2006

Sergey V. Iordanski (Landau Inst. Theor. Physics, Moskow), October 9-10, 2006

Grigory Astrakharchik (UPC, Barcelona), October 2-7, 2006

Raoul Santachiara (Univ. Strasbourg and ENS-Paris), October 1-2, 2006

Kostas Sakkos (UPC, Barcelona), September 24-30, 2006

Matteo Rizzi (SNS, Pisa), September 20-22, 2006

Francesca Maria Marchetti (Univ. Cambridge, UK), September 20-21, 2006

Simone De Liberato (ENS, Paris), September 20-21, 2006

René Stock (Univ. Calgary), June 26 - July 15, 2006
Events and outreach

Conferences organized by the BEC Center

Fourth International Workshop on
Theory of Quantum Gases
and Quantum Coherence
Grenoble, June 3-7, 2008

This is the fourth in a series of workshops dedicated to the theoretical challenges in the area of quantum gases (including strongly correlated systems, disorder, spinors), with a strong connection to condensed matter physics. In this conference the speakers and the participants are to mainly young researchers from Europe and overseas, with some overview lectures delivered by leading Senior Scientists. The BEC center has co-organized the series of workshops (Salerno 2001, Levico 2003, Cortona 2005, and Grenoble 2008). The Grenoble edition has been organized by R. Citro (LPMMC, Grenoble and University of Salerno), C. Menotti (BEC, Trento), A. Minguzzi (LPMMC, Grenoble), and P. Vignolo (INLN, Nice).

International Workshop on
Coherence, Squeezing and Entanglement
for Precision Measurements
with Quantum Gases
Levico (Trento), 3-5 April, 2008

In the last few years we have witnessed crucial theoretical and experimental advances in our understanding and control of cold and degenerate gases. We are now at the verge
of exploiting the quantum nature of these novel systems to develop new concepts and technologies for ultra-sensitive measurements. This new generation of devices will be characterized by long coherence times and lengths, accurate quantum states tailoring and, in a broader perspective, by the possibility to create and exploit entanglement for Heisenberg limited interferometry. This conference is among the first to focus on precision measurements with degenerate atomic gases. Organized by: A. Smerzi (chair), A. Aspect, M. Inguscio, W. Phillips, S. Stringari.

Workshop on
Quantum Monte Carlo Methods
Sardagna, Trento, 2 December 2006

This small workshop was intended to bring together the Italian scientific community working in different fields of theoretical physics and using quantum Monte Carlo methods as a common numerical technique. The topics covered various aspects of solid state physics, condensed matter physics, quantum liquids, quantum chemistry and biophysics. The number of participants was about 30 people, mainly young researchers at the level of PhD students or young research associates. The programme included 7 oral contributions and a poster session.

Group meetings and seminars at the BEC Center

2008

Monday, September 22nd 2008 at 15.00
Informal meeting with Sandro Stringari
*Density and spin response in a polarized Fermi gas at unitarity*

Friday, July 17th 2008 at 11.00
Informal meeting with Ed Taylor
*Spin-polarized Fermi superfluids as Bose-Fermi mixtures*

Thursday, July 17th 2008 at 11.00
Christian Schunck (MIT-Harvard Center for Ultracold Atoms)
Events and outreach

*Resonant spin-changing collisions in spinor Fermi gases*

Friday, June 6th 2008 at 11.00
Philipp Hyllus (University of Hannover)

*Resonant spin-changing collisions in spinor Fermi gases*

Tuesday, June 3rd 2008 at 16.00
David Clement (LENS, Florence)

*Localization phenomena in disordered media: recent experiments with gaseous matter-waves*

Monday, May 26th 2008 at 16.00
Sebastiano Anderloni (Università di Trieste)

*Quantum measuring processes for trapped ultracold gases*

Friday, May 16th 2008 at 11.00
Informal meeting with Georg Bruun and Ed Taylor

*Recent work on superfluid dipolar Fermi gases*

Tuesday, May 6th 2008 at 16.00
Michiel Wouters (EPFL, Lausanne, Switzerland)

