Collapse and expansion of a Bose-Fermi mixture

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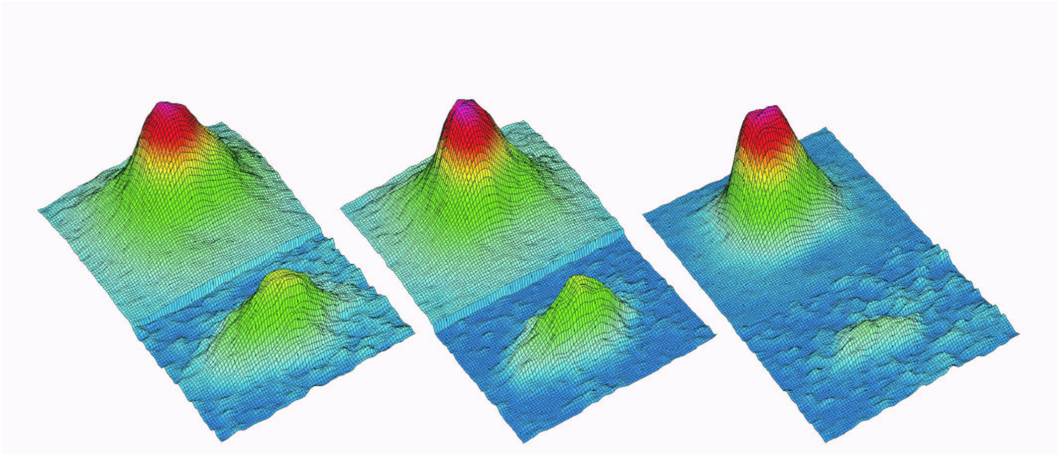
Motivations

- **Experiments with ^{40}K-^{87}Rb mixtures @ LENS:**



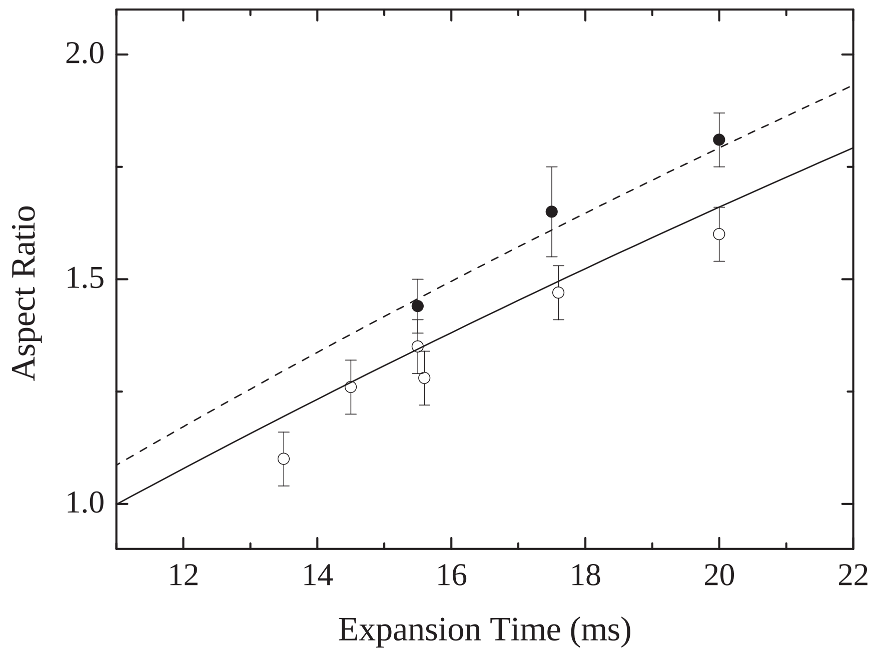
☞ **Attractive Fermi-Bose interaction**

[A. Simoni, F. Ferlaino, G. Roati, G. Modugno, and M. Inguscio, cond-mat/0301159]

