COLLECTIVE OSCILLATIONS IN TRAPPED SUPERFLUID FERMI GASES

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HARMONIC TRAPS:
Test of hydrodynamics and of equation of state

ROTATING TRAPS:
Superfluid vs collisionless hydrodynamics

1D OPTICAL LATTICES:
Coherent tunneling vs umklapp processes
HARMONIC TRAPS:
Test of hydrodynamics and of equation of state

T=0 superfluidity $\rightarrow$ irrotational hydrodynamics

**Surface** modes: unaffected by equation of state

**Compression** ($m=0$) modes sensitive to equation of state $\mu(n)$ of uniform matter

$\mu =$ chemical potential, $n =$ density

Explore behaviour of collective frequencies in the BCS-BEC crossover, near Feshbach resonance

(S. Stringari, cond-mat/0312614, Europhys. Lett. 65, 749 (2004))
Molecular BEC regime

\[ a > 0, \ a << d \]
\[ a = \text{scattering length}, \ d = \text{interparticle distance} \]

\[ T_c = 0.83 \hbar \omega_{ho} N_m^{1/3} \] critical temperature
\[ \omega_{ho} \] geometrical trapping freq.
\[ N_m = N / 2 \] number of molecules

Eq. of state of molecular BEC:

\[ \mu(n_m) = g_m n_m \]

\[ g_m = \frac{4 \pi \hbar^2 a_m}{2m} \]
\[ a_m = 0.6a \]

molecular scattering length

(Petrov et al. 2003
see also Pieri, Strinati 2000)
Molecular BEC regime

Collective frequencies in harmonic traps do not depend on coupling constant (Thomas-Fermi regime $R >> a_{ho}$)

Example: cigar-like traps ($\omega_z << \omega$)

Radial compression mode

$$\omega = 2\omega_\perp$$

Axial compression mode

$$\omega = \sqrt{\frac{5}{2}}\omega_z$$

(well checked in BEC gases)
Radial compression mode

\[ \frac{\omega}{\omega_{\perp}} \]

\[ (k_F a)^{-1} \]

Molecular BEC
BEC + beyond mean field effects

If diluteness condition $n_m a_m^3 << 1$ is not satisfied Bolgoliubov equation of state is changed:

$$\mu(n_m) = g_m n_m \left( 1 + \frac{32}{3 \sqrt{\pi}} \sqrt{n_m a_m^3} \right)$$

Huang, Yang, Lee, 1957

In harmonic traps molecular gas parameter is related to atomic parameters ($a_{ho} = \sqrt{\hbar / m \omega_{ho}}$) by

$$\sqrt{n_m a_m^3} = 0.13 (k_F a)^{6/5} \text{ with } k_F a = 1.7 N^{1/6} \frac{a}{a_{ho}}$$
BEC + beyond mean field effects in harmonic trap

Radial compression frequency becomes

(Pitaevskii, Stringari 1998; Braaten, Pearson 1999)

$$\omega = 2\omega_\perp \left(1 + \frac{105\sqrt{\pi}}{256} \sqrt{n_m a_m^3}\right)$$

In terms of Fermi momentum:

$$\omega = 2\omega_\perp \left(1 + 0.09(k_F a)^{6/5}\right)$$
Radial compression mode

\[ \omega / \omega_\perp \]

Beyond mean field

Molecular BEC

\[ (k_F a)^{-1} \]
Unitarity limit in harmonic trap

When $k_F|\alpha| >> 1$ the system exhibits universal behaviour, independent of sign of scattering length, (Heiselberg, 2001):

$$\mu(n) = (1 + \beta) \frac{\hbar^2}{2m} (6\pi^2)^{2/3} n^{2/3}$$

(holds at $T=0$ and for short range forces)

$\beta = -0.56$ (Carlson et al., 2002)

Radial mode: $\omega = \sqrt{\frac{10}{3}} \omega_\perp$

Axial mode: $\omega = \sqrt{\frac{12}{5}} \omega_z$
Radial compression mode

\[ \frac{\omega}{\omega_\perp} \]