*Mean field theory for exciton-polariton condensates*

Tuesday, April 22nd, 2008 at 16:00
Grigory E. Astrakharchik (Universitat Politenica de Catalunya)

*Quasi-condensation of pairs of attractive Fermions in one-dimensional geometry*

Thursday, April 17th 2008 at 15.00, Aula Seminari
Francesco Bariani (CNR-INFM BEC, Trento),
Tung-Lam Dao (Polytechnique, Paris),
Dario Gerace (ETHZ and Univ. PV)

*Brave young opticians: new developments in optics at BEC*

Thursday, April 10th, 2008 at 15:30
Francesco Piazza (CNR-INFM BEC, Trento)

*Phase Cat States with Bose-Einstein Condensates*

Wednesday, April 2nd, 2008 at 14:30
William D. Phillips (NIST)

*Spinning atoms with light: a new twist on coherent deBroglie-wave optics*

Monday, March 31st at 15.00, Aula Seminari
Informal discussion with Ed Taylor

*News from the APS meeting*

Monday, March 17th at 15.00, Aula Seminari
Informal discussion with Georg Bruun

*Collisional properties of a Fermi gas in the extreme imbalanced limit*

Friday, March 7th at 11.00, Aula Seminari
Informal discussion with Iacopo Carusotto

*Hawking radiation from acoustic black holes*

Tuesday, March 4th, 2008 at 16:00, Aula Seminari
Thierry Giamarchi (DPMC-MaNEP, University of Geneva)

*Quasi-one dimensional physics in cold atomic gases*

Thursday, February 28th at 16:30, Aula 21
Sebastiano Pilati (CNR BEC-INFM)

*Studies of ultracold gases using quantum Monte Carlo Techniques*

Tuesday, February 26th at 15.00, Aula n. 20
Kumar Das (University of Calcutta)

*Research at the Department of Physics of the University of Calcutta*

Friday, February 22nd, 2008 at 09:30, Aula Seminari
Christophe Salomon (Laboratoire Kastler Brossel, ENS)

*Informal discussions*

Tuesday, February 19th, 2008 at 11:15, Aula Seminari
Tapan Kumar Das (University of Calcutta)

*Hyperspherical many-body approach for Bose-Einstein condensation*

Friday, February 15th, 2008 at 16:00, Aula Seminari
Edward Taylor (BEC CNR-INFM)

*Josephson’s other relation: an exact identity for the superfluid density in the BCS-BEC crossover*

Tuesday, February 12th, 2008 at 16:00, Aula Seminari
Sebastiano Pilati (BEC CNR-INFM)

*Phase separation in a polarized Fermi gas at zero temperature*

Thursday, February 7th, 2008 at 17:00, Aula Seminari
Nicolas Pavloff (LPTMS, Orsay, France)
**Superfluidity versus Anderson localization in a dilute Bose gas**

Tuesday, January 29th, 2008 at 16:00, Aula Seminari
Paul Eastham (Imperial College London)

*Beyond Bose Condensation of Polaritons: Non-Equilibrium Quantum Condensation without Relaxation in a Semiconductor Microcavity*

Tuesday, January 22nd, 2008 at 16:00, Aula Seminari
Georg M. Bruun (BEC CNR-INFM and Niels Bohr Institute)

*Magnetic and superfluid phases of confined fermions in two-dimensional optical lattices*

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**2007**

Friday, December 7th, 2007 at 16:00, Aula Seminari
Ed Taylor (BEC CNR-INFM)

*Two-fluid hydrodynamic modes in trapped Fermi superfluids at unitarity*

Wednesday, November 21st, 2007 at 14:30, Aula 21
Alain Aspect (LCFIO and Univ. Paris-Sud)

*From Einstein intuitions to quantum bits: a new quantum age?*

Tuesday, November 13th, 2007 at 17:00, Aula seminar
Georg M. Bruun (Niels Bohr Institute)

*Problems in Cold Fermi Gases*

Tuesday, November 6th, 2007 at 16:00, Aula seminar
Michal Kolar (Palacky University)

*Orbital angular momentum single photon interferometry with which-path detection*

Wednesday, October 17th, 2007 at 16:00, Aula seminar
Anna Posazhennikova (Karlsruhe University)