$$\mathbf{a_{bf} \simeq -410 \pm 80\ a_0} \qquad (9/2, 9/2) \times (2, 2)$$

☞ Effects of a_{bf} on the **expansion of bosons**

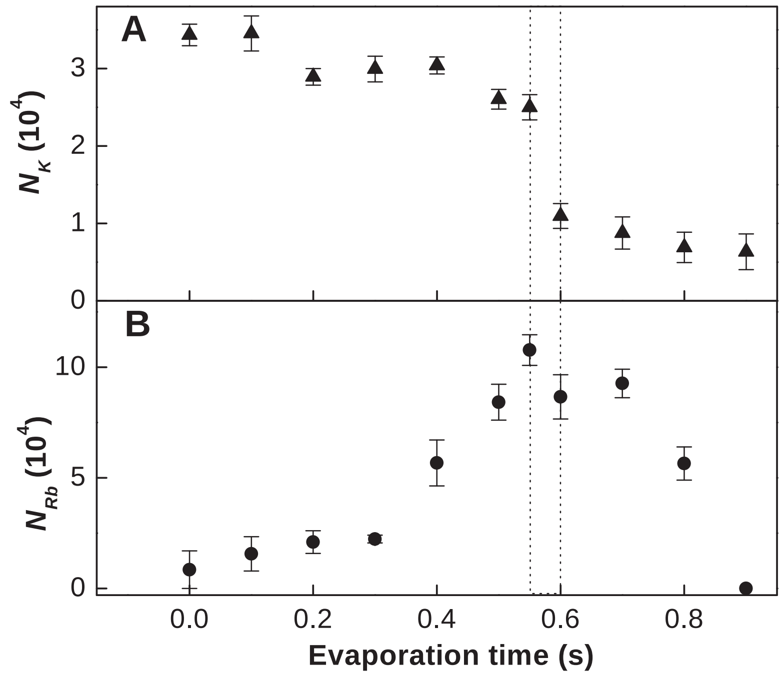
[G. Roati, F. Riboli, G. Modugno, and M. Inguscio, Phys. Rev. Lett. **89**, 150403 (2002)]



Boson aspect ratio during the expansion

☞ Attractive interaction $\Longrightarrow$ **Collapse**

[G. Modugno, G. Roati, F. Riboli, G. Modugno, F. Ferlaino, R. J. Brecha, and M. Inguscio, Science **297**, 2240 (2002)]



Evolution of the number of atoms in the mixture during the evaporative cooling.

Outline of the talk

☞ **Mean-field analysis of**:

✗ **stability and collapse** of the mixture
with: Exp. group @ LENS

☞ **estimate of a_{bf}**

✗ effects of the interaction on the **expansion of fermions and bosons**
with: H. Hu (Post-Doc @ ICTP, Trieste) and X.-J. Liu (Post-Doc @ LENS)
cond-mat/0301182

☞ **scaling approach**

System geometry

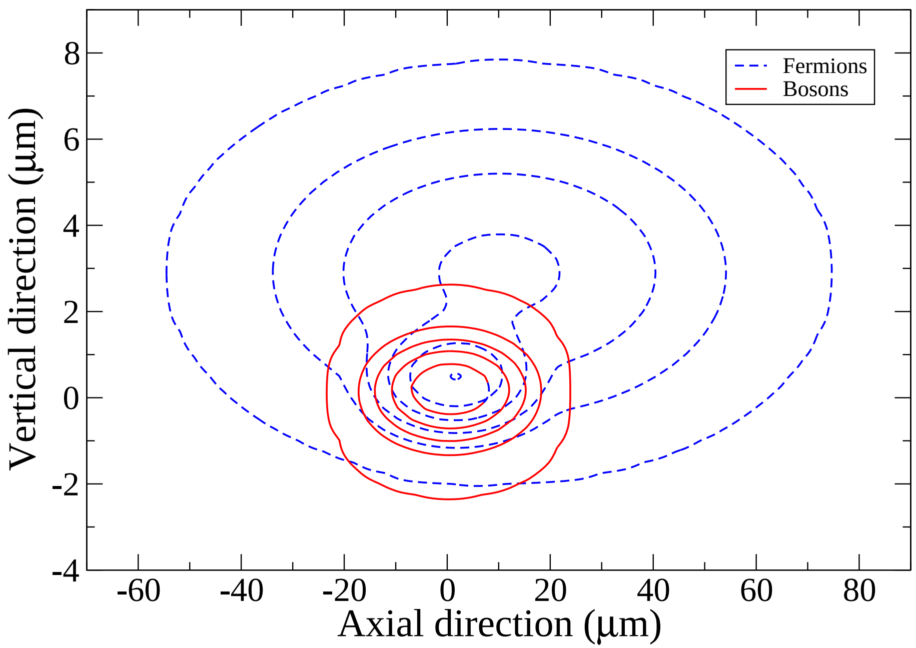
☞ Trapping potentials:

$$V_b(\mathbf{x}) = \frac{1}{2} m_b \omega_{b\perp}^2 \left[(x^2 + y^2) + \lambda^2 z^2 \right]$$
$$V_f(\mathbf{x}) = \frac{1}{2} m_f \omega_{f\perp}^2 \left[(x^2 + (y - y_0)^2) + \lambda^2 (z - z_0)^2 \right]$$