- beyond mean field
- molecular BEC
- unitary limit

\[ (k_F a)^{-1} \]
BCS regime (dilute Fermi gas, $a<0$)

$$\mu(n) = \frac{\hbar^2}{2m} (6\pi^2)^{2/3} n^{2/3}$$

(neglected interaction corrections to eq. of state)
regime difficult to reach (very low critical Temp)

Same predictions for collective frequencies as in unitarity limit

**BCS + mean field** (radial mode):

$$\omega = \sqrt{\frac{10}{3}} \omega_\perp (1 + 0.04 k_F a)$$
Radial compression mode

(S. Stringari, cond-mat/0312614, Europhys. Lett. 65, 749 (2004))

[Diagram showing a graph with axes labeled \( (k_F a)^{-1} \) and \( \omega / \omega_\perp \), with regions marked as BCS+mean field, molecular BEC, and beyond mean field, and the unitary limit.]
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ROTATIONAL EFFECTS

Probe transverse response
Crucial test of superfluidity

Irrotationality

- Quenching of moment of inertia at low angular velocity (scissors mode)
- Quantization of circulation (quantized vortices)
Scissors mode
Scissors at Oxford
Marago’et al, PRL 84, 2056 (2000)

above $T_c$
rigid value of
moment of inertia

below $T_c$
irrot. value of
moment of inertia
IS THE OBSERVATION OF THE SCISSORS MODE AT FREQUENCY 
\[ \omega^2 = \omega_x^2 + \omega_y^2 \]

DIRECT PROOF OF SUPERFLUIDITY?

- **YES** IF NORMAL GAS IS COLLISIONLESS (OXFORD EXP)
- **NO** IF NORMAL GAS IS COLLISIONAL AND NON ROTATING
- **YES** IF NORMAL GAS IS COLLISIONAL AND ROTATING

INVESTIGATE COLLECTIVE MODES IN THE PRESENCE OF ROTATING TRAPS!!
Rotating Anisotropic Trap

\[ V_{\text{ext}} = \frac{M}{2} \left( \omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2 \right) = \frac{M}{2} \left[ \omega_\parallel^2 (1 + \varepsilon) x^2 + \omega_\parallel^2 (1 - \varepsilon) y^2 + \omega_z^2 z^2 \right] \]

Rigid rotation \( \mathbf{v}_0 = \Omega \wedge \mathbf{r} \): Irrotational flow \( \mathbf{v}_0 = \alpha \nabla (xy) \):

normal  superfluid
TESTS OF SUPERFLUIDITY
(able to distinguish between superfluid and collisional hydrodynamics)

(Cozzini, Stringari, PRL 91, 070401 (2003))

• SCISSORS MODE IN ROTATING GAS

• PRECESSION OF QUADRUPOLE OSCILLATION IN ROTATING GAS
scissors mode (stopping rotation of the trap)

Superfluid (T=0)

Normal (collisional) \((\Omega \gg \varepsilon^2 \omega_\perp)\)

- \(\Omega = 0.2 \omega_\perp\)
- \(\varepsilon = 0.2\)

Normal (collisionless)
Precession of quadrupole oscillation after switching off deformation of rotating trap

\[ \dot{\theta} = \frac{l_z}{2m\omega_z^2 \langle r^2 \rangle} = \frac{\Omega}{2} \]
QUANTIZED VORTEICES

- **Quantum of circulation** in a Fermi superfluid is $\hbar/2$
- **Density change** on the vortex line **less pronounced** than in a BEC (see Bulgac, 2003 in unitarity limit)
- **Splitting of quadrupole frequencies** useful test of vortices

$$\omega_+ - \omega_- = \frac{2}{M} \frac{\langle l_z \rangle}{\langle r_{\perp}^2 \rangle}$$

Zambelli, Stringari 1998
Bruun, Viverit 2001
Measurement of angular momentum in BEC gas (Chevy et al., PRL 85, 2223 (2000))

**FIG. 2.** Variation of the angular momentum deduced from (1) as a function of the stirring frequency $\Omega$ for $\omega_{\perp}/2\pi = 175$ Hz and $2.5 (\pm 0.6) \times 10^5$ atoms.
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Center of mass oscillation in harmonic trap

Frequency: \[ \omega = \omega_z \]

Independent of interactions, temperature, quantum statistics (Kohn’s theorem)
What happens in the presence of 1D lattice?