"Shuttle" for cold atoms: atomic quantum dot inside Bose Josephson junction

Monday, October 1st, 2007 at 16:00, Aula seminar
Boris A. Malomed (Tel Aviv University)

*Localized states in a superfluid fermion gas in optical lattices*

Thursday, September 25th, 2007 at 16:00, Aula seminar
Petra Scudo (SISSA)

*Entanglement in the quantum Ising model*

Thursday, September 11th, 2007 at 16:00, Aula seminar
Carlos A. R. Sa de Melo (JQI, University of Maryland/NIST)

*Mixtures of Ultra-cold Fermions with Unequal Masses*

Thursday, September 6th, 2007 at 15:00, Aula seminari

Nir Barnea (The Hebrew University - Jerusalem)

*The Fermi Gas in Dynamic Mean Field Approximation*

Wednesday, July 11th, 2007 at 11:00 a.m., Aula seminari

Jean-Noel Fuchs (LPS, University Paris Sud, France)

*Introduction to Graphene*

Friday, July 6th, 2007 at 16:00, Aula seminari

Shyamal Biswas (Department of Theoretical Physics, Jadavpur, India)

*Temperature Dependence of Critical Number of Particles for Attracting Atomic Bose Gas*

Thursday, May 31th, 2007 at 16:00, Aula seminari

Francesco Piazza (INFM-CNR BEC)

*On the Importance to be Odd*

Monday, May 21st, 2007 at 16.00, Aula Seminari

Francesco Buonsante (Dipartimento di Fisica, Politecnico di Torino)

*Ground-State Fidelity and Bipartite Entanglement in the Bose-Hubbard Model*

Thursday, May 17th, 2007 at 15.00, Aula Seminari

Giuliano Benenti (Universita’ dell’ Insubria, Como)

*Quantum ratchets for periodically kicked cold atoms and Bose-Einstein condensates*

Monday, May 14th, 2007 at 16.00, Aula Seminari

Sergio Gaudio (Univerista’ la Sapienza, Roma)

*Fermi liquid behavior in two component Fermi systems close to a diverging scattering length*

Thursday, May 10th, 2007 at 16.00, Aula Seminari

Gabriele De Chiara (CNR-INFM BEC, Trento)

*Optimal control of single atom motion in optical lattices*

Monday, May 7th, 2007 at 16.00, Aula Seminari

Gentaro Watanabe (CNR-INFM BEC, Trento)

*Vortices and Vortex Lattices in Trapped Cold Bose Gases*

Thursday, April 26th, 2007 at 16.00, Aula Seminari
Francesco Nesi (Institut for Theoretical Physics, Regensburg)
Spin-boson dynamics: A unified approach from weak to strong coupling
Friday, April 13th, 2007 at 16.00, Aula Seminari

Francesco Bariani (BEC CNR-INFM Trento)
Radiation-matter interaction in Mott insulator phase: a fully quantum approach
Wednesday, March 28th at 15:00, Aula seminari
Informal meeting with Piero Pieri
Monday, March 26th at 14:30, Aula seminari
Informal meeting with Chris Pethick
Wednesday, March 21, 2007 at 15:00, Aula 21, Joint Colloquium
Randy Hulet (Rice University)
Fermion Pairing with Ultracold Atoms
Wednesday, March 14th, 2007 at 16.00, Aula Seminari
Ippei Danshita (NIST, Gaithersburg, USA)
Superfluid-to-Mott insulator transition of ultracold bosons in double-well optical lattices
Monday, March 5th, 2007 at 16.00, Aula Seminari
Davide Sarchi (EPFL, Lausanne)
Bose-Einstein condensation of microcavity polaritons
Monday, February 19th, 2007 at 16.00, Aula Seminari
Olivier Morsch (University of Pisa)
Resonantly enhanced tunneling in optical lattices
Tuesday, February 13th, 2007, at 14.30, Aula Seminari
Informal Meeting with Markus Oberthaler
Tuesday, February 13th, 2007, at 10.30, Aula 21
Luca Pezzè (PhD defense)
Quantum Interferometry
Friday, February 9th, 2007, at 16.00, Aula Seminari
Guido Pupillo (University of Innsbruck)
2D Self-Assembled Crystals with Polar Molecules: Shaping the Interaction Potentials
Monday, January 22th, 2007, at 16.00, Aula Seminari
Maria Luisa Chiofalo (SNS, Pisa)
Evidence of Luttinger liquid behavior in one-dimensional dipolar quantum gases
Monday, January 12th, 2007, at 16.00, Aula Seminari
Simone Montangero (SNS, Pisa)