☞ $\omega_{f\perp} = 2\pi \times 317$ Hz, $\lambda^{-1} \simeq 13.2$

$\omega_{b\perp}$ a factor $\sqrt{m_b/m_f} \simeq 1.47$ smaller

✗ horizontal and vertical **gravitational sag:** $y_0 \simeq 3$ μ**m** and $z_0 \simeq 10$ μ**m**



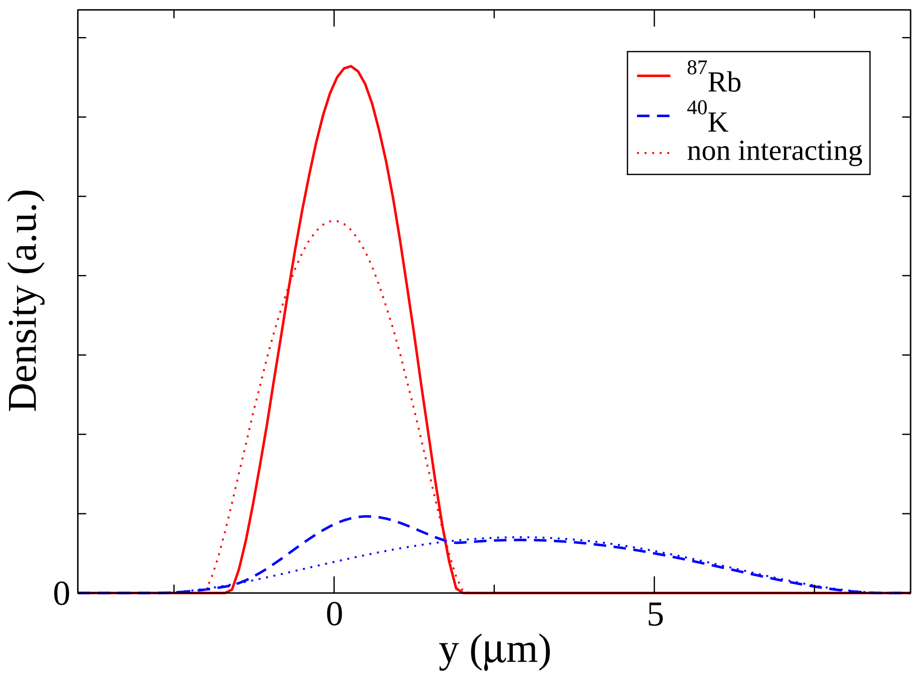
Typical geometry in the experiments @ LENS