Different regimes

Different answers
COLLISIONLESS NON SUPERFLUID
(single spin species)

L. Pezze' et al. cond-mat/0401643
(Florence-Trento collaboration see poster 2.13)

- Due to non quadratic dispersion law particles with different quasi-momentum have different effective mass
- Dephasing of single particle orbits
- Damping of oscillation
COLLISIONLESS NON SUPERFLUID
(single spin species)

- If Fermi energy is larger than band width

\[ T_F > 2\delta \]

\[ \varepsilon(p_z) = 2\delta \sin^2 \left( \frac{p_z d}{2\hbar} \right) \]

- Single particle orbits at the Fermi surface are open (the gas does not oscillate around minimum of potential)

- **off-set of the oscillation** (localization)
open orbits

$E_F$

$2\delta$
Damping and offset of center of mass oscillation theory vs experiment (Lens)

![Graph showing damping and offset comparison between theory and experiment with and without lattice.](image-url)
COLLISIONAL NON SUPERFLUID  
(two spin species, HD regime)  
(G. Orso et al. cond-mat/0401643 poster n. 2.11)  

Equations for center of mass oscillation in hydrodynamic regime (local equilibrium ansatz)

\[ f(r, p, t) = f_0((\varepsilon(r, p) + u(t)p_z)) \]

Boltzmann eqs. yield:

\[ \frac{\partial}{\partial t}Z = \frac{P_z}{m^*} \]
\[ \frac{\partial}{\partial t}P_z + m\omega_z^2Z = -\frac{P_z}{\tau_{uk}} \]
Center of mass relaxation time

\[
\frac{1}{\tau_{uk}} = -\frac{1}{kT} \int p_{1z} (p_{1z} + p_{2z} - p_{3z} - p_{4z}) D dp_1 dp_2 dp_3 dp_4 dr \\
\int p_z^2 \frac{\partial f_0}{\partial \epsilon(p_z)} dp dr
\]

Numerator vanishes if momentum is conserved
Relaxation time determined by umklapp collisions

\[
p_1 + p_2 = p_3 + p_4 + 2\hbar q_B
\]

\[
q_B = \frac{\pi}{d}
\]

Bragg momentum
Ratio between umklapp and normal collisional rate depends on band filling factor $T_F / 2\delta$

If $T_F << 2\delta$ umklapp collisions are rare and center of mass oscillates undamped

If $T_F >> 2\delta$ umklapp and normal rates are comparable and center of mass oscillation is overdamped in HD regime
Conclusion: if $T_F > 2\delta$

a normal Fermi gas in 1D optical lattice cannot exhibit center of mass oscillation:

In the collisionless regime due to localization (LENS experiment)

In the collisional regime overdamping due to umklapp collisions
In the Fermi superfluid phase one expects undamped Josephson-like oscillation of center of mass (Wouters et al. Cond-mat/0312154, see poster n. 2.27) similar to the case of bosons (LENS, 2001)

In BCS regime one predicts

$$\omega = \sqrt{\frac{m}{m^*}} \omega_z$$

with

$$m^* = -\frac{2}{Nh^3} \int p_z^2 \frac{\partial f_0}{\partial \varepsilon(p_z)} dp dr$$

$$\tilde{m}/m = \frac{5T_F}{3E_R}$$

$$s = 8$$

$$s = 5$$

$T_F/E_R$
MAIN CONCLUSIONS

- Compression modes sensitive to equation of state (non trivial predictions in unitarity limit)
- Oscillations with rotating and periodic traps permit to distinguish between superfluid and normal regimes.

OPEN PROBLEMS

- Equation of state along the BCS-BEC crossover
- Collective oscillations at finite $T$. 