*Entanglement Entropy dynamics in Heisenberg chains*

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2006

Wednesday, December 20th, 2006, at 16.00, Aula Seminari
Andrea Trombettoni (SISSA, Trieste)

*Inhomogeneous Networks of Bosonic and Superconducting Josephson Junctions*

Monday, December 18th, 2006, at 15.00, Aula Seminari
Roberto Onofrio (Dartmouth College, NH, and University of Padova)

*Ultracold Fermi-Bose mixtures in light-assisted magnetic traps*

Monday, December 4th, 2006, at 15.45, Aula Seminari
Massimo Boninsegni (University of Alberta)

*Superfluidity and Quantum Melting of para-Hydrogen clusters*

Monday, November 27th, 2006, at 15.45, Aula Seminari
Sebastiano Pilati

*Path-integral Monte-Carlo study of an interacting Bose gas*

Tuesday, November 14th, 2006, at 15:00, Aula 21
Eric Cornell (JILA-NIST Boulder, Colorado)

*Colloquium: Progress towards an improved limit on the electron electric dipole moment using trapped molecules*

Monday, November 13th, 2006, at 15:00, Aula 20
Mauro Antezza (CNR-INFM-BEC)

*PhD Defense: Thermal dependence of the Casimir-Polder-Lifshitz force and its effect on ultracold gases*

Monday 6th November, 2006, at 15:45, Aula Seminari
Ryan Kalas (Washington State University)

*Dilute Bose and Fermi gases with large generalized scattering lengths*

Monday 30th October, 2006, at 17:00, Aula Seminari
Vittorio Giovannetti (Scuola Normale Superiore, Pisa)

*Quantum limits in parameter estimation*

Monday 23th October, 2006, at 15.45, Aula Seminari
Doerte Blume (Washington State University)
Events and outreach

Two- and three-dimensional bosonic clusters near threshold
Monday 16th October, 2006, at 15.45, Aula Seminari
Massimo Boninsegni (University of Alberta, Canada)

Phases of Hard Core Bosons in Optical Lattices
Monday 16th October, 2006, at 11.00, Aula Seminari
Francesco Bariani

Struttura delle bande fotoniche in un sistema di atomi freddi intrappolati in un reticolo ottico
Monday 9th October, 2006, at 15.45, Aula Seminari
Sergey V. Iordanski (Landau Institute for Theoretical Physics, Moskow)

On the relaxation of order parameter in BCS model
Thursday 5th October, 2006, Aula Seminari
Sebastiano Pilati, PhD seminar

Weakly Interacting Bose Gas: PIMC and Worm Algorithm Studies
Monday 2nd October, 2006, at 15.45, Aula Seminari
Grigori Astrakharchik (Universitat Politecnica de Catalunya, Barcelona)

Quantum phase transition in a 2D system of dipoles
Doerte Blume (Washington State University)

Two- and many-body physics of interacting dipoles
Monday 11th September, 2006, at 17.00, Aula Seminari
Alessio Recati

Some recent results obtained for "polarized" ultra-cold Fermi gases unitarity
Monday 3rd July, 2006, at 15.00, Aula Seminari
René Stock (Institute for Quantum Information Science, Univ. Calgary)

How to Sort out the Pseudopotential Mess - Generalizing Pseudopotentials Beyond s-wave Interactions
Tuesday 6th June, 2006, at 15.00, ITC-Irst, Main Conference Hall
Shunri Oda (Tokyo Institute of Technology)

Nano-Silicon for Novel Quantum Dot Based Electronic and Photonic Devices
Monday 8th May, 2006, at 11.00, Aula Seminari
Christophe Salomon (Laboratoire Kastler-Brossel, ENS, Paris)
Events and outreach

*Experiments in the BEC-BCS Crossover with Lithium Atoms*

Friday 5th May, 2006, at 11.00, Aula Seminari
Raffaella Burioni (University of Parma)

*Quantum Particles on Inhomogeneous Networks*

Thursday 4th May, 2006, at 11.00, Aula Seminari
Gershon Kurizki (Weizmann Institute of Science, Israel)

*Can we protect Quantum Information from Decoherence?*

Friday 28th April, 2006, at 15.00, Aula Seminari
Vukics Andras (Institut fuer Teoretische Physik)

*Cooling and Trapping of Atoms in Optical Resonators*

Thursday 20th April, 2006, at 11.00, Aula Seminari, Talk by Ingrid Bausmerth (University of Karlsruhe)

*Atomic Quantum Dot Coupled to Trapped Bose Einstein Condensates*

Monday 10th April, 2006, at 10.00, Aula Seminari
Raul Santachiara (LPT Strasbourg)

*Increasing of Entanglement Entropy from Pure to Random Quantum Critical Chains*

Friday 7th April, 2006, at 15.00, Aula Seminari Matematici
Group meeting: Chiara Menotti talks about her recent activities in Barcelona

Wednesday 5th April, 2006, at 15.00, Aula Seminari
Francesco Riboli (University of Trento)

*Optical Modes Induced Force between Coupled Waveguides*

Monday 3rd April, 2006, at 10.00, Aula Seminari
Giacomo Roati (LENS, University of Florence)

*Measurement of the Thermal Casimir-Polder Force with Degenerate Bosons*

Monday 6th March, 2006, at 15.00, Aula Seminari
Joaquin E. Drut (University of Washington)

*Spin 1/2 Fermions in the Unitary Regime at Finite Temperature*

Monday 27th Feb., 2006, at 15.00, Aula Seminari
Nikolai Prokof'ev (University of Massachusetts, Amherst, USA)

*Worm Algorithm for Path-Integral Monte Carlo and new results for the supersolid He-4*

Tuesday, 21st Feb., 2006, at 15.00 (Room 21),
*Joint Colloquium*, Lev P. Pitaevsky
Surface-atom Casimir force out of thermal equilibrium

Monday, 20th Feb., 2006, at 15:00, Aula Seminari
Nikolai Prokof’ev (University of Massachusetts, Amherst, USA)

Worm Algorithms for classical and quantum models

Friday 17th Feb., 2006, at 10.00 (Room 21)
Nathan L. Harshman (American University Washington, DC, USA)

Dynamical Entanglement in Particle Systems

Tuesday 14th Feb., 2006, at 17.00, Aula Seminari
Michiel Wouters presents some concepts of the
theory of pattern formation

Monday, 13th Feb., 2006, at 15:00, Aula Seminari
Nikolai Prokof’ev (University of Massachusetts, Amherst, USA)

Diagrammatic quantum Monte Carlo: from polarons to fermions with contact interactions

Thursday 9th Feb., 2006, at 15:00, ECT* Seminar (Villa Tambosi)
Sebastian Diehl (Nuclear University of Heidelberg)

Universality in the BCS - BEC: Crossover in Cold Fermion Gases

Monday, 6th Feb., 2006, at 15:00, Aula Seminari
Nikolai Prokof’ev (University of Massachusetts, Amherst, USA) on

Quasicondensate mean-field for the weakly interacting Bose gas

Monday 30th January, 2006, at 15:00, Aula Seminari
Nikolai Prokof’ev (University of Massachusetts, Amherst, USA) on

Weak first-order superfluid-solid quantum transitions: is deconfined criticality a self-consistent theory?

Monday 23rd January, 2006, at 15:00, Aula Seminari
Gabriele De Chiara

Density Matrix Renormalization Group: introduction and two applications in condensed matter physics
Education and training

The BEC Center has contributed to the PhD programme of the Physics Department of the University of Trento, by funding several fellowships. Several students are presently preparing their doctoral thesis under the supervision of members of the BEC center and/or in the framework of international collaborations.