A mean-field approach

☞ **GPE for bosons, Thomas-Fermi for fermions**

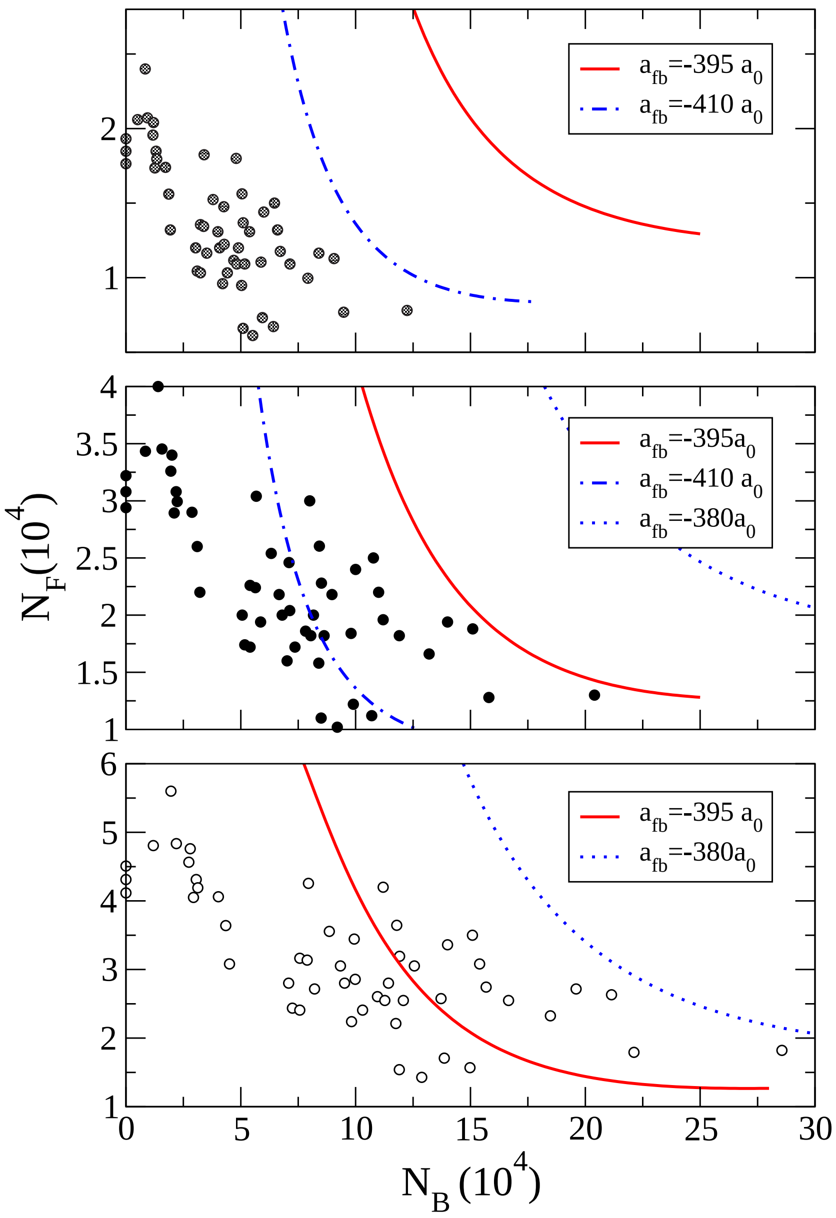
[R. Roth, Phys. Rev. A **66**, 013614 (2002): spherical symmetry, effective confinement]

$$\begin{cases} \left[-\frac{\hbar^2}{2m_b}\nabla^2 + V_b + g_{bb}n_b + g_{bf}n_f \right] \phi = \mu_b \phi \\ n_f = \frac{(2m_f)^{3/2}}{6\pi^2\hbar^3} (\epsilon_f - V_f - g_{bf}n_b)^{3/2} \end{cases}$$



Ground-state of the mixture: density profiles along the vertical y direction.

Stability and collapse



a_{fb}=-395 a_0
a_{fb}=-410 a_0
a_{fb}=-395a_0
a_{fb}=-410 a_0
a_{fb}=-380a_0
a_{fb}=-395 a_0
a_{fb}=-380a_0
$N_F(10^4)$
$N_B\,(10^4)$

☞ Mean-field estimate for a_{bf}

$$a_{bf}^{mf} \simeq -395 \pm 15 \; a_0$$

in agreement with $a_{bf}^{coll} \simeq -410 \pm 80 \; a_0$

☞ Effects of 3D geometry: 3% **correction** to the **"effective spherical" value of** a_{bf}^{mf}

☞ **Beyond mean-field........**

[A. Albus, F. Illuminati, M. Wilkens, `cond-mat/0211060`]:

exchange-correlations $\sim +a_{bf}^{2}$

Expansion

☞ BEC: Thomas-Fermi hydrodynamic equations

$$\begin{cases} \dfrac{\partial n_b}{\partial t} + \nabla \cdot (n_b \mathbf{v}_b) = 0 \\ m_b \dfrac{\partial \mathbf{v}_b}{\partial t} + \nabla \left(\dfrac{1}{2} m_b \mathbf{v}_b^2 + V_b + g_{bb} n_b + g_{bf} n_f \right) = 0 \end{cases}$$

☞ Fermi gas: Boltzmann-Vlasov kinetic equation

$$\frac{\partial f}{\partial t} + \mathbf{v}_f \cdot \frac{\partial f}{\partial \mathbf{r}} - \frac{1}{m_f} \frac{\partial V_f}{\partial \mathbf{r}} \cdot \frac{\partial f}{\partial \mathbf{v}_f} - \frac{g_{bf}}{m_f} \frac{\partial n_b}{\partial \mathbf{r}} \cdot \frac{\partial f}{\partial \mathbf{v}_f} = 0$$

☞ Ground state: Thomas-Fermi approximation

$$\begin{cases} V_b(\mathbf{r}) + g_{bb} n_b^0(\mathbf{r}) + g_{bf} n_f^0(\mathbf{r}) = \mu_b \\ \\ \dfrac{\hbar^2}{2m_f}\left(6\pi^2 n_f^0(\mathbf{r})\right)^{2/3} + V_f(\mathbf{r}) + g_{bf} n_b^0(\mathbf{r}) = \mu_f \end{cases}$$

☞ Simplifying assumption: *concentric* configuration

$$V_{b,f}(\mathbf{r}) = \frac{1}{2} m_{b,f} \omega_{\perp b,f}^2 \left(\rho^2 + \lambda^2 z^2\right)$$

$g_{bf} = 0$: simple **scaling solution**

☞ for the **condensate:**

$$\begin{cases} n_b(\mathbf{r},t) = \dfrac{1}{\prod_j b_j(t)} n_b^0\left(\dfrac{r_i}{b_i(t)}\right) \\ v_{bi}(\mathbf{r},t) = \dfrac{1}{b_i(t)}\dfrac{db_i(t)}{dt} r_i \end{cases}$$

✗ scaling parameters:

$$\ddot{b}_i(t) + \omega_{bi}^2(t) b_i(t) - \frac{\omega_{bi}^2(0)}{b_i(t)\prod_j b_j(t)} = 0$$

☞ and for the **degenerate Fermi gas:**

$$\begin{cases} f\left(\mathbf{r}, \mathbf{v}_f, t\right) = f_0\left(\dfrac{r_i}{\gamma_i(t)}\mathbf{V}(\mathbf{r}, t)\right) \\ \\ V_i(\mathbf{r}, t) = \gamma_i(t) v_{fi} - \dfrac{d\gamma_i(t)}{dt} r_i \end{cases}$$

✗ scaling parameters:

$$\ddot{\gamma}_i(t) + \omega_{fi}^2(t)\gamma_i(t) - \frac{\omega_{fi}^2(0)}{\gamma_i^3(t)} = 0$$

General case $g_{bf} \neq 0$

☞ **We require the scaling ansatz to be valid on average**

[D. Guéry-Odelin, Phys. Rev. A **66**, 033613 (2002)]: Collective oscillations in a classical gas

[C. Menotti, P. Pedri and S. Stringari, Phys. Rev. Lett. **89**, 250402 (2002)]: Expansion of an interacting Fermi gas

[X.-J. Liu and H. Hu Phys. Rev. A **67**, 023613 (2003)]: Collective oscillations in a Bose-Fermi mixture

☞ **the proper shapes of the density distributions do not enter directly the equations**

☞ only the knowledge of the **initial equilibrium density distribution** is required!

☞ **Bosons:**

$$\ddot{b}_i(t) + \omega_{bi}^2(t) b_i(t) - \frac{\omega_{bi}^2(0)}{b_i(t) \prod_j b_j(t)} - \frac{g_{bf}}{m_b N_b \langle r_i^2 \rangle_b} \frac{1}{b_i \prod_j b_j} \int d^3\mathbf{r} \frac{\partial n_f^0}{\partial r_i} r_i n_b^0$$

$$+ \frac{g_{bf}}{m_b N_b \langle r_i^2 \rangle_b} \frac{1}{b_i \prod_j b_j} \int d^3\mathbf{r} \frac{\partial n_f^0}{\partial r_i} r_i n_b^0 (\frac{\gamma_i}{b_i} r_i) = 0$$

☞ **Fermions:**

$$\ddot{\gamma}_i(t) + \omega_{fi}^2(t) \gamma_i(t) - \frac{\omega_{fi}^2(0)}{\gamma_i^3(t)} - \frac{g_{bf}}{m_f N_f \langle r_i^2 \rangle_f} \frac{1}{\gamma_i^3} \int d^3\mathbf{r} \frac{\partial n_b^0}{\partial r_i} r_i n_f^0$$

$$+ \frac{g_{bf}}{m_f N_f \langle r_i^2 \rangle_f} \frac{1}{\gamma_i \prod_j \gamma_j} \int d^3\mathbf{r} \frac{\partial n_b^0}{\partial r_i} r_i n_f^0 (\frac{b_i}{\gamma_i} r_i) = 0$$

☞ **static effect on the ground state profiles**:

attractive interaction: both the densities are remarkably enhanced within the overlap region.

$\Longrightarrow$ **tighter confinement**: if considered alone would lead to a **faster expansion** for both species.

☞ **dynamical effect during the early stages of the expansion**: with the attractive interaction both species will feel a ***running confinement*** which reduces their expansion rate.

☞ Due to statistics the Fermi distribution is wide even for small N: we **expand n_f^0 around the center** $\longrightarrow$ for the bosons:

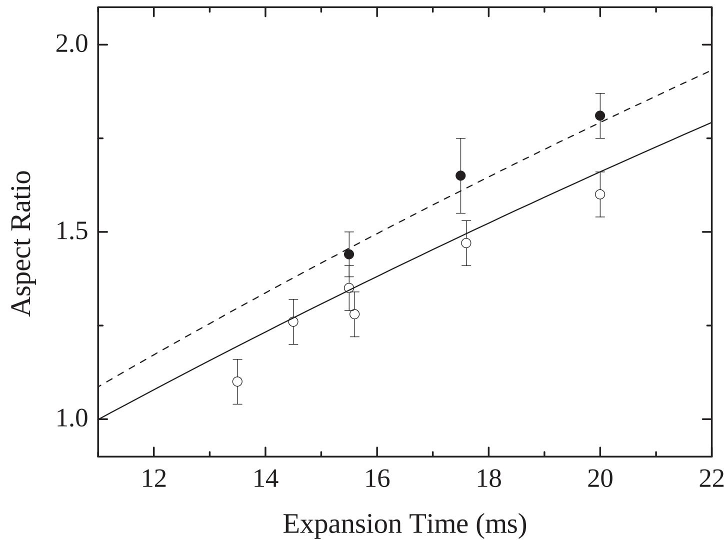
$$\ddot{b}_i(t) - \frac{\omega_{bi}^2(1+A)}{b_i(t)\prod_j b_j(t)} + \frac{A\omega_{bi}^2 b_i(t)}{\gamma_i^2(t)\prod_j \gamma_j(t)} \approx 0$$

$$A \approx \frac{g_{bf}}{m_b N_b \langle r_i^2 \rangle_b \omega_{bi}^2} \int d^3\mathbf{r} \frac{\partial n_f^0(\mathbf{r})}{\partial r_i} r_i n_b^0(\mathbf{r})$$

☞ **Static effect on bosons**:

$$A \approx 40\% \longrightarrow \Delta\omega/\omega \approx 20\%$$

☞ two times larger than that used in the experimental fit (10%)



Boson aspect ratio during the expansion $(N_b = 2 \times 10^4,\ N_f = 10^4)$.

☞ we expect the "dynamical effect" to give important corrections: the Fermi-Bose interaction during the early stages of the expansion is **not negligible!**

☞ Scaling solution for elongated traps at first order in $\lambda \equiv \omega_z/\omega_\perp$

$$b_\perp(\tau) = \sqrt{1 + (1+\delta)^2\tau^2}, \qquad b_z(\tau) \approx 1$$

$$\gamma_\perp(\tau) \approx \sqrt{1 + \beta^2\tau^2}, \qquad \gamma_z(\tau) \approx 1$$

☞ **the *net* increase δ of the boson trapping frequencies is** $(\beta \equiv \omega_f/\omega_b)$

$$\delta = \beta \left[\frac{\left(\beta^4 + 4A(1+A)\right)^{1/2} - \beta^2}{2A} \right]^{1/2} - 1$$

☞ $\boldsymbol{\delta} = 12\%$, in good agreement with the exp. fitting value of 10%

☞ **Expansion of** $^{87}\mathrm{Rb}$: the **static effect** $(+20\%)$ **dominates** over the **dynamical** one (-8%)

☞ **Full solution of the scaling equations:**

✗ **Expansion of** $^{40}\mathrm{K}$: the dynamical effect always dominates and the **aspect ratio is less than that of a pure Fermi gas**.

✗ **Dependence** of the aspect ratio **on the scattering lenght**

✗ **Dependence** of the aspect ratio **on the trap geometry**

Summary

☞ **Mean-field analysis of stability and collapse of a Bose-Fermi system**: **^{40}K-^{87}Rb mixtures @ LENS:**

✗ **estimate of** $a_{bf}^{mf} \simeq -395 \pm 15\ a_0$

☞ **Scaling approach to the expansion of the mixture:**

✗ **increase of the boson aspect ratio**
✗ **reduction on the fermion aspect ratio**

with respect to the noninteracting case.