Superfluidity in Bose-Fermi Mixtures

Omjyoti Dutta

ICFO-BARCELONA
Outline

- Motivation.
- Fermionic superfluidity in Bose-Fermi mixtures.
- Exotic superfluidity in nature.
- Dipolar Bose-Fermi Mixtures.
- Vortex Excitations.
- Future Work.
Motivation

- Non-standard pairing symmetry.
- High enough interaction strength.
- Possible breaking of time-reversal symmetry.
- Excitations obeying non-abelian statistics.
Bose-Fermi Mixture

- Different kind of phenomena:
  
  - Pairing of fermions mediated by bosons.
  
  - Novel pairing between bosons and fermions.
  
  - Different exotic quantum phases.
Fermionic Superfluidity in Bose-Fermi Mixture

- Analogous to phonon-mediated superfluidity in metals.
- Induced interaction depends on momentum.
- Stability of the mixed state: \( \frac{g_{bf}^2 N_0}{g_{bb}} < 1 \)
- Interaction strength for p-wave \( \sim 0.1 \).

\[
T_c \sim T_f \exp(-10) \sim 10^{-5} T_f
\]

Hard to observe experimentally with ultracold atoms.
Unconventional pairing in nature

- Various phases in He³.
- Possible p-wave pairing in Sr₂RuO₄.
- P-wave pairing in ultracold atoms using Feshbach resonances.
- D-wave pairing in Cuprates.
Higher order pairing

- F-wave pairing in a honeycomb optical lattice.

- Superconductivity in Na$_x$CoO$_2$·yH$_2$O.

- Hubbard models with disconnected Fermi surfaces.
Our system

- Dipolar bosons.
- Single component fermions.
- Trapped along the $z$ direction and homogeneous along the $x$-$y$ direction.
Experimental possibility

- Ultracold samples of Cr$^{52}$ with s-wave scattering length tuned to zero-Magnetic dipole moment.

- Quantum degenerate samples of Heteronuclear bosonic molecules-Electric dipole moment.

- Fermionic atoms like, K$^{40}$, Li$^{6}$, Cr$^{53}$ etc.
Dipolar Particles

- Anisotropic interaction.

\[ g_{dd} \frac{\mu_1 \cdot \mu_2 - 3(\mu_1 \cdot \hat{r})(\mu_2 \cdot \hat{r})}{r^3} \]

- Long-ranged: \( g_{dd} \)

- \( g_{dd} \) = Dipolar interaction strength.

- Trapping stabilizes the condensate against collapse.

FERMIX-2009
Dipolar condensate

- **Bosonic density**: \( n_b(x, y, z) = \frac{3n_b}{4R_z} \left( 1 - \frac{z^2}{R_z^2} \right) \).

- **Thomas-Fermi approximation**.

- **Minimize the mean field energy**.

- **Hamiltonian**: \( H_b = \sum_{k_\perp} \Omega (k_\perp) b_{k_\perp}^\dagger b_{k_\perp} \).

- **Excitation Spectrum**: \( \Omega_{k_\perp} \).

- **Bogoliubov operators**: \( b_{k_\perp}, b_{k_\perp}^\dagger \).
Excitation Spectrum

\[ g_{3d} = \frac{8\pi m_b g_{dd} n_b l_0}{5\hbar^2} \]

\( l_0 = \) Oscillator length.

\( n_b = \) Bosonic density

Phonons

Roton instability.
Degenerate fermions

- Single component fermions.

- Fermionic Density: \( n_f(x, y, z) = \frac{n_f}{\pi \ell_f^2} \exp \left[ - \frac{x^2 + y^2}{\ell_f^2} \right] \)

- Width of the Gaussian = Oscillator length.

- Assume a two-dimensional Fermi surface.
Boson-fermion interaction

- Condensate-fermion Hamiltonian:

\[ H_{bf} = \frac{3g_{bf}}{4\sqrt{\pi} R_z} \sum_{k_1, q_1} \gamma(k_1) c_{\bar{k}_1}^\dagger c_{\bar{q}_1 - \bar{k}_1} \left[ b_{\bar{k}_1}^\dagger + b_{-\bar{k}_1}^\dagger \right]. \]

\[ g_{bf} = \text{Boson-fermion contact interaction.} \]

\[ \gamma(k_1) = \sqrt{\frac{2n_b \varepsilon_b(k_1)}{\Omega(k_1)}} \]

Kinetic energy for bosons.
Effective Interaction

- Integrate out the bosonic degrees of freedom.
- Momentum exchanges occur around the Fermi surface.
- Expand the interaction in angular momentum basis:

\[
V_{\text{eff}}(\phi) = \frac{3g_{bf}^2 N_0}{8\pi g_{dd}^1} \sum_{m=-1,0}^{1,\ldots} \lambda_m e^{im\phi}
\]

interaction at channel \(m\)
Effective Interaction

Interaction at channel $m$.

- Dimensionality parameter: $\eta = \frac{\epsilon_F (\text{Fermi energy})}{\hbar\omega_0 (\text{Trapping frequency})}$

- Dimensionless dipolar interaction: $g_{3d} = \frac{8\pi m_b g_{dd} n_b l_0}{5\hbar^2}$

\[
\frac{R_z}{l_0} \approx \left( 2.5g_{3d} \right)^{\frac{1}{3}}
\]

\[
\frac{R_z}{\ell_f} = \frac{R_z}{l_0} \times \frac{m_f}{m_b}
\]
Effective interaction

\[ \lambda_m = \alpha^2 \int_0^{2\pi} \frac{\exp[im\phi]d\phi/2\pi}{\eta R^2 \frac{g_{3d}}{\ell_f^2} (1 - \cos \phi) + \frac{R^2}{\ell_f^2} \mathcal{V}(\sqrt{\frac{R^2}{\ell_f^2} \eta (1 - \cos \phi)})} \]

\[ \mathcal{V}(\tilde{k}_\perp) = \frac{1}{\tilde{k}_\perp^5} \left[ 4\tilde{k}_\perp^3 - 6\tilde{k}_\perp^2 - 6(1 + \tilde{k}_\perp^2) \exp(-2\tilde{k}_\perp) + 6 \right] - \frac{8}{15} \]

Non-trivial angular dependence!!
P-wave interaction

\[ \eta = \frac{\varepsilon_F (\text{Fermi energy})}{\hbar \omega_0 (\text{Trapping frequency})} \]

\[ g_{3d} = \frac{8\pi m_b g_{dd} n_b l_0}{5\hbar^2} \]

Reactive interaction
p-, f-, h- wave interactions

\[ g_{3d} = 3.1 \]

\[ \eta = \frac{\varepsilon_F}{\hbar \omega_0} \]

Transition from p-wave to h-wave!!
Chirality

P-wave: \( k_x + i k_y \)

f-wave: \( (k_x + i k_y)^3 \)

h-wave: \( (k_x + i k_y)^5 \)

Excitations:
\[
\sqrt{\left(\frac{k^2}{2m_f} - \mu\right)^2 + \Delta^2} k^{2m}, \quad m = 1, 3, 5
\]
Vortex Excitations

Order Parameter: \( \Delta = \Delta_0(\vec{r}) \left( \frac{k}{k_f} \right)^m e^{im\theta} \)

Vortex structure:
\[
\begin{align*}
\Delta_0(\vec{r}) &= 0 & r \leq \xi \\
\Delta_0(\vec{r}) &= \Delta_0 e^{i\phi} & r > \xi
\end{align*}
\]

Single vorticity
Quasi-particle excitations inside a vortex

For \( r \gg \xi \)

\[
H_0 u_m + (-i)^m \frac{\Delta_0}{k_f} e^{i\phi/2} \left[ e^{-i\phi} \left( \frac{\partial}{\partial r} - \frac{i}{r} \frac{\partial}{\partial \phi} \right) \right]^m e^{i\phi/2} v_m = E u_m
\]

\[
-H_0 v_m + (i)^m \frac{\Delta_0}{k_f} e^{-i\phi/2} \left[ e^{i\phi} \left( \frac{\partial}{\partial r} + \frac{i}{r} \frac{\partial}{\partial \phi} \right) \right]^m e^{-i\phi/2} u_m = E v_m,
\]

\[
\begin{pmatrix}
u_m \\
u_m
\end{pmatrix} = \text{Excitation amplitudes for } m = 1, 3, 5.
\]

Zero-energy solution: \( u_m^* = v_m \)

Energy: \( E_n = n\omega_0, \quad \omega_0 \sim \frac{\Delta_0^2}{\epsilon_F}, \quad n = \text{integers.} \)
Zero-energy modes

For \( r \gg \xi \)

Quasi-particle operator: \( \gamma_m = \int d^2r \left[ u_m c^\dagger(r) + v_m c(r) \right] \)

Majorana Fermions: \( \gamma_m = \gamma_m^\dagger \)

Ground state degeneracy
Topological properties

- Two well-separated vortices $i$ and $j$.
- Each vortex carries a Majorana zero-energy mode.
- Exchange of two such vortices: $|0\rangle \rightarrow \gamma^i_m \gamma^j_m |0\rangle$.
- Non-abelian nature: $\gamma^i_m \gamma^j_m \neq \gamma^j_m \gamma^i_m$.
- $2n$ such vortices have degeneracy $2^n$.

(Ivanov-2001)
Conclusion

- Superfluidity in Bose-Fermi mixture.
- Effect of roton minimum on the pairing symmetry.
- Feshbach resonance like character.
- Possibility of non-standard f-wave and h-wave superfluidity.
- Non-Abelian statistics.
Future Study

- Identification of different pairing symmetries.
- Ways to create and manipulate vortices.
- Identification of the statistics of the excitations.
- Looking into the different pairing possibilities within each angular momentum channel.
- Stability of Fermi surface.
Acknowledgement

- Prof. Maciej Lewenstein.

Funding Agencies: **FERMIX, QOIT, TOQATA